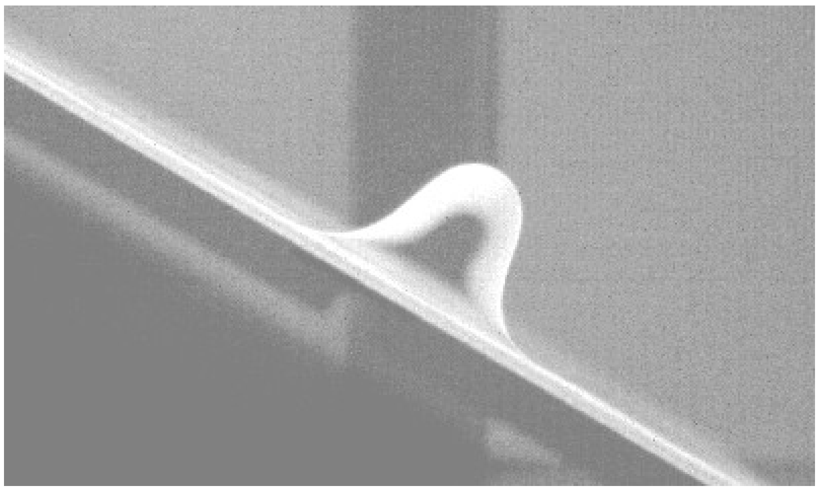
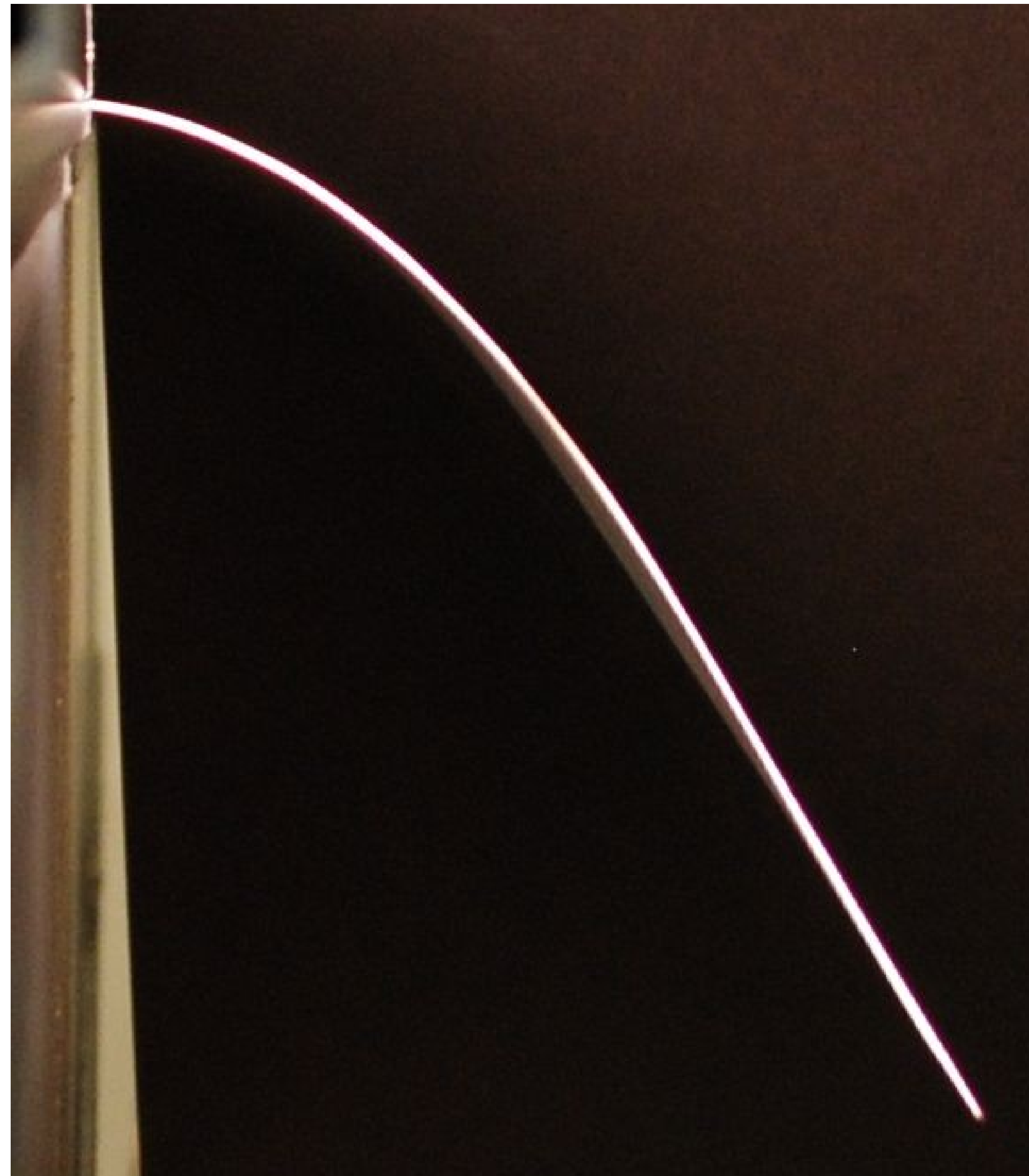
Elastic Sheets: A Real Cliffhanger!





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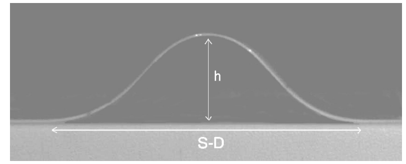
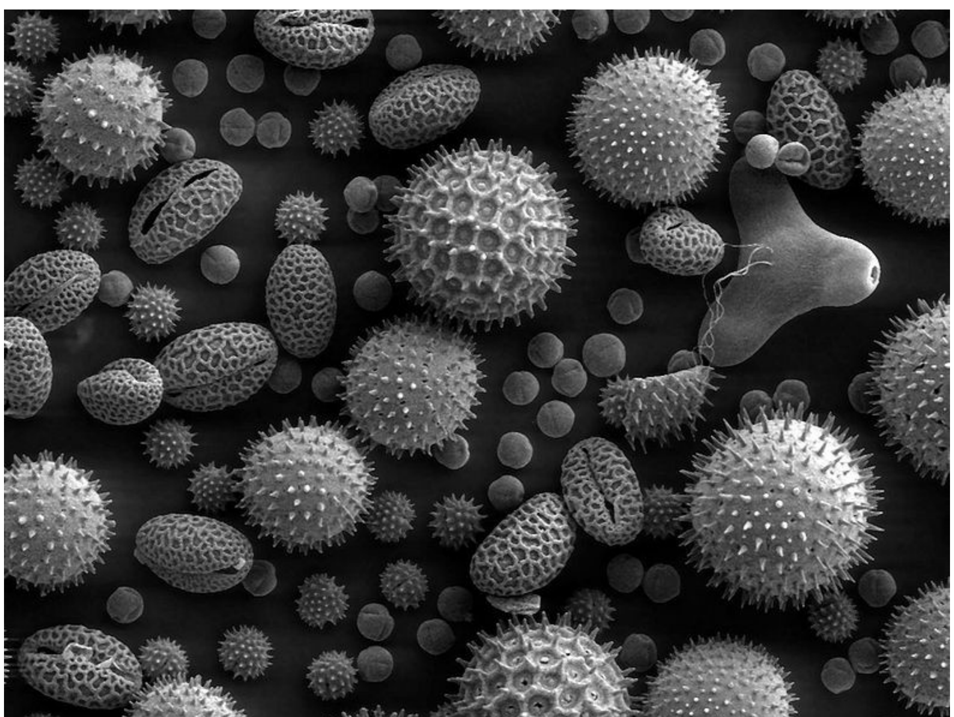
Non-linearity in Elastic Sheets

Wrinkled sheets: from Flickr (milkshakepants)

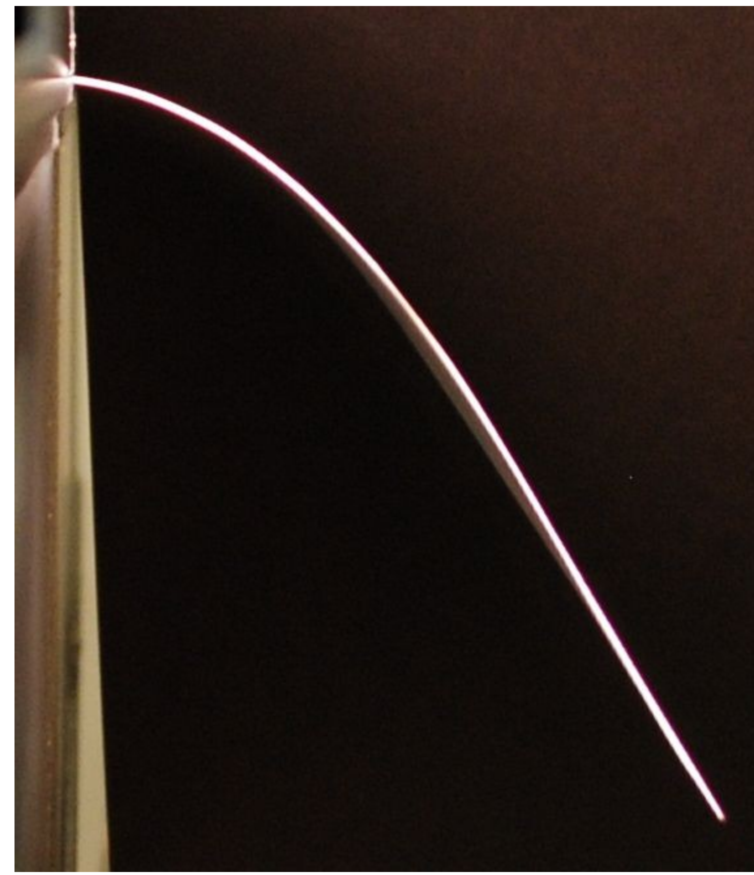


Elastic
Ripples

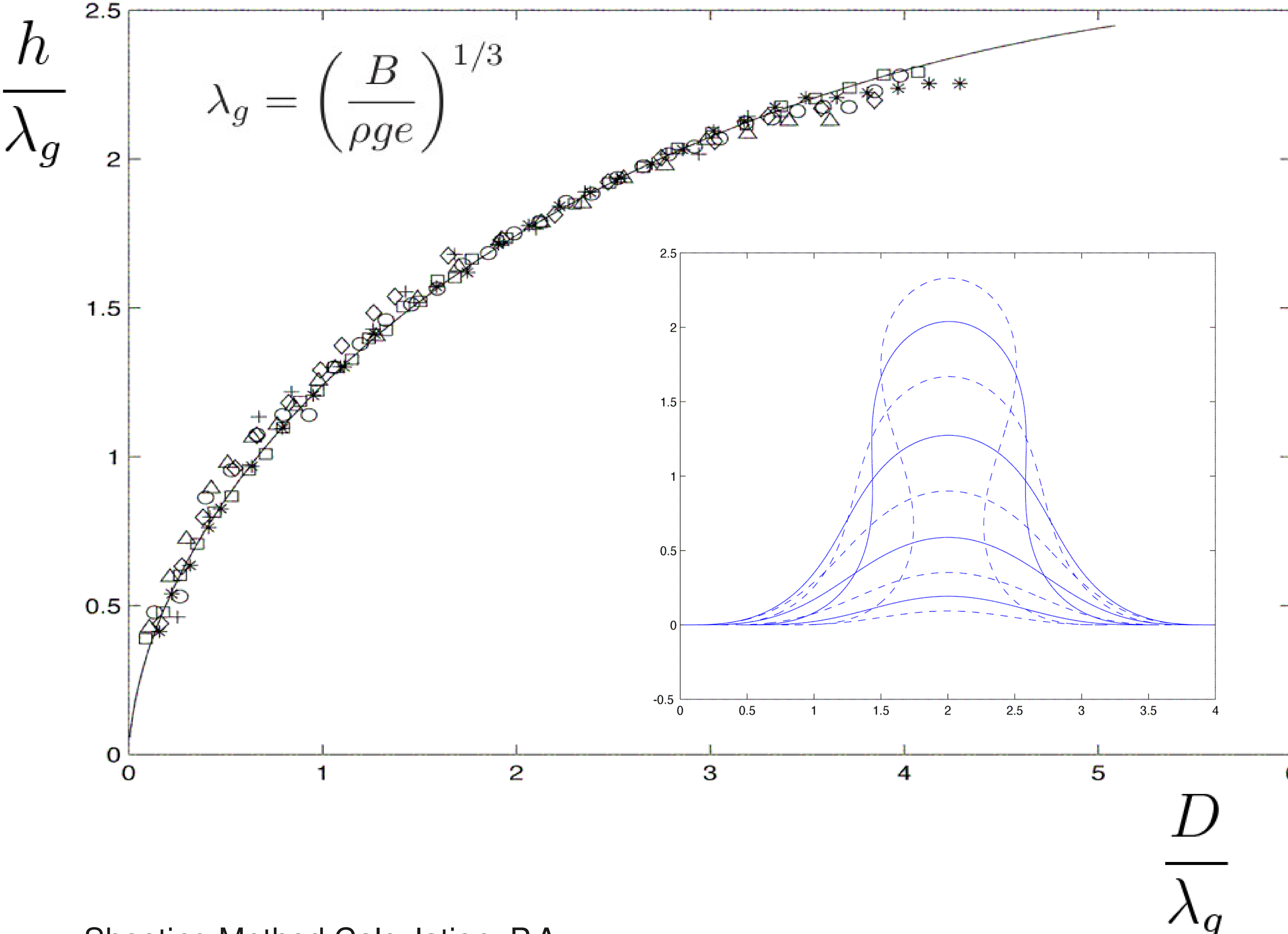
Leaves



Crinkled Pollen
Grains



Cliff-hanging sheets

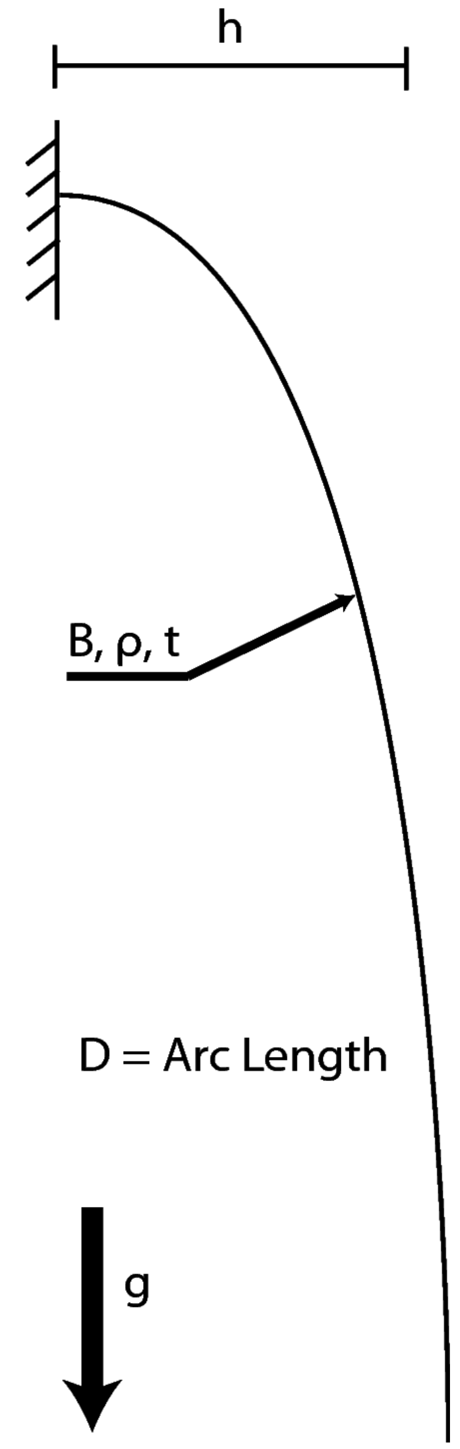


Shooting Method Calculation: P.A.

Free Body Diagram

-Cantilevered sheet

-Large, non-linear deformation



Finite-Element Problem Set-up

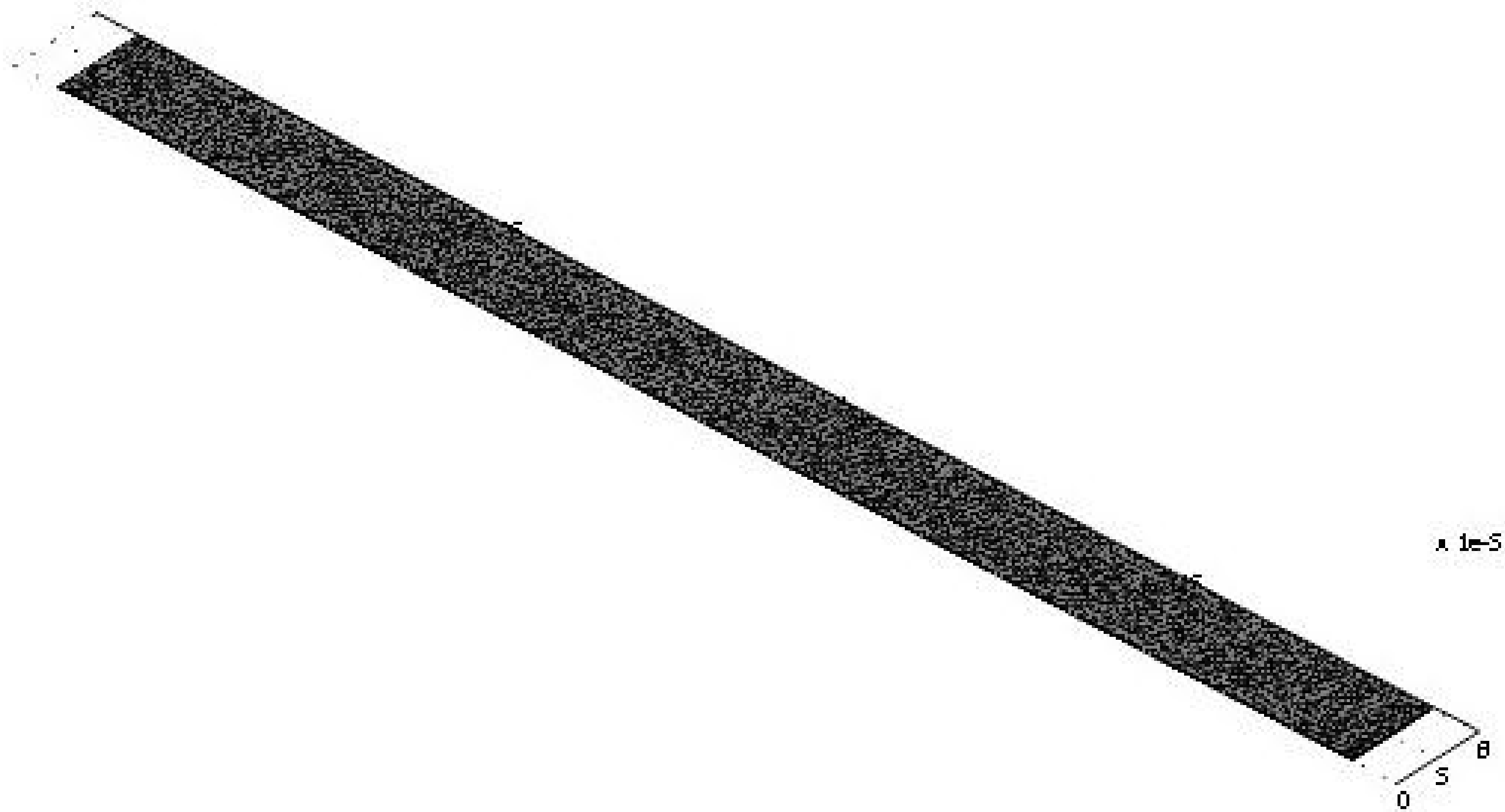
-Non-linear elements

-Thin sheet (0.19 m x 0.01 m x 0.0001 m)

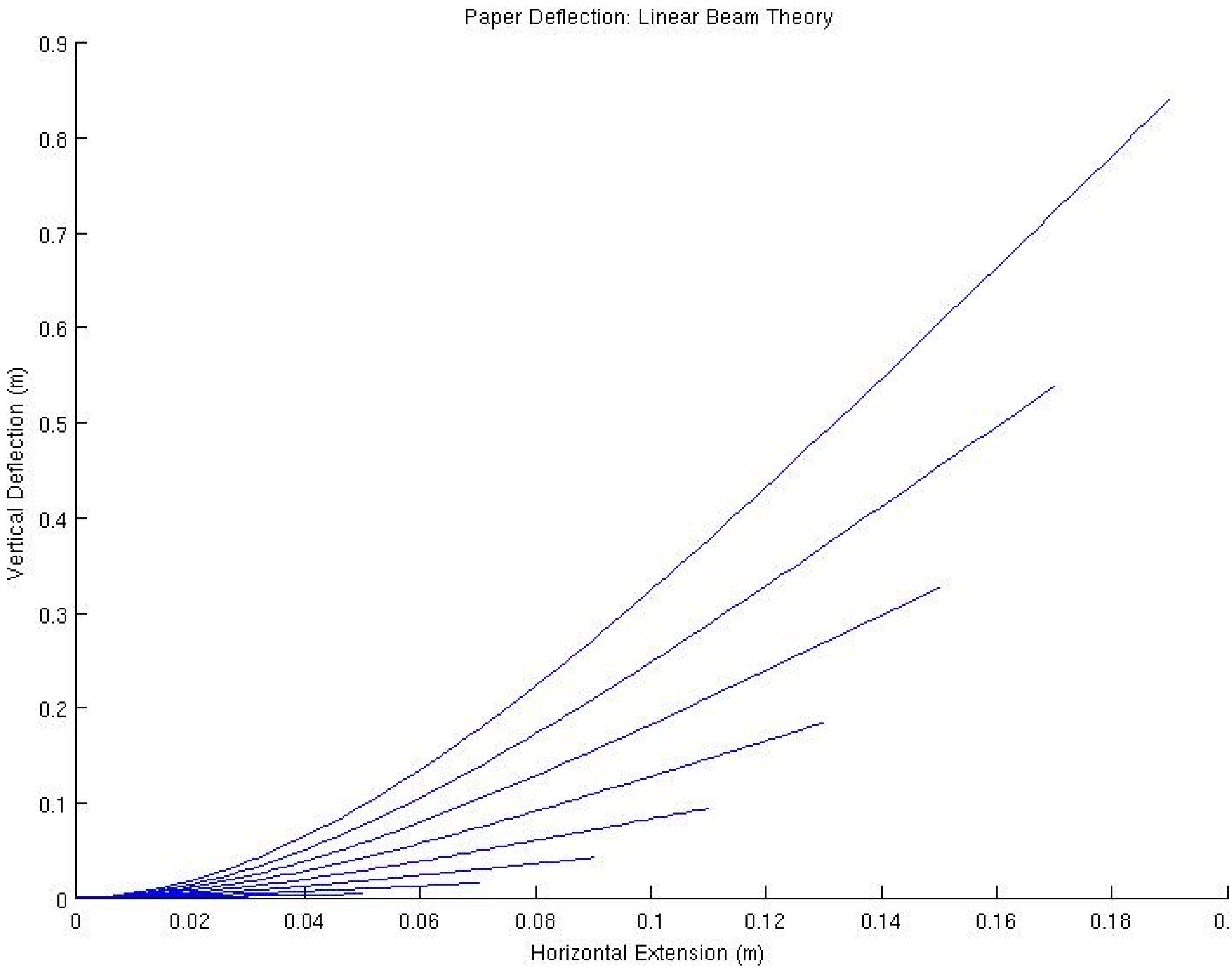
-Density = 880 kg/m^3

-E = 2 Gpa

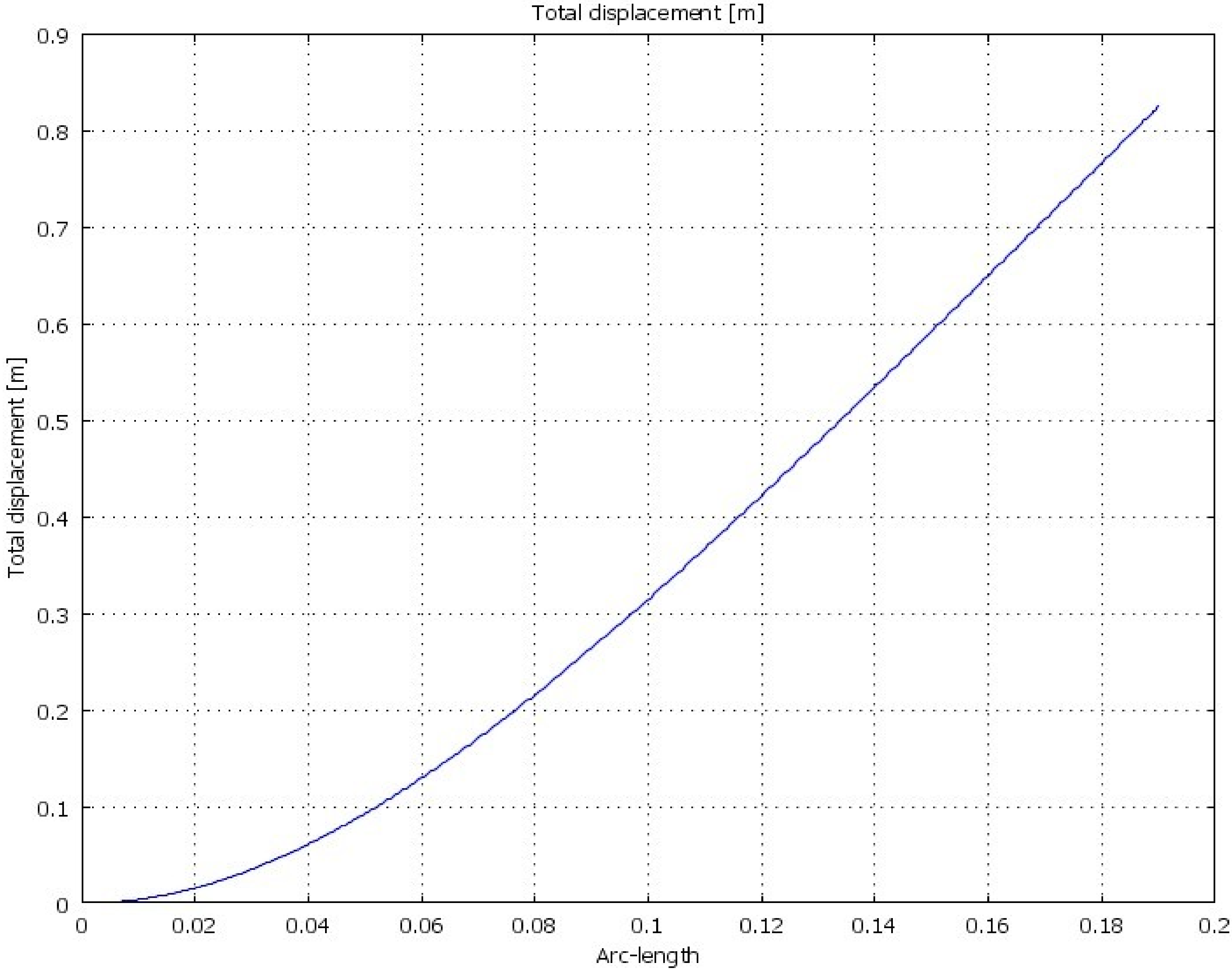
-g = 9.81 m/s^2



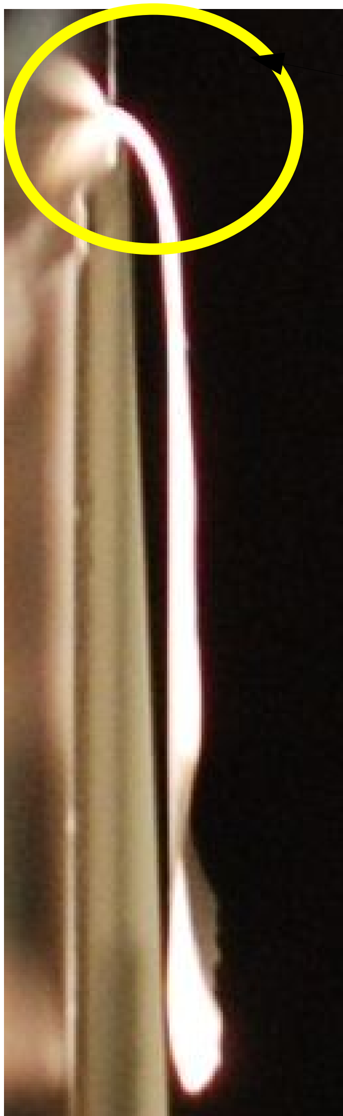
Cliffhanging Sheets: Linear Beam Theory vs. COMSOL FEM Calculation



COMSOL Non-Linear Calculation

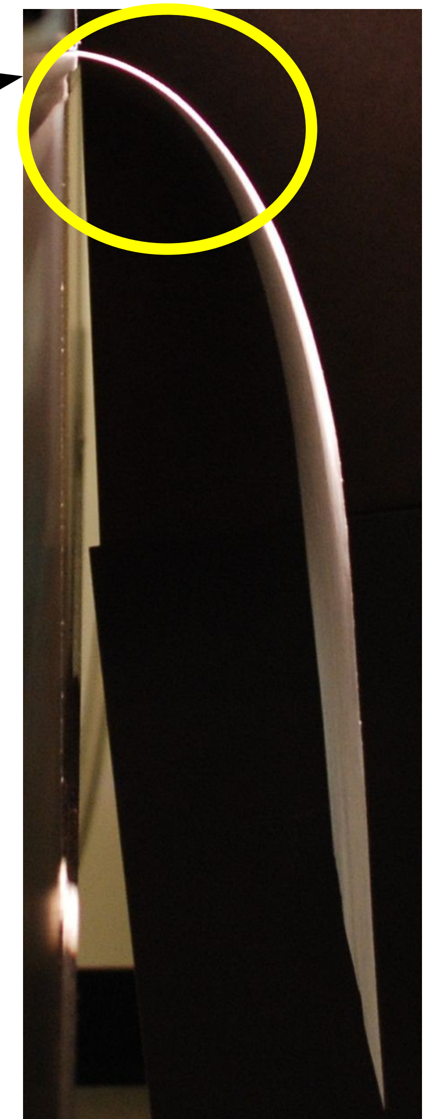


The Ultimate FEM Solver: Experimentation!
~10^23 elements
~Infinite order interpolation, etc.



Curvature Condensation: A Boundary Layer

Sheet Drapes in this Region



Latex Sheet
D ~ 11 cm

Paper Sheet
D ~ 25 cm

A Boundary-Layer Theory

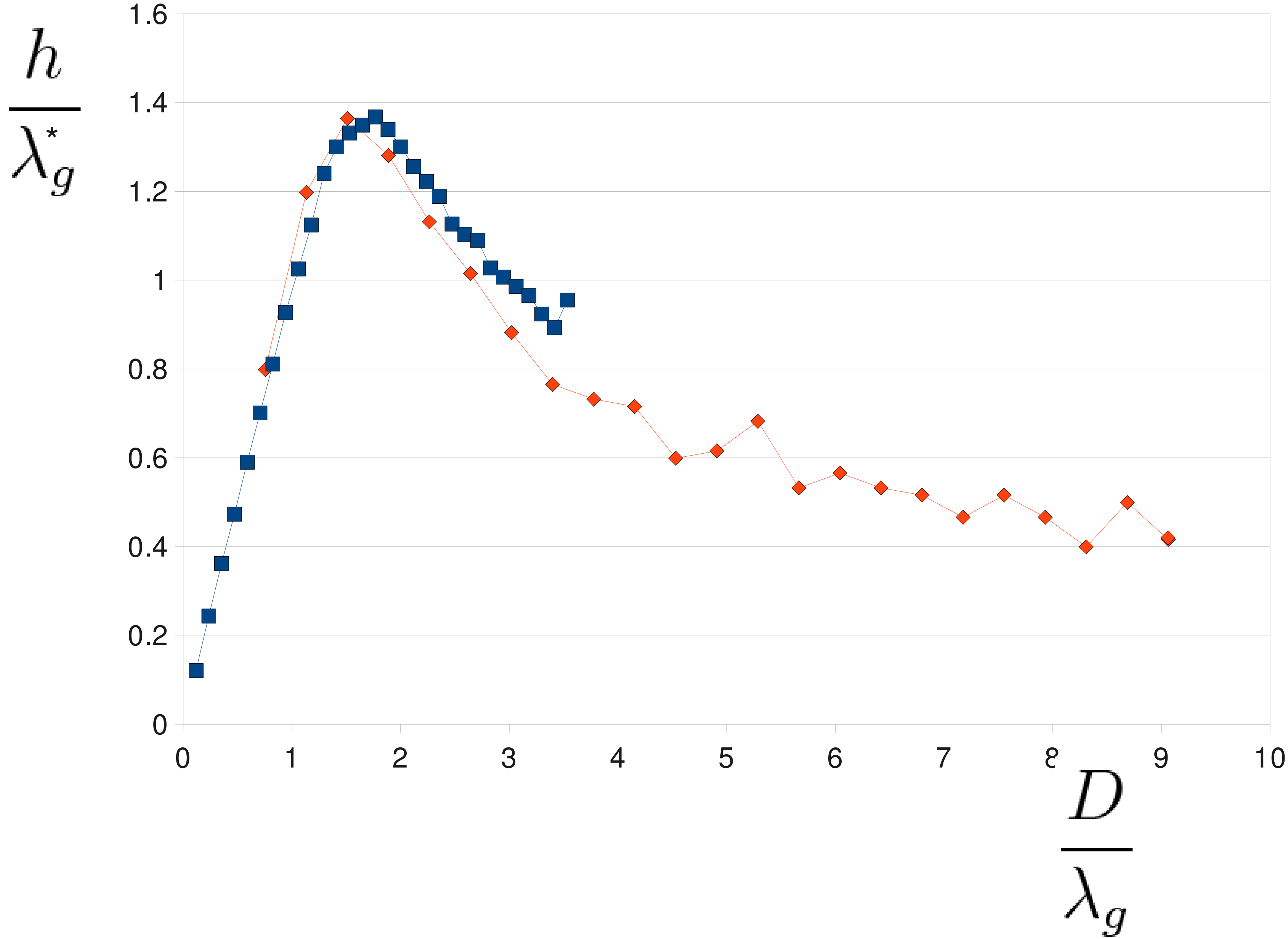
Elastic Gravity Length $\lambda_g = \left(\frac{B}{\rho g e} \right)^{1/3}$

Boundary Layer for
Curvature Condensation*: $\epsilon = \lambda_g^{3/2} / L^{1/2}$

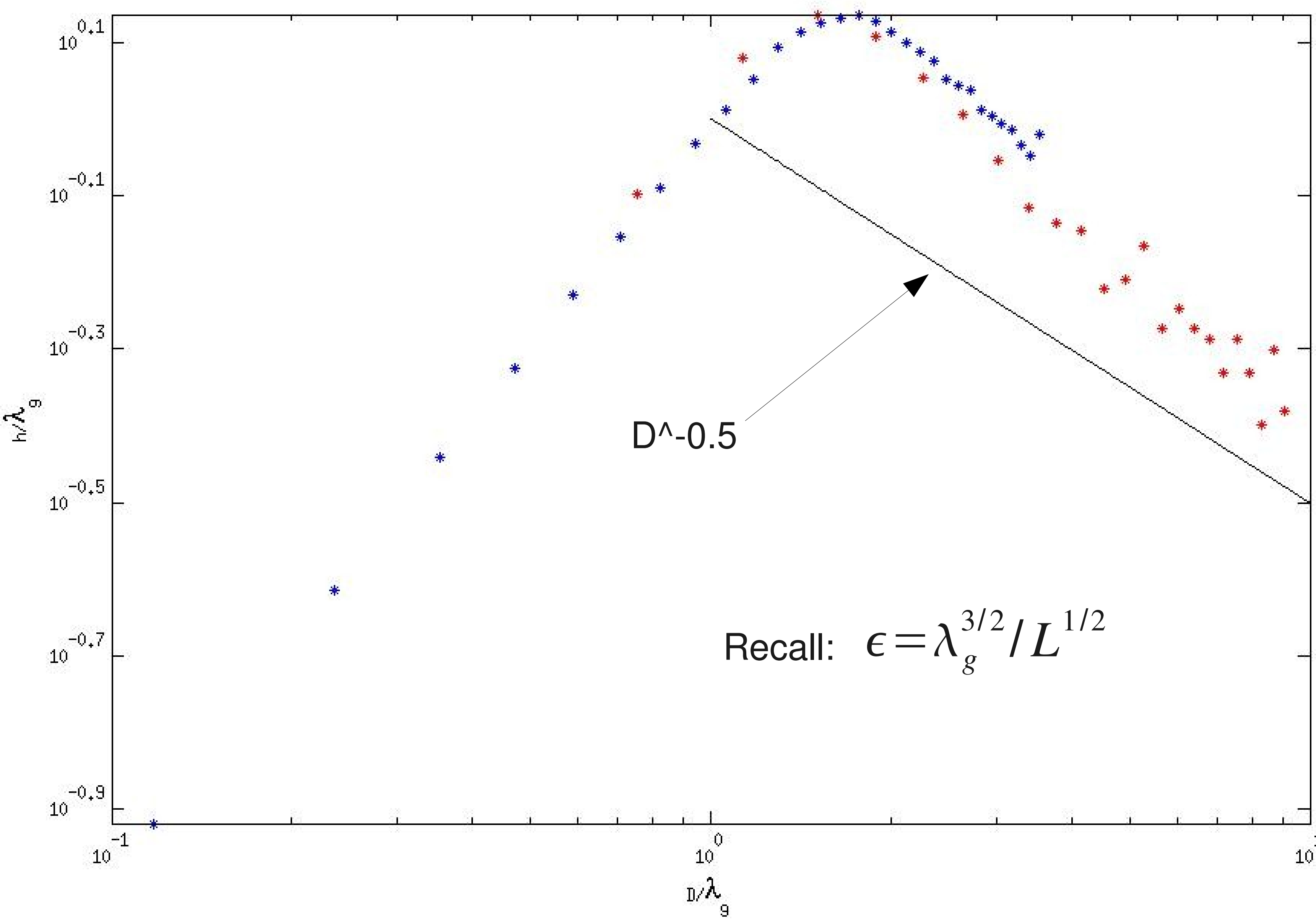
In Principal, this could be used for our calculation: Additional draped weight is included as a point mass on the end of a cantilevered beam of length epsilon in the large-L limit

*L.M.

Experimental Results

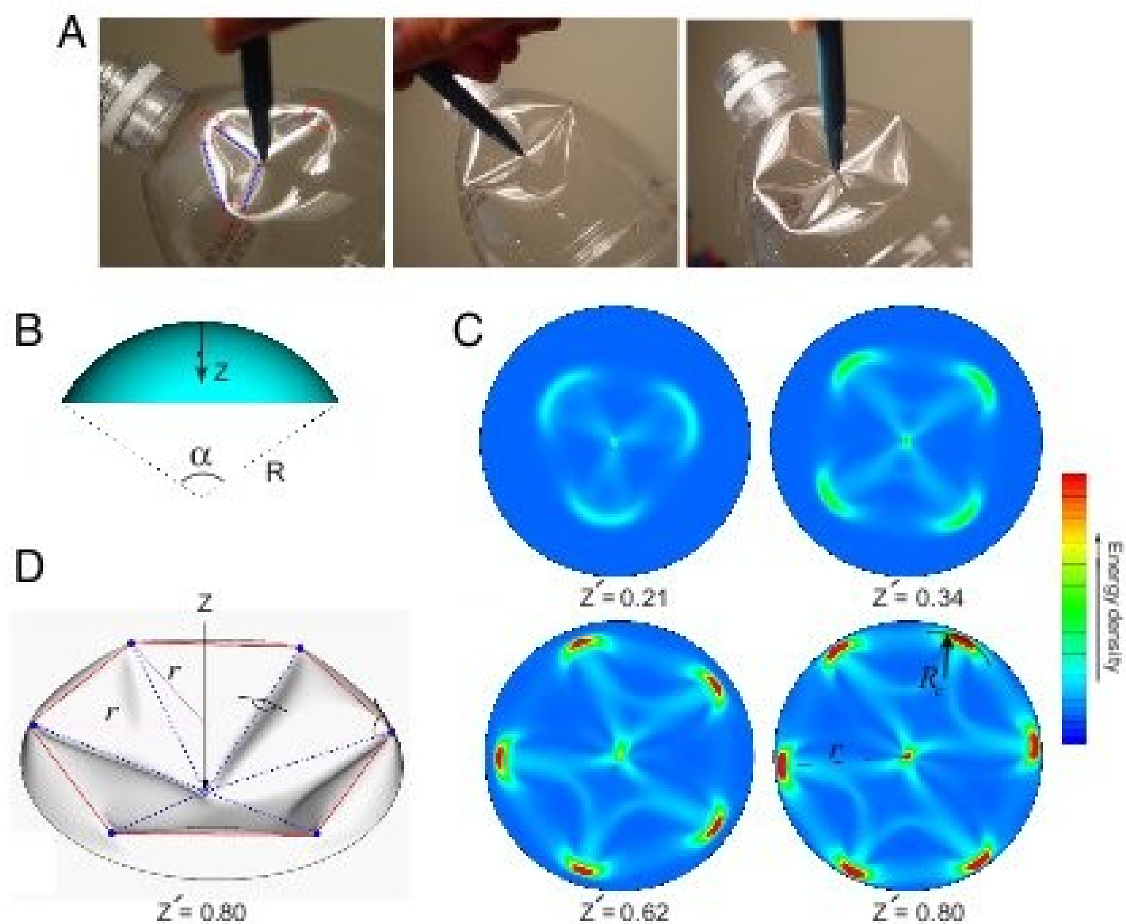


Ff: 2.66



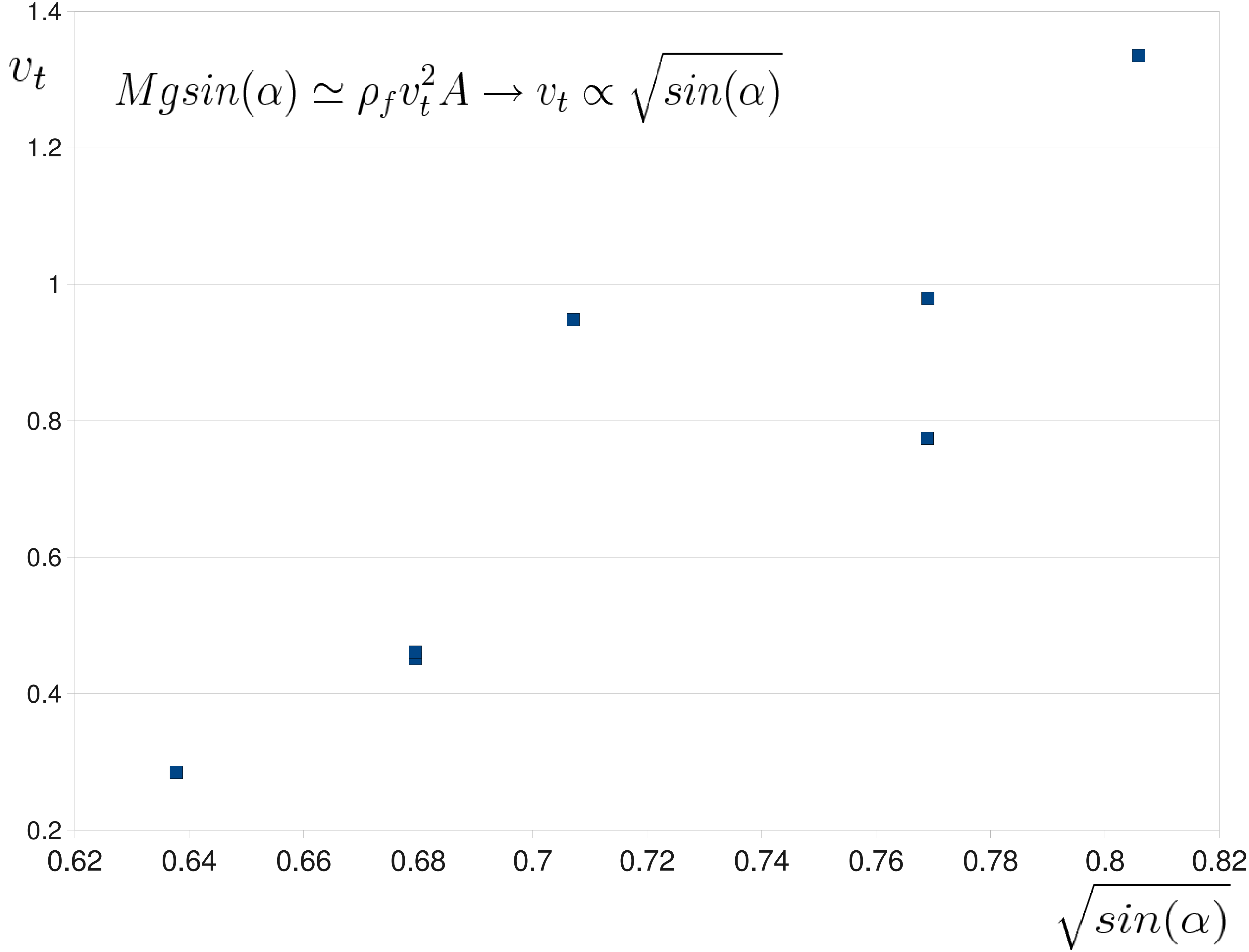
$10^{0.1}$
$10^{-0.1}$
$10^{-0.3}$
$10^{-0.5}$
$10^{-0.7}$
$10^{-0.9}$
10^{-1}
10^{0}
10^{1}
h/λ_g
D/λ_g
D^-0.5
Recall: $\epsilon = \lambda_g^{3/2} / L^{1/2}$

More on Curvature Condensation:



See e.g. Localized and Extended Deformations of Elastic Shells. Vaziri, A. Mahadevan, L.

Thanks to Ben Jordan for help with COMSOL


v_t
$Mgsin(\alpha) \simeq \rho_f v_t^2 A \rightarrow v_t \propto \sqrt{sin(\alpha)}$
1.4
1.2
1
0.8
0.6
0.4
0.2
0.62
0.64
0.66
0.68
0.7
0.72
0.74
0.76
0.78
0.8
0.82
$\sqrt{sin(\alpha)}$