Anton Ishmurzin, 6th of April, 2010

1 Derivative $\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}}$

Here I am going to describe the possible problem with the derivation of the aforementioned derivative $\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}}$ in the paper by Masatsugu Yaguchi and Yukio Takahashi "Ratchetting of viscoplastic material with cyclic softening, part 2: application of constitutive models", International Journal of Plasticity 21 (2005) 835-860, when taking into account the tension–compression asymmetry. It is also possible though that it is just my misunderstanding of the derivation. If it is the later, could you please give some details on where a flaw might be in my derivations?
The term that takes into account the tension=-compression asymmetry is

$$J_{\mathrm{m}}(\underline{\sigma}) = \eta |\mathrm{tr}\ \underline{\sigma}|^{l} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}), \tag{1}$$

where η and l are constants.
Then later in the paper its derivative with respect to $\underline{\sigma}$ is taken:

$$\underline{\dot{\epsilon}}_{\mathrm{in[m]}} = \dot{p} \frac{l J_{\mathrm{m}}(\underline{\sigma})}{|\mathrm{tr}\ \underline{\sigma}|} \underline{I} = \dot{p} \frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}} = \dot{p} l \eta |\mathrm{tr}\ \underline{\sigma}|^{l-1} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \underline{I}. \tag{2}$$

So according to the paper

$$\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}} = l \eta |\mathrm{tr}\ \underline{\sigma}|^{l-1} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \underline{I}. \tag{3}$$

When I tried to repeat the derivation, I came up with

$$\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}} = \frac{\partial}{\partial \underline{\sigma}} \left[\eta |\mathrm{tr}\ \underline{\sigma}|^{l} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \right] = \eta \left[\frac{\partial}{\partial \underline{\sigma}} \left(|\mathrm{tr}\ \underline{\sigma}|^{l} \right) \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) + |\mathrm{tr}\ \underline{\sigma}|^{l} \frac{\partial}{\partial \underline{\sigma}} \left(\mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \right) \right]. \tag{4}$$

Where

$$\frac{\partial}{\partial \underline{\sigma}} \left(|\mathrm{tr}\ \underline{\sigma}|^{l} \right) = l |\mathrm{tr}\ \underline{\sigma}|^{l-1} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \underline{I} \tag{5}$$

and

$$\frac{\partial}{\partial \underline{\sigma}} \left(\mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \right) = 2\delta(\mathrm{tr}\ \underline{\sigma})) \underline{I}, \tag{6}$$

where δ is the Dirac delta function. Alternatively, we can say that $\frac{\partial}{\partial \underline{\sigma}} \left(\mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \right)$ is $\underline{0}$ (tensor with all components equal to zero) everywhere where $(\mathrm{tr}\ \underline{\sigma})$ is non-zero.
Combining (4), (5) and (6) we get

$$\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}} = \eta l |\mathrm{tr}\ \underline{\sigma}|^{l-1} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) \underline{I} \mathrm{sgn}(\mathrm{tr}\ \underline{\sigma}) + \eta |\mathrm{tr}\ \underline{\sigma}|^{l} 2\delta(\mathrm{tr}\ \underline{\sigma})) \underline{I}. \tag{7}$$

Taking into account the relationship $(\mathrm{tr}\ x)^2 = 1$ (for any $x \neq 0$) and the fact that $\delta(x) = 0$, for $x \neq 0$ we can simplify (7) to

$$\frac{\partial J_{\mathrm{m}}(\underline{\sigma})}{\partial \underline{\sigma}} = \eta l |\mathrm{tr}\ \underline{\sigma}|^{l-1} \underline{I}. \tag{8}$$

As you can see, the derivative (3) obtained in the paper differ from the derivative (8) obtained here.
Is the derivation correct?