ME 7953: Simulations in Materials

Fall 2002

Problem Set 5 (Friday, 10/18/2002)

Problems are due at the beginning of the class, Friday, 10/25/2002.

Material Point Method (MPM) simulation is the focus of this week's problems. MPM utilizes two representations of the continuum - one based on a collection of material points and the other based on a computational grid. In the method, the material is discretized into a finite collection of material points. The material points are followed throughout the deformation of the material and provide a Lagrangian description that is not subject to mesh tangling. Information from the material points is projected onto a background computational grid, where evolution equations are solved. The solution on the grid is then used to update the material points.

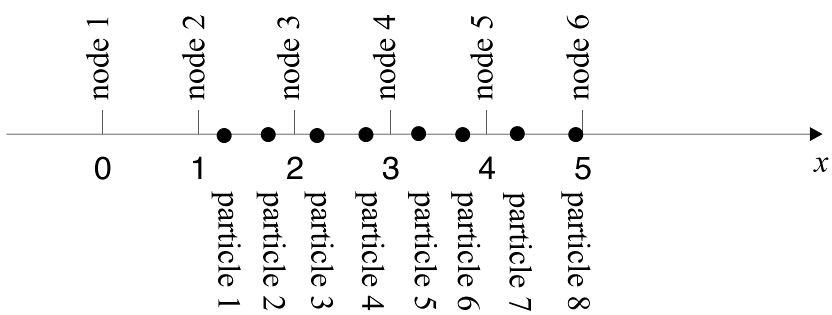
1) Shape function

Consider a one-dimensional MPM problem with background grid at x_g:

```
x_g=[0.0, 1.0, 2.0, 3.0, 4.0, 5.0];
```

The body is divided into 8 particles. The coordinates of particles are in x_p:

```
x_p=[1.24, 1.7, 2.2, 2.6, 3.4, 3.8, 4.5, 4.9];
```



Answer following questions:

(A) Write a MATLAB function to calculate $N^{(n)}(x)$.

```
function [N] = shape(n,x)
```

(B) Write a MATLAB function to calculate $\frac{d}{dx}N^{(n)}(x)$.

```
function [dN] = dshape(n,x)
```

(C) Calculate the value of $N^{(3,2)}$, which means a shape function between node 3 (coordinate is 2.0) and particle 2 (coordinate is 1.7)?

(D) For each particle *p*, calculate all the shape function between particle *p* and grid nodes. Show that $\sum_n N^{(n,p)} = 1$ for *p*=1,…,8.

2) Mass matrix

For the above grid and particles, assume all the particle masses are the same to be 1. The mass-matrix on the grid is defined as

$$\overline{m}^{(n,n')} = \sum_p N^{(n',p)} N^{(n,p)} m^{(p)}, \qquad n,\ n' = 1,\ldots,6, \qquad p = 1,\ldots,8$$

where $m^{(p)}$ is the mass of particle *p*, and $N^{(n,p)} = N^{(n)}(x^{(p)})$ is the shape function of node *n* at the location of particle *p*.

Answer the following questions with the help of MATLAB.

(A) Compute the details of the 6 by 6 mass-matrix.

(B) Compute the lumped mass $\overline{M}^{(n)}$ at each node *n*. Lumped mass is defined as $\overline{M}^{(n)} = \sum_p N^{(n,p)} m^{(p)}$.

(C) Show that $\overline{M}^{(n)} = \sum_{n'} \overline{m}^{(n,n')}$ for every grid node *n*.

3) Solving dynamic equations

For the above problem, assume the current stresses developed in particles $\sigma^{(p)}$ are

```
sig_p=[-5.1, -4.7, -4.1, -3.5, -2.9, -2.4, -1.95, -1.45];
```

The body-force densities for all particles are the same, $b^{(p)} = 10$, *p*=1,…,8.

Answer the following questions with the help of MATLAB.

(A) Compute the external force $\overline{F}^{(n)}$ at each node *n*. $\overline{F}^{(n)}$ is defined as $\overline{F}^{(n)} = \sum_p N^{(n,p)} m^{(p)} b^{(p)}$.

(B) Compute the internal force $\bar{f}^{(n)}$ at each node *n*. $\bar{f}^{(n)}$ is defined as

$$\bar{f}^{(n)} = -\sum_p V^{(p)} \frac{dN^{(n,p)}}{dx} \sigma^{(p)}.$$

Assume the volume of all the particles are the same, $V^{(p)} = 2$, for all *p*.

(C) Using the lumped mass, calculate the acceleration $\bar{a}^{(n)}$ at each node.

$$\bar{a}^{(n)} = \frac{\bar{F}^{(n)} + \bar{f}^{(n)}}{\bar{M}^{(n)}}$$

3) System updating

For the above problem, the initial particle velocities $v^{(p)}$ are

```
v_p=[1.25, 1.1, 0.9, 0.8, 0.45, 0.1, -0.9, -0.7];
```

Answer the following questions with the help of MATLAB. Take time step $\Delta t = 0.01$.

(A) Map the velocities from particles to grid. Compute the grid velocity $\bar{v}^{(n)}$ at each node n. $\bar{v}^{(n)}$ is defined as $\bar{v}^{(n)} = \frac{1}{\bar{M}^{(n)}} \sum_p N^{(n,p)} m^{(p)} v^{(p)}$.

(B) Update the grid velocities using $\bar{v}^{(n)} \leftarrow \bar{v}^{(n)} + \bar{a}^{(n)} \Delta t$.

(C) Update the particle velocities using $\bar{v}^{(p)} \leftarrow \bar{v}^{(p)} + \Delta t \sum_n \bar{a}^{(n)} N^{(n,p)}$.

(D) Update the particle positions using $\bar{x}^{(p)} \leftarrow \bar{x}^{(p)} + \frac{\Delta t}{2} \left(v^{(p)} + \sum_n \bar{v}^{(n)} N^{(n,p)} \right)$.

(E) Calculate the strain rate at particles using $\dot{\varepsilon}^{(p)} = \sum_n \bar{v}^{(n)} \frac{dN^{(n,p)}}{dx}$.

(F) Update the particle stresses using $\sigma^{(p)} \leftarrow \sigma^{(p)} + E \Delta t \dot{\varepsilon}$. Take Young's modulus to be $E = 100$.