

Physics of Explosions

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1 Introduction

Equation of state (EOS) is a relation between state variables of the system. It is used to describe the material response under dynamic loading. Shock waves have been used to measure EOS of materials at extreme conditions through Hugoniot equations.

1.1 State of matter

We constantly deal with three states of matter:

- Gases are compressible fluids. Their molecules are widely separated.
- Liquids are relatively incompressible fluids. Their molecules are more tightly packed.
- Solids are nearly incompressible. Their molecules or ions are in close contact and do not move.

1.2 State variables

State variables are state parameters or thermodynamic variables that describe the momentary condition of a system. Change of the state variables are path independent, i.e., regardless of the sequence of intermediate states.

There are many state variables; usually in fluid and gas dynamics we talk about pressure (P), temperature (T), mass density (ρ) and internal energy density (e), because these variables appear in the equations of motion.

1.3 Equations of state

Equation of state is a relation between state variables (such as its temperature, pressure, volume, or internal energy) of the system.

EOS can be used to predict the state of gases and liquids. One of the simplest equations of state for this purpose is the ideal gas law, which is roughly accurate for gases at low pressures and high temperatures. However, this equation becomes increasingly inaccurate at higher pressures and lower temperatures.

EOS can also be used to predict the state of solids.

1.3.1 Ideal gas

Simple systems in equilibrium can be fully specified by two properties such as temperature T and pressure P ; all the other state variables, such as volume V , are functions of these. In an ideal gas assumption, the equation of state can be expressed as

$$PV = nRT, \quad (1)$$

where n is the number of moles of a substance, and R is the ideal gas constant $8.31 \text{ J}/(\text{mol}\times\text{K})$. The basic assumptions are:

- (1) identical point particles without internal structure;
- (2) no interaction between particles.

Since the EOS for ideal gas, Eq. (1), neglects both molecular size and interactions (attractions or repulsions) among molecules, it is only accurate for gases at high temperatures and low pressures.

Problem: By colliding two streams of liquids together (450 mph), intense local heating will be generated with a transient temperature of about 5000 K and a pressure of about 1000 MPa. (Prof. Suslick at the University of Illinois at Urbana-Champaign). Estimate the average distance between molecules in an ideal gas assumption.

Solution: Using the ideal gas assumption, the volume for 1 mol of molecules is $V = RT/P$, which gives $V = 42\text{cm}^3$. The average distance between these molecules is $\sqrt[3]{V}$ divided by the Avogadro number 6.02×10^{23} , which gives $6\times 10^{-17} \text{ nm}$. In reality, distance between molecules is on the scale of nanometer; this is much larger than the estimation based on the idea gas assumption. The reason is that the inter-atomic repulsion is neglected in idea gas assumption.

1.3.2 Simple metal

Figure 1 shows a $P(V,T)$ surface that describes all possible states attainable for a simple metal (for example, aluminum, magnesium, calcium, zinc, and lead). This is the material's equation of state. To complete the EOS we also need to relate the internal energy to volume and temperature.

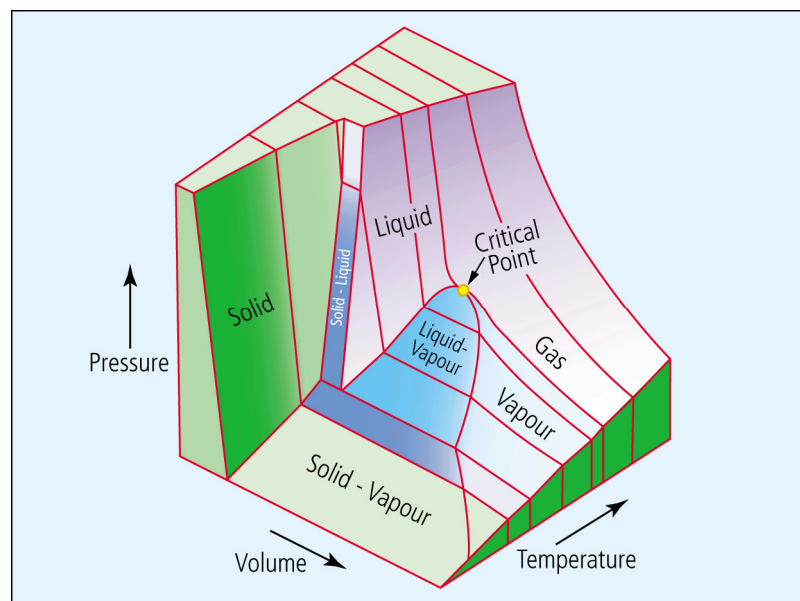


Figure 1. The 3D phase diagram for a simple metal.

Two observations:

- The $P(V,T)$ relation is close to that of an ideal gas when the temperature is high and the pressure is low.
- The volumes for solid and liquid show small changes.

1.4 Material subject to high pressure

Bumper shields for spacecrafts: In designing bumper shields for spacecrafts, it is necessary to consider the effects of hypervelocity impacts of 20-40 km/s which give rise to a requirement for material information in the TPa regime.

Treatment of kidney stones: In the medical area shocks of around 1GPa are used for the treatment of kidney stones, necessitating an accurate knowledge of the material response for biological samples.

1.5 Measurement of EOS

It is difficult to make direct measurements of EOS parameters, e.g., internal energy and density, at high pressure. Use of shock wave permits us to replace these by velocity measurements, which can be made relatively easily to high accuracy.

2 Shock Wave

Shock waves have long been used to study the equation of state of materials at extreme conditions. In addition, experiments on shock-induced plasticity and fracture have provided useful insight into material deformation and failure.

2.1 What is shock wave?

Shock waves are characterized by an abrupt, nearly discontinuous change in the mechanical or thermodynamic properties of the medium: pressure, mass density, internal energy density, temperature, particle velocity. A shock can propagate through solids, liquids or gases.

Shock waves are made by a rapid, continuous push. The mechanism of shock compression of a material is illustrated in Figure 2 by the image of a row of skiers piling up in front of obstacles.



Figure 2. Schematic picture of continuous pushing of shock wave (from Courant and Friedrichs, 1950).

Figure 3 shows the shock waves in the air. This is a photograph of an air blast during a micro-explosion, an explosion generated by an explosive magazine of about 1 cm in size. The micro-explosion technique allows us to monitor and measure the propagation of an air blast in a more simple and easy way with a high degree of accuracy.

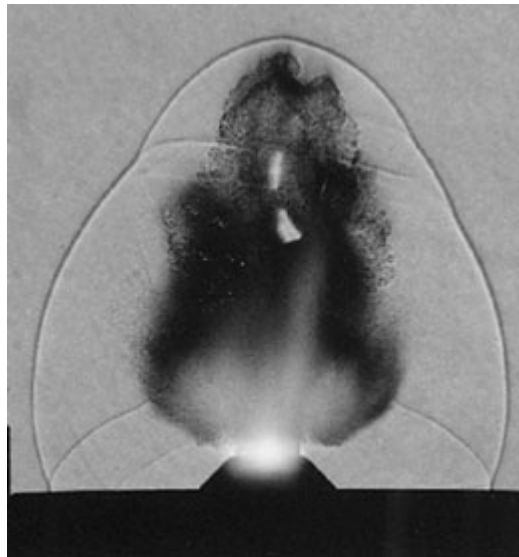


Figure 3. Photograph showing the shock waves in the air. (From The Research Center for Explosion Safety, National Advanced Industrial Science and Technology, Japan.)

3 Rankine-Hugoniot Equation

Assumes that the shock is a single steady wave, which means the jump conditions can be calculated by the black-box treatment: The entire shock front is considered a black box of fixed thickness which moves at the shock speed; ahead of the box is materials in the initial equilibrium state and behind the box is uniformly moving material in the

final equilibrium state. Without knowing any details of the shock structure it is still possible to apply conservation laws: What ever flows into the box must flow out. In this way conservation of mass, momentum, and energy can be established.

In Figure 4, a rigid piston with velocity u drives into an un-deformable tube containing a material at rest of density ρ_0 , temperature T_0 , and internal energy density (energy per unit volume) e_0 . The “piled-up” shock compressed material moving with the same velocity as the piston has density $\rho_1 (> \rho_0)$, pressure P , and internal energy density $e_1 (> e_0)$. The sudden shock transition from state 0 to state 1 takes place at the shock front which moves into the material at rest with velocity D and gives the shocked material the velocity u .

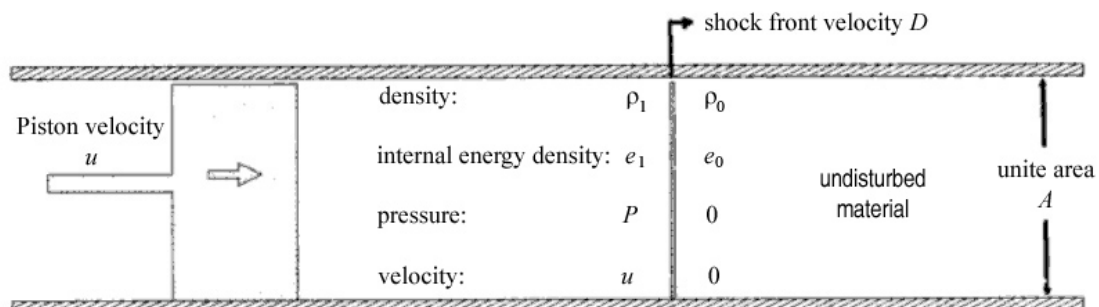


Figure 4. Constant velocity piston-generated shock transition from state 0 to state 1.

3.1 Conservation of mass

We use “shock” coordinate system, where we view the material behaviours as if we were sitting on the crest of the shock wave. In the following, “shock” coordinate system is used to derive the basic jump equations that describe the shock phenomena.

$$\text{Mass in: } \Delta m_{\text{in}} = \rho_0 D \cdot \Delta t \cdot A$$

$$\text{Mass out: } \Delta m_{\text{out}} = \rho_1 (D - u) \cdot \Delta t \cdot A$$

where A is the area of the cross section as shown in Figure 4.

Conservation of mass requires that the amount of mass per unit time and unit area which passes from the initial low pressure region into the shock front must be equal to the amount of mass per unit time and unit area which passes from the shock front into the high pressure region. This requires that

$$\Delta m_{\text{out}} = \Delta m_{\text{in}} \equiv \Delta m, \tag{2}$$

which gives

$$\rho_1 = \rho_0 \frac{D}{D - u}. \tag{3}$$

Problem: For the case $u = D/5$, what is ρ_1 / ρ_0 ?

Solution: Using Eq. (3), one has $\rho_1 / \rho_0 = 1.25$.

3.2 Conservation of momentum

Change of momentum for the mass Δm , from the momentum before the shock (which is 0) to the momentum after the shock (which is $\Delta m \cdot u$) is

$$\Delta M = \Delta m \cdot u . \quad (4)$$

The force acted on the mass is

$$F = PA . \quad (5)$$

Conservation of momentum implies that the rate of the change of the momentum equals to the resultant force applied to it, i.e.,

$$F = \frac{\Delta M}{\Delta t} , \quad (6)$$

which gives

$$P = \rho_0 Du . \quad (7)$$

3.3 Conservation of energy

Specific energy (energy per unit mass) is denoted as e . During the shocking process,

- energy of mass Δm before shock is $E_0 = e_0 \cdot \Delta m$,
- energy of mass Δm after shock is $E_1 = e_1 \cdot \Delta m + \frac{1}{2} \Delta m \cdot u^2$,
- work being done on mass Δm is $W = P \cdot A \cdot u \cdot \Delta t$.

Conservation of energy is that the rate of energy increase equals to the rate of work being done on it,

$$W = E_1 - E_0 , \quad (8)$$

which gives

$$e_1 = e_0 + \frac{1}{2} u^2 . \quad (9)$$

References

Courant, R., and Friedrichs K.O. (1948) Supersonic flow and shock waves, Interscience Publishers, New York.