On the connection between Palmgren-Miner's rule and crack propagation laws

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January 10, 2018

Abstract

The classical Palmgren-Miner's rule (PM), despite clearly approximate, is commonly applied for the case of variable amplitude loading and to date, there is no simple alternative. In the literature, previous authors have commented that the PM hypothesis is based on an exponential fatigue crack growth law i.e. when da/dN is proportional to the crack size a, the case which includes also Paris law for $m = 2$, in particular. This is because they applied it by updating the damage estimate during the crack growth.

It is here shown that applying PM to the "initial" and nominal SN curve of a cracked structure, results *exactly* in the integration of the simple Paris' power-law equation, and more in general to any crack law in the form $da/dN = H\Delta\sigma^h a^n$. This leads to an interesting new interpretation of PM rule. Indeed, the fact that PM rule is often considered to be quite inaccurate pertains more to the general case when propagation cannot be simplified to this form (like when there are distinct initiation and propagation phases), rather than in long crack propagation. Indeed, results from well known round-robin experiments under spectrum loading confirm that even using modified Paris' laws for crack propagation, the results of the "non-interaction" models, neglecting retardation and other crack closure or plasticity effects due to overloads, are quite satisfactory, and these correspond indeed very closely to applying PM, at least when geometrical factors can be neglected. The use of generalized exponential crack growth, even in the context of spectrum loading, seems to imply the PM rule applies. Therefore, this seems closely related to the so called "lead crack fatigue lifing framework". The connection means however that the same sort of accuracy is expected from PM rule and from assuming exponential crack growth for the entire lifetime.

Keywords: Fatigue, variable amplitude loading, Palmgren-Miner' law, linear damage cumulation

1 Introduction

The classical approach to Variable Amplitude (VA) loading is to apply the Palmgren-Miner's (PM) linear damage rule, suggested by Miner [1] at Douglas Aircraft in 1945, 21 years after Palmgren [2], which suggests for a given block with a total number of cycles per block N_c that damage will be

$$D = \sum \frac{n_i}{N_i} \tag{1}$$

where n_i is the number of cycles spent at level i on the stress amplitude, and N_i is the total number of cycles the specimen could resist at that level of stress, according to the constant amplitude (CA) SN Wöhler curve. Failure according to PM should occur at critical damage of $D_c = 1$. PM linear rule is obviously quite approximate, does not consider load sequence or memory effects, and could be both on the unsafe or on the safe side, but it is by far the most well known and used damage summation law (see review in [3]). To design on the safe side, handbooks suggest to simply assume a lower D_c. For example, FKM-Guideline [4] recommends $D_c = 0.3$ for steels, steel castings, aluminum alloys, while $D_c = 1$ for ductile iron, grey cast iron malleable cast iron (for which therefore PM seems to work quite well in general). Attempts to generalize PM rule like e.g. Miller & Zachariah [5], Manson & Halford [6] have had limited success, and anyway become very cumbersome when large number of blocks or indeed random loading is considered.

For cracked structures, PM has been applied much less, because full integration of crack growth curves is generally attempted in the hope to be more accurate [7, 8, 9], and the ultimate goal would be to obtain the full SN curve from integration as a total-life analysis based solely on crack propagation, i.e. including starting from very short cracks. This would hope to shed light on the old problem of distinguishing between the initiation and propagation phases, which generally has only a vague solution (the threshold "has often been defined as a macrocrack, visible in a low-power microscope" [10]). However, unsatisfactory prediction quality at times "stems from an inadequate conception of the constraint factors incorporated in the NASGRO models" [7].

Clearly, PM is not the way forward for very advanced designs of light structures, although it remains the basis, for example, for the application where it started from, rolling bearings, and many other applications. It is also commonly used in design of welded joints (see [4] where variants are proposed to account for cycles below the fatigue limit, if it exists).

In any case, it is appropriate to draw some possible conclusions, because some confusion may have originated from the conclusion of some authors who have noticed, like Miller & Zachariah [5], that "*the Palmgren-Miner hypothesis can be stated to be based on an exponential fatigue crack growth law*". Indeed, [5] notice that a law of the type

$$da/dN = L\Delta\sigma^h a \tag{2}$$

where in their case they refer to the type of equations originally proposed by Frost and Dugdale [11], leads to PM rule. It should be noted immediately that the "exponential crack growth" was proposed even earlier by Shanley [12] in his eqt.4 in order to justify SN curves also under spectrum loading and his reference to $h \simeq 8$ without much reference to actual real crack, is rather similar to the equations by few authors (Nisitani [13] Nisitani and Goto [14] Nisitani et al. [15], Murakami et al. [16,17], see also Pugno et al. [17]).

Notice that integration of this type of crack growth laws leads to a SN curve $\Delta\sigma^h N = const$ where the typical value of h is of the order of that known in textbooks as correspond to the Basquin exponent in the SN curves. The logarithmic dependence on initial crack size which results from the integration has been obviously not observed in the classical studies and textbooks (except of course that empirical factors do take into account, for example, of surface finish which may be an indication of a size of initial crack), until recently when it was indeed observed [17]. The exponential crack growth is also recalled sometimes in more general attempts to see unified procedures for crack growth under spectrum loading [19, 20], but this requires some explanation. Indeed, the USAF report [17] refers to exponential fits either for crack sizes $a < 0.005in$, which would be probably called indeed short cracks, or as an approximation in small increments of propagation. Indeed, we read explicitly (pag.A10) that exponential fits are assumed only over small increments "*Incremental crack growth is determined through log-linear interpolation of the crack growth curve. Crack growth curves typically increase at about the same rate as an exponential function. That is, although an exponential function may not fit the crack growth curve exactly, over a short interval the rate of increase of the crack growth curve is nearly exponential. Crack growth calculation errors can occur using linear interpolation even when a large number of points are included in the crack growth table.*" Hence, it should not be concluded that exponential crack growth can be assumed for the entire lifetime.

The present note starts from showing a disagreement with the Miller-Zachariah [5] statement that PM should be valid only with exponential crack growth. This erroneous conclusion is shown to clearly come from updating the damage in each block starting from the initial crack size. This is not the correct interpretation of the PM rule, which in general never updates the damage during the calculation — indeed, this simplification is the basis for the simplicity of PM rule. A more correct interpretation of PM in the context of long cracks, leads to quite more general conclusions. Indeed, it is here shown that a correct application of PM rule follows directly from any propagation law of the form

$$da/dN = L\Delta\sigma^h a^n \tag{3}$$

which covers the equations for short cracks which we referred to as special cases, as well as the more well known Paris' law defines the advancement of crack in terms of the range of stress intensity factor K

$$\frac{da}{dN} = C\left(\Delta K\right)^m \tag{4}$$

where C and m are experimentally determined "material parameters". This is of course just the basic form of the Paris equation, and strictly speaking, PM does not follow from more elaborate versions, and from application of the PM rule in range where Paris' law show these deviations, like near threshold or near static failure.

The note then concludes with a discussion about what may be the reasons why PM doesn't seem to work so well in general, and when instead could work.

2 PM rule as a consequence of crack propagation laws

We consider applying the PM rule to a crack of characteristic size a. We remind that Paris' rule or law defines the advancement of crack in terms of the range of stress intensity factor $\Delta K = g\Delta\sigma\sqrt{\pi a}$ (4). The Paris crack growth law is considered first, but then it will be recognized that this is a special case of a more general set of propagation laws. Integrating Paris' law (4) (assuming geometrical factor g is constant) within a block i gives a propagation from a_i to a_{i+1} (assuming $m > 2$)

$$\pi^{m/2}\left(m/2-1\right)C\left(g\Delta\sigma_i\right)^m N_i = a_i^{1-m/2} - a_{i+1}^{1-m/2} \tag{5}$$

and hence summing up to failure, all the intermediate values of the crack cancel out (we are neglecting interaction and retardation effects of course)

$$\sum \Delta\sigma_i^m N_i = \frac{a_1^{1-m/2} - a_f^{1-m/2}}{\pi^{m/2}\left(m/2-1\right)Cg^m} \tag{6}$$

Writing the factors $\alpha_i = \frac{N_{fi}}{N_*} = \frac{n_i}{N_c}$ which is the proportion of cycles n_i spent at level i on the total number of cycles of the base spectrum N_c, or else the proportion of the total life N_{fi} spent at level i with respect to the total life N^*, we get a sort of Gassner SN curve

$$\left(\Delta\sigma^*\right)^m N^* = \frac{a_1^{1-m/2} - a_f^{1-m/2}}{\pi^{m/2}\left(m/2-1\right)Cg^m}\frac{1}{G} \tag{7}$$

where it can be considered that an amplification factor for the base spectrum β is defined such that $\Delta\sigma^* = \beta\Delta\sigma_{\max}$ is a Gassner stress range, where $\Delta\sigma_{\max}$ is the largest stress range of the spectrum, and so that each individual block stress range is $\Delta\sigma_i = \beta\Delta\sigma_i^*$. Finally, that the multiplicative factor

$$G = \sum\left(\frac{\Delta\sigma_i}{\Delta\sigma^*}\right)^m \alpha_i = \sum\left(\frac{\Delta\sigma_i^*}{\Delta\sigma_{\max}}\right)^m \alpha_i \tag{8}$$

can be computed from the base spectrum.

This result looks identical to the result that was recently obtained for the SN "Gassner" curve (i.e. the Wöhler SN curve for spectrum loading), see [21], where it was found that Gassner curve are simply shifted CA curves starting from power laws for the CA case, like Basquin's law

$$N\left[\Delta\sigma\left(N\right)\right]^{k}=C_{W} \tag{9}$$

and even much more in general. The integrated form of Paris' law for CA (7) with $G=1$, is also a power law of the Basquin type, where $m=k$ and $C_W=\frac{a_1^{1-m/2}-a_f^{1-m/2}}{\pi^{m/2}(m/2-1)Cg^m}$. Hence, our result is exactly the same as obtained from applying the linear damage sum rule of PM for which damage sum will be given by

$$D=\sum\frac{n_i}{N_i}=\sum\frac{n_i}{N_c}\frac{N_c}{N_i}=N_c\sum\frac{\alpha_i}{N_i}=\frac{N_c}{C_W}\sum\alpha_i\Delta\sigma_i^m \tag{10}$$

The life under the sum of all i blocks is N^*,

$$\frac{1}{N^*}=\frac{D}{N_c}=\frac{1}{C_W}\sum\alpha_i\Delta\sigma_i^m \tag{11}$$

and the result follows.

If $G=1$, one simply has the SN curve for the Constant Amplitude (CA) case. Indeed, in [21] it was also found that starting from a power law SN curve, and applying PM, gives a Gassner VA curve which is shifted from the CA curve, but here dealing with specimen with a long crack we have also the independent integration of Paris' law which results in the same final equation. For a plain specimen or a notched one (if critical distance methods could be applied), [21] proved only that Gassner curves were shifted CA curves with the factor G.

An important consideration is that the same procedure we just outlined can be generalized for the law (3) as the exponent in $\Delta\sigma$ plays no role in the summation, and the result carries over to the more general crack growth curves.

Therefore, it is concluded that in this general class of crack propagation laws, including many short crack proposed in the past, as well as Frost-Dugdale, PM follows naturally and is equivalent to integration of the crack growth. Further, that the Gassner curve is a shifted CA curve with shift given by the factor G.

2.1 Applying PM in a refined sense?

The reason [5] suggested PM rule stems from the exponential crack growth, and not from Paris' law, may be that [5] computes the damage by considering at the denominator the number of cycles $\widehat{N_i}$ which would lead to failure at the given stress range level,

$$\pi^{m/2}\left(m/2-1\right)C\left(g\Delta\sigma_i\right)^m\widehat{N_i}=a_i^{1-m/2}-a_{fi}^{1-m/2}\simeq a_i^{1-m/2} \tag{12}$$

where we supposed that the final size is large enough for the term to be neglected.

Hence, dividing the actual number of cycles spent at each level N_i, by $\widehat{N_i}$ we obtain the total damage as

$$D = \sum \frac{N_i}{\widehat{N_i}} = \sum \frac{a_i^{1-m/2} - a_{i+1}^{1-m/2}}{a_i^{1-m/2}} \tag{13}$$

which *does not sum to 1.* If we take small increments $a_{i+1} = a_i + da_i$, then expanding in Taylor series

$$D \simeq \sum \frac{1}{2} \frac{a_i^{-m/2}(m-2)}{a_i^{1-m/2}} da_i = \frac{(m-2)}{2} \sum \frac{da_i}{a_i} = \frac{(m-2)}{2} \int_{a_1}^{a_f} \frac{da}{a} = \frac{(m-2)}{2} \log \frac{a_f}{a_1} \tag{14}$$

which in general is $>> 1$. For example, if $a_f = 1000 a_1$, then $\log \frac{a_f}{a_1} = \log 1000 = 6.91$. This is only valid for $m > 2$, and approximately as we have expanded in Taylor series, and neglected the final values of the crack at failure at each stress range level. For example, for $m = 4$, this means a spurious artificial increase of damage of 7. There could be also an effect of load sequence for discrete spectra, if we removed these approximations.

Miller & Zachariah [5] mention this interpretation of the PM rule, and therefore state that the PM hypothesis is based on an exponential fatigue crack growth law i.e. Paris for $m = 2$, as in this case, repeating the process just completed for $m > 2$, we obtain simply (like in their eqt.3)

$$D = \sum \frac{N_i}{\widehat{N_i}} = \sum \frac{\log a_i / a_1}{\log a_{fi} / a_1} \tag{15}$$

and neglecting the change of a_{fi} with $\Delta\sigma$ (considering a given final crack size which is dictated by static failure $K_{Ic}(1-R) = g\Delta\sigma_i\sqrt{\pi a_{fi}}$ where $R = \sigma_{\min}/\sigma_{\max}$ is the load ratio, and K_{Ic} is static toughness, and neglecting the influence of $\Delta\sigma_i$ on a_{fi})

$$D = \frac{1}{\log a_f / a_1} \sum \log a_{i+1}/a_i = \frac{1}{\log a_f / a_1} \left(\log a_2/a_1 + \log a_3/a_2 + ... \log a_{f1}/a_{n-1} \right) = 1 \tag{16}$$

which satisfies PM.

3 Discussion - validity of PM

In view of the general correct interpretation, PM should apply rather commonly, including the case of short cracks. However, since we cannot state that "fatigue life is dominated" by initiation, as it is often believed for constant amplitude loading at low levels of stress range, in the case of short cracks we inevitably have the sum of the two fatigue phases (initiation and propagation), and probably this makes the PM invalid. Indeed, as reported in [22] [23] and see Fig.1, Damage sum D_{real} can be as low as $D_{\text{real}} = 0.001$ in extreme cases, or as large as $D_{\text{real}} = 10$ in other extreme cases, although the distribution is rather of the extreme values type, so that only 10% of cases, for example, $D_{\text{real}} < 0.1$, or in another 10% of cases, it is $D_{\text{real}} > 1$. It is found that the median value is rather 0.4, from which the standards obtain the safety factors suggested for design purposes, which we mentioned in the introduction.

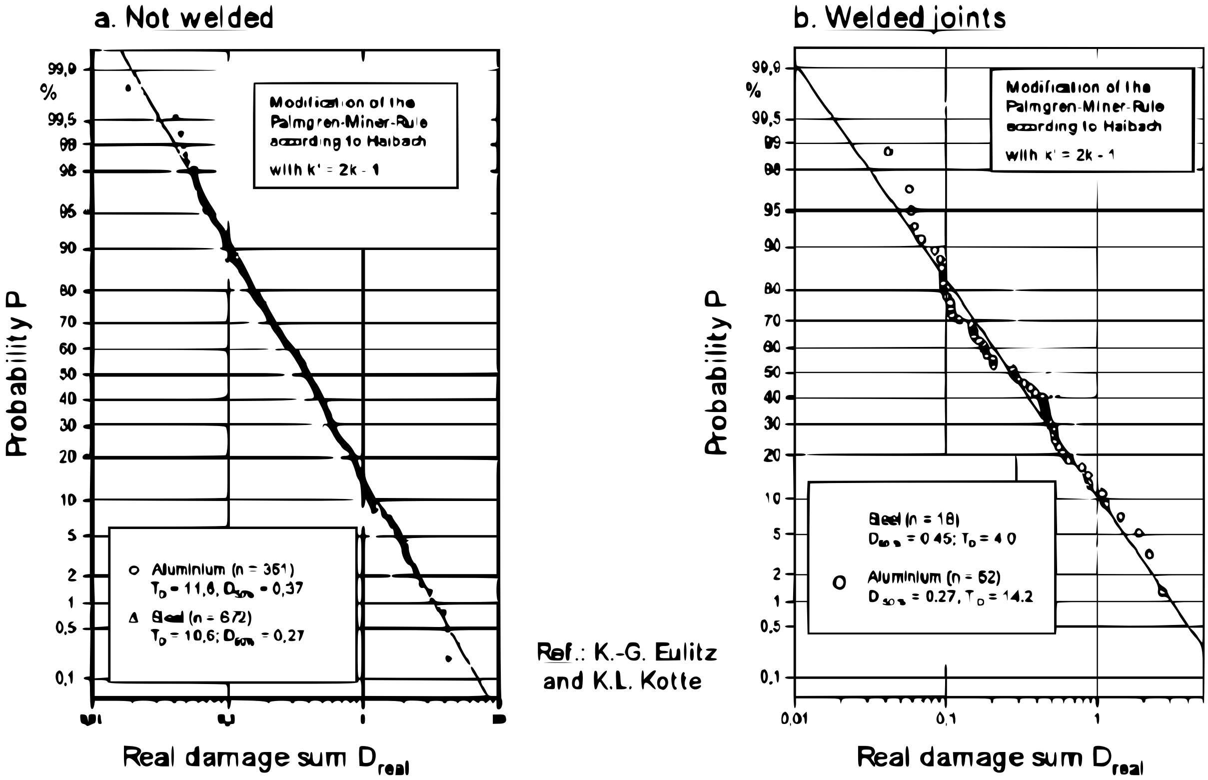

Fig. 1 – Adapted from [22] [23] - Damage sum D_{real} can be as low as $D_{\text{real}} = 0.001$ in extreme cases, or as large as $D_{\text{real}} = 10$ in other extreme cases.

It is interesting to note that in double linear damage rules like Miller Zachariah [5], but also Manson and Halford [6], it is said that cycles at high strains tend to *decrease* the initiation life at cycles at lower strains, so they introduce a negative effect, accelerating failure. In crack propagation, the *opposite* is normally found, as overloads give "crack retardation" effects. This means that even double linear damage rules would need to be adapted to crack propagation effects and tuned appropriately — and it is not clear if this could be done in the general context. In the most common case, when there is crack initiation and propagation, it is unclear what their benefit could be, and indeed, this may explain their very limited success.

When dealing with long cracks, there have been very few investigations of PM rule, perhaps because overload effects were found and an attempt was immediately started to deal with them accurately. Not only Miller and Zachariah [5], but also in this context early authors (Schijve & Broek [24]) interpreted PM rule in the "refined sense", and therefore exacerbated the increase of damage. Indeed, in [24] the ratio of final to initial crack length is $\frac{30}{5} = 6$ and hence the spurious artificial increase of damage (14) is (for their material having $m = 4$)

$$\frac{(m-2)}{2} \log \frac{a_f}{a_1} \simeq 1.8$$

Therefore, their result that gust load fatigue tests the predicted damage at failure was fairly high, viz. of the order of 2 to 4, should be significantly reconsidered to be not so high, of the order of 1 to 2, which will be very close to values of a round-robin excercise we are about to describe in Tab.1. Therefore, despite clearly there are overload effects as it is well known, and we are not saying this is not a real effect, they do not produce such tremendous effect on the PM sum. A fortiori one of the conclusions from Schijve & Broek [24] is valid, that "The Palmgren-Miner rule will give conservative crack rate predictions ...it can be a useful tool to fix inspection periods for 'fail-safe' aircraft structure".

Today, NASGRO models [8] are considered to be among the best possible calculation methods. They do attempt to consider load sequence effects with "interaction models" of various complexity. However, as it can be read in the paragraph "2.1.7.6 Notes on using the Load Interaction Models": *"In general, caution should be exercised when these models are used because they can be unconservative compared to the non interaction model. This is so because the dominant effect modelled is retardation, even if accelerated growth is predicted in a few cases. Before applying these models for life predictions, it is recommended that the user gain sufficient experience and fine tune the various model parameters based on comparisons with test data for the kind of spectra relevant to the usage."*

Obviously, a calculation based on PM rule would work similarly to integration of Paris' law with "no-interaction" models, at least in the sense that no overload effect and other memory effects are considered and within the assumption that geometrical factors in the crack size do not change during propagation, which is largely satisfied in cases when cracks are relatively small for much of their life. Hence, with respect to a full NASGRO calculation without load interaction, the PM rule will often be almost equivalent.

Based on Tab.4 of the NASGRO manual [8], which in turn reports a large set of data from a round-robin exercise [9] with a material having a rather good form of Paris law crack propagation law, with m very close to 3 (2219-T851 aluminum), some findings in the present paper seem confirmed. The paper deals with a Center-Cracked Tension Specimens (so that geometrical factors are probably constant over much of the fatigue propagation), under Random Spectrum Loading of interest of a typical fighter aircraft Air-to-Air (A-A), Air-to-Ground (A-G), Instrumentation and Navigation (I-N), and Composite missions.

Spec. No.	Loading Spectrum	Stress ksi	Test Cycles	Non Int. NASGRO	Non Int. Walker	Willenborg Walker $R = 3.0$	Willenborg Generalized $R = 3.0$	Willenborg Modified Phi0 = 0.4	Strip Yeld	Constant Closure Cfspec = 0.5
M-81	3*Air-Air	20	115700	0.78	0.71	0.49	0.54	0.54	0.62	0.70
M-82		30	58585	1.27	0.92	0.62	0.87	0.92	1.27	1.15
M-83		40	18612	1.28	0.79	0.54	0.88	0.94	1.71	1.16
M-84	3*Air-Grn	20	268908	0.86	0.77	0.51	0.57	0.55	0.52	0.60
M-85		30	95642	1.25	0.98	0.63	0.81	0.82	0.98	0.87
M-86		40	36397	1.24	1.04	0.67	0.99	1.04	1.64	1.06
M-88	2*Ins-Nav	30	380443	1.36	1.19	0.56	0.66	0.59	0.51	1.23
M-89		40	164738	1.73	1.37	0.63	0.80	0.79	0.92	1.56

Table 1: Ratio test over predicted life N_{test}/N_{pred} for ASTM Round Robin Spectra. Material 2219-T851, L-T AL, Data in Tab.4 of NASGRO® manual [8] and in turn based on [9]. The non-interaction models prediction (which is a form very close to applying PM rule to crack propagation) is extremely close to real tests, and often conservative by a little margin. Instead, interaction models predict often unconservative results and sometimes by larger margins.

Indeed, even though the round-robin exercise was made with slightly different forms of crack propagation law than Paris law, the results are very encouraging, and they are relevant for spectrum loading of engineering interest, and with high quality data, although since 1981 of course the crack propagation codes may have improved. As in [8,9], load interaction models, despite their complexity, lead to unconservative results unless the parameters are finely tuned, the non-interaction assumption seem of more appeal (as indeed recommended in the NASGRO manual after all). Indeed, they lead to errors which are certainly not very large, *with total life predicted very close to the real life measured*, as reported in Tab.1 (see second and third column). And what is important to notice, with respect to the PM rule, is that they are much closer to correct than the real damage sum D_{real} from Fig.1 was suggesting. Remarkably the damage sums in this round-robin excercise are not too different from those of Schijve & Broek [24], once the latter are corrected for the misinterpretation of PM damage rule.

Hence, in general, the error in applying the PM rule in the most general cases, could be attributed to various factors:

- Either the laws governing crack propagations are not correctly of the form above (separate variables power law forms) and in particular not of the exponential type
- or they are correct for short cracks, but differ largely when propagation stage is reached (and probably they are no longer exponential, but rather

of the form expected from the integration of Paris' law), in which case a single exponential law should not be used. These effects tend to make the damage sum too low

- there are strong sequence effects (and in particular, overloads) which make the damage sum too high

4 Conclusions

There has been a suggestion in the literature that PM should follow from exponential crack growth only (Miller and Zachariah, [5]). This is shown to stem perhaps from an incorrect interpretation of PM rule, updating the damage during the damage sum, which was repeated also in some early assessment of PM rule for long cracks (Schijve & Broek [24]). In the correct version, PM follow instead directly from a much more general crack growth law, where the crack growth is proportional to the product of powers of stress amplitude and crack length. This includes a number of laws proposed in the past for short or long crack growth, and obviously Paris' law. The reason why PM doesn't apply very accurately in general may stem from its use in context involving a change from initiation to propagation laws (which tends to make the damage sum too low), or obviously also sequence effects and in particular overloads effects in long crack propagation, which tend to make the damage sum too high.

When correcting for the "refined" interpretation of PM rule for long crack, PM may work not too bad, as for some important round-robin data on spectrum loading of an aluminum alloy of military aircrafts, neglecting interaction effects was found to be perhaps better than including them.

The application of PM as suggested by Ciavarella D'Antuono & Demelio [21] leads to VA SN curves which are shifted power laws of the CA curve, similarly to what was found plain specimen having power law SN curves (Basquin's law) but also for notched cases, using critical distance approaches. Therefore, this provides a general framework to consider Gassner curves.

The use of generalized exponential crack growth during the entire lifetime (which seems closely related to the "lead crack fatigue lifing framework" [20]), even in the context of spectrum loading, seems to imply the PM rule applies. Therefore, the same sort of accuracy is expected as PM rule.

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