Numerical Results for shear-lock free finite elements based on Mindlin-Reissner plate and Timoshenko beam theories

Plate Elements

N x N	Mesh Size (Full Plate)
Exact Solution	Srinivasa Rao and AK Rao and Theory of Plates and Shells
CF	Convergence Factor
ASR	Aspect Ratio
MR_FE_1 MR_FE_2 MR_FE_3 MR_FE_4	Finite Elements based on Mindlin-Reissner theory using new shape functions
Material:	E=1.092E06; $\upsilon = 0.3$;
Geometry:	Square plate a=10.0; h=2.0, 1.4, 1.0, 0.5;
Type of plate	Simply supported (SS)
Load:	UDL=10.0

Timoshenko beam elements

N	Number of Elements (Full beam)
Exact Solution	Theory of Elasticity
CF	Convergence Factor
ASR	Aspect Ratio
Timo_FE_1 Timo_FE_2 Timo_FE_3 Timo_FE_4	Finite Elements based on Timoshenko beam theory using new shape functions
Material:	29,000; $\upsilon = 0.3$;
Geometry:	L=5,10, 25,100, 200, 400; h=1.0; b=1.0;
Load:	Concentrated Load q=100.0

Higher order beam elements (capable of accurately predicting three dimensional stresses) based on the higher order shear deformation theories developed by me

HFE_1 -	based on Lagrangian polynomials
HFE_3 -	based on Lagrangian polynomials
FE_NSF_1 -	based on new shape functions
FE_NSF_3 -	based on new shape functions

Beam

Cantilever beam with tip load for ASR = 5.0

error in deflection at centre

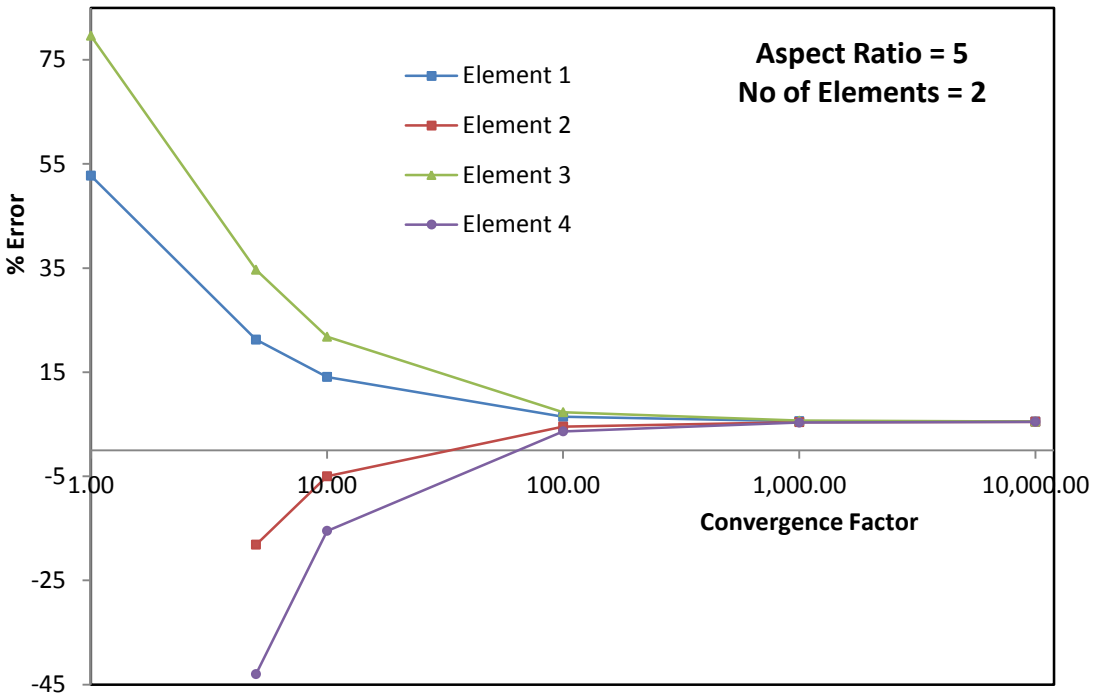


Fig.1

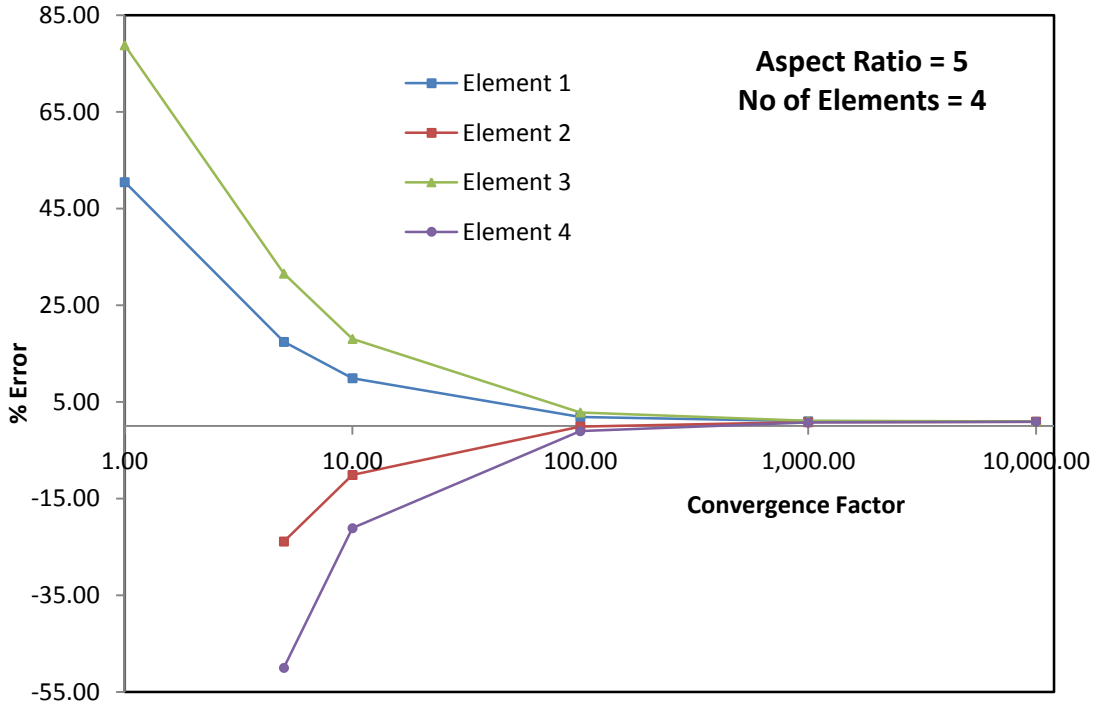


Fig.2

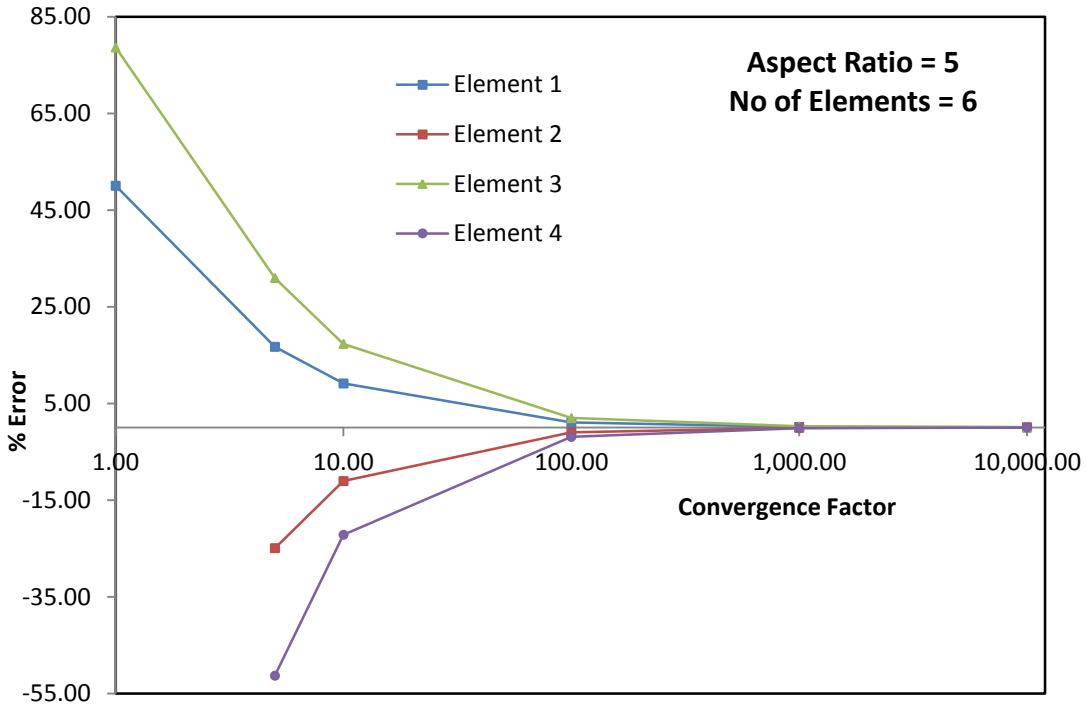

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 5
No of Elements = 6

Fig.3

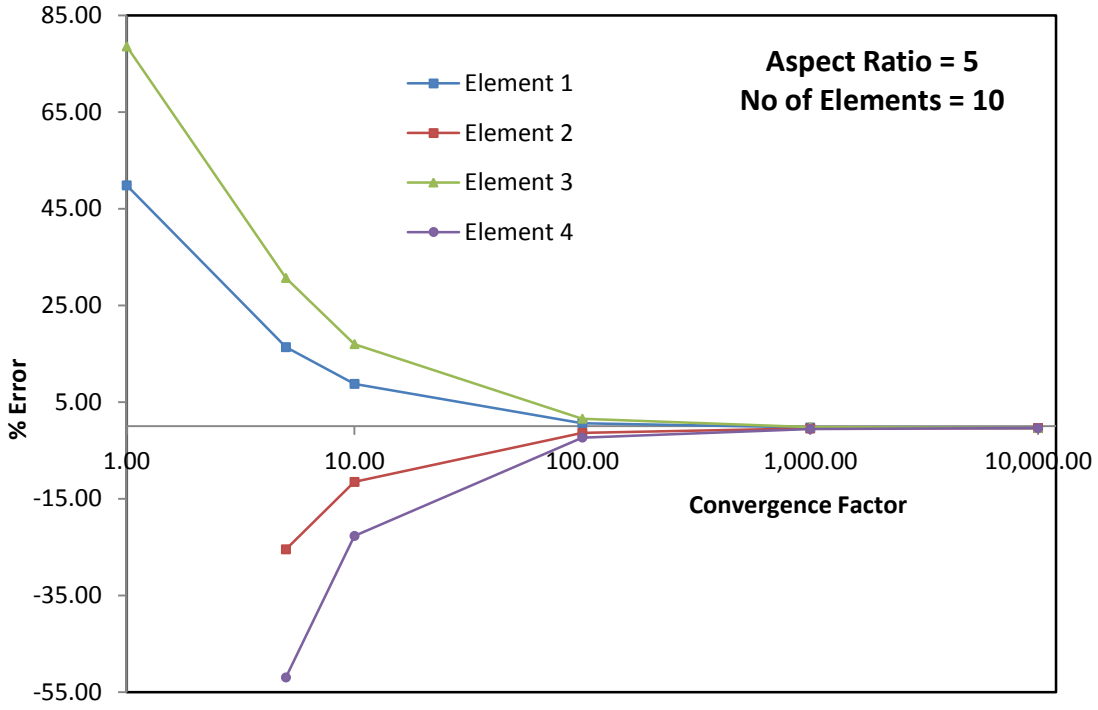

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 5
No of Elements = 10

Fig.4

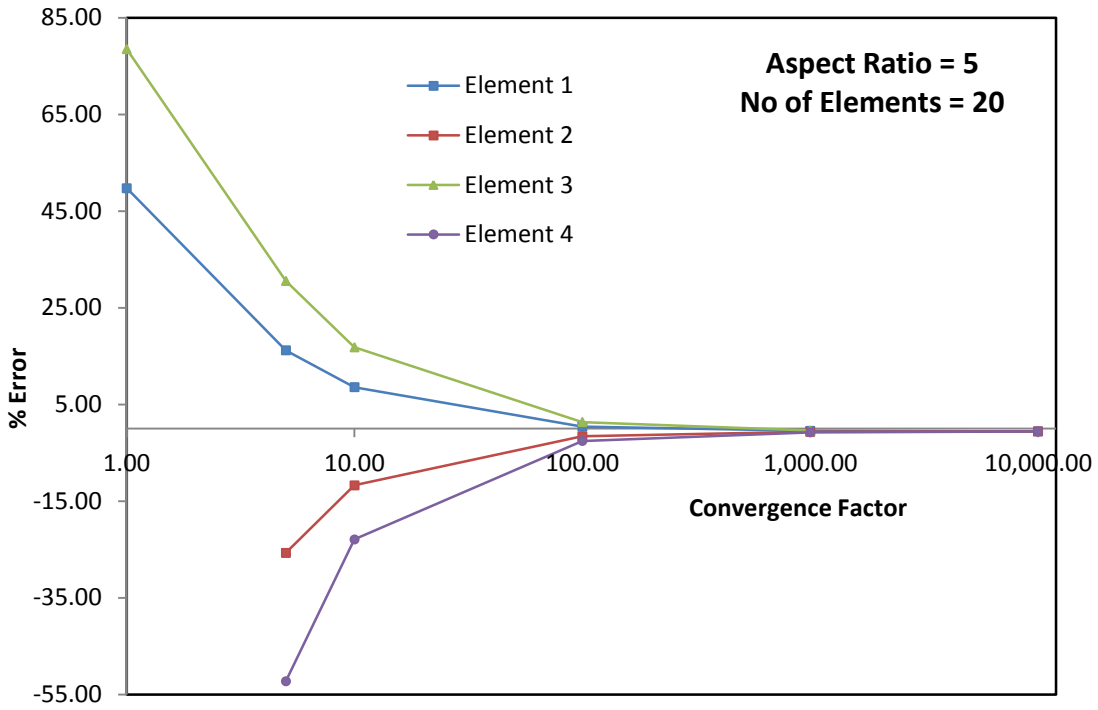


Fig.5

Cantilever beam with tip load for ASR = 10.0
error in deflection at centre

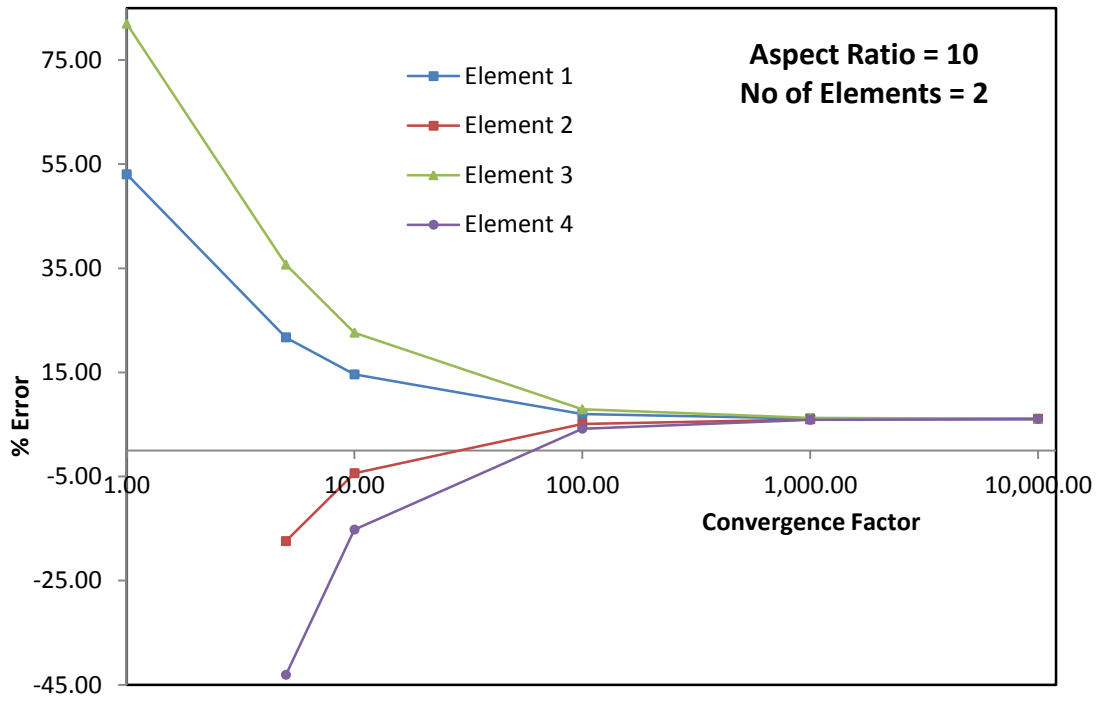


Fig.1

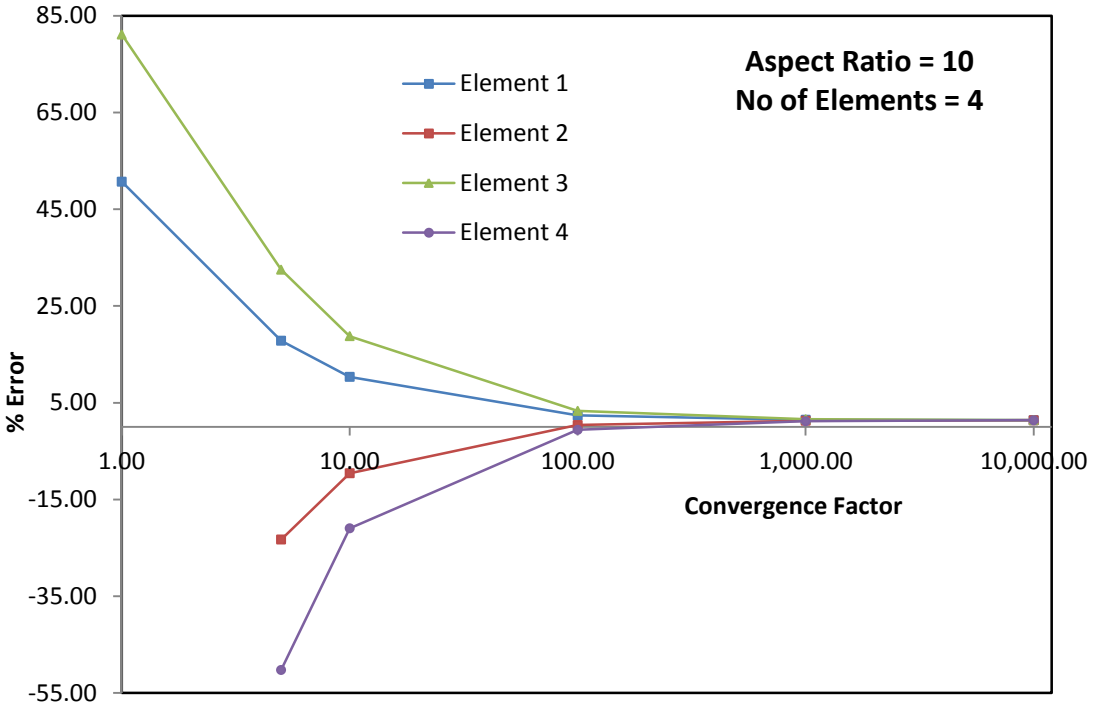

Aspect Ratio = 10
No of Elements = 4
Element 1
Element 2
Element 3
Element 4
% Error
Convergence Factor
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00

Fig.2

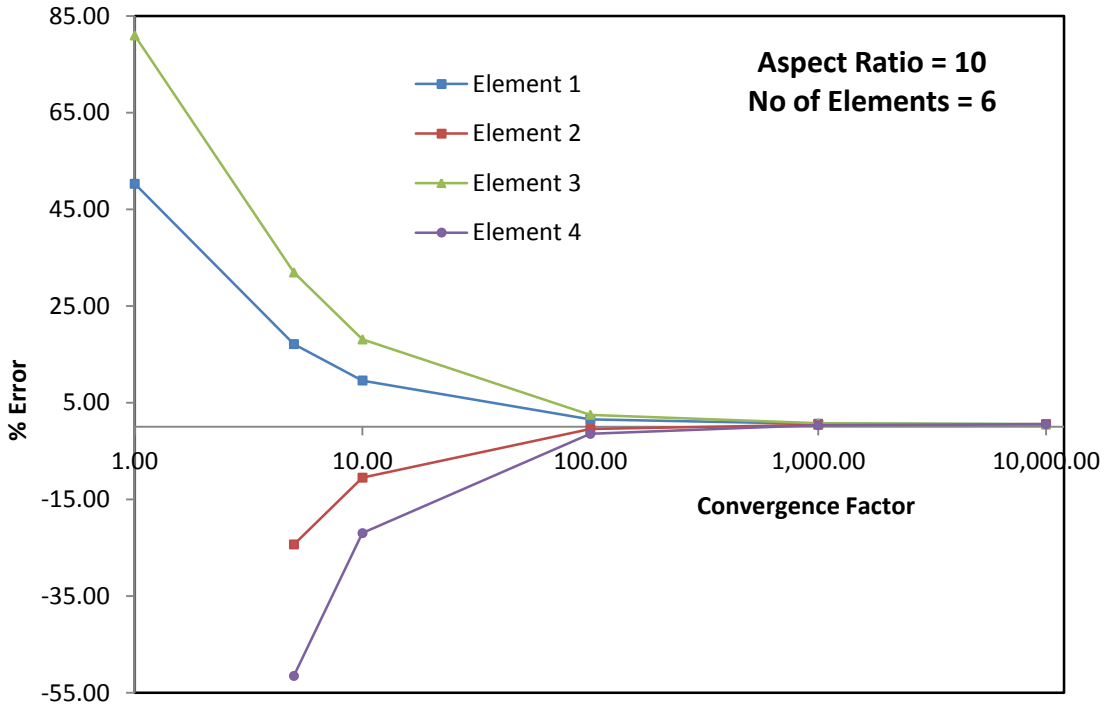

Aspect Ratio = 10
No of Elements = 6
Element 1
Element 2
Element 3
Element 4
% Error
Convergence Factor
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00

Fig.3

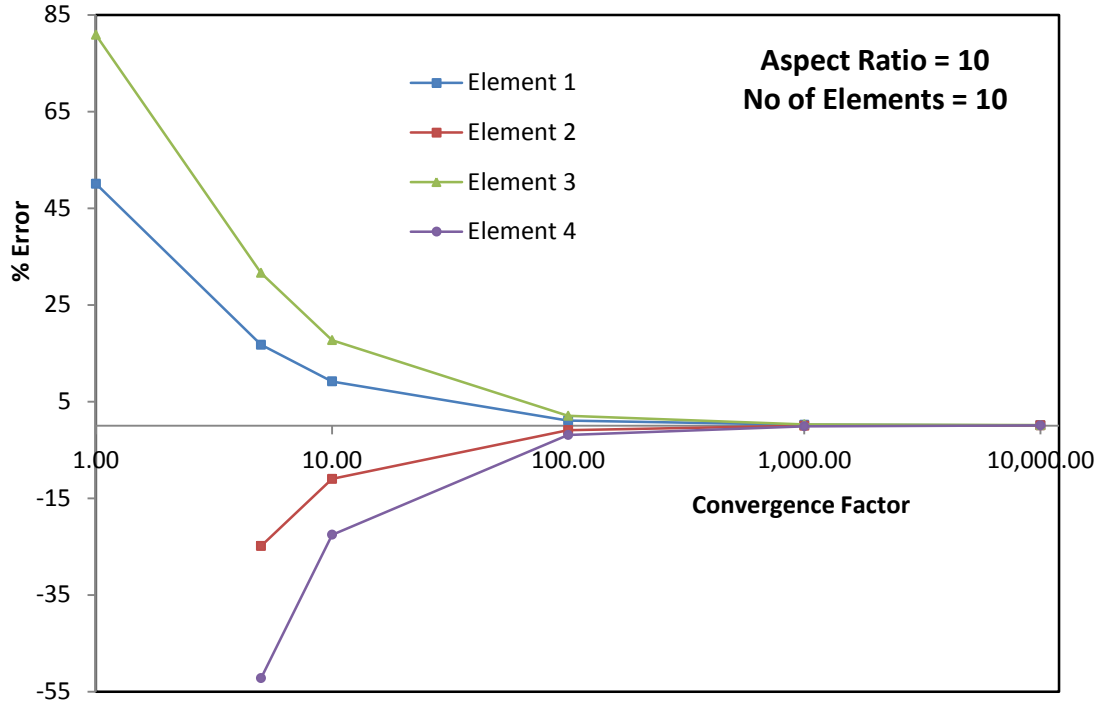

Aspect Ratio = 10
No of Elements = 10
Element 1
Element 2
Element 3
Element 4
% Error
85
65
45
25
5
-15
-35
-55
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.4

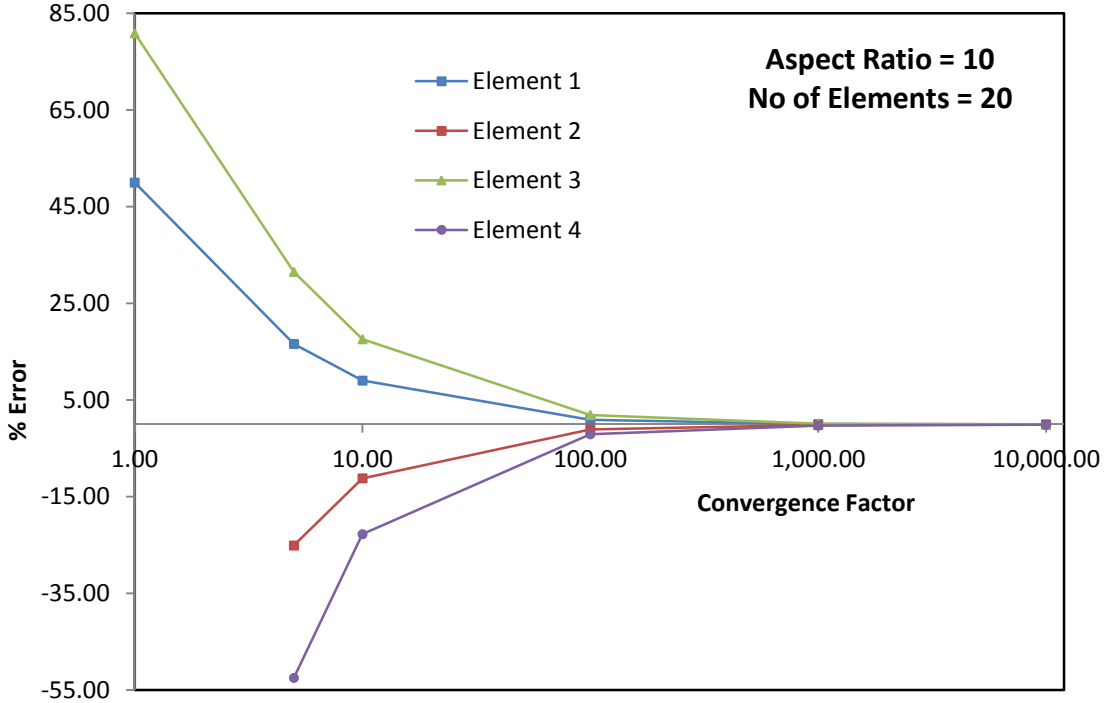

Aspect Ratio = 10
No of Elements = 20
Element 1
Element 2
Element 3
Element 4
% Error
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.5

Cantilever beam with tip load for ASR =25.0
error in deflection at centre

Fig.1

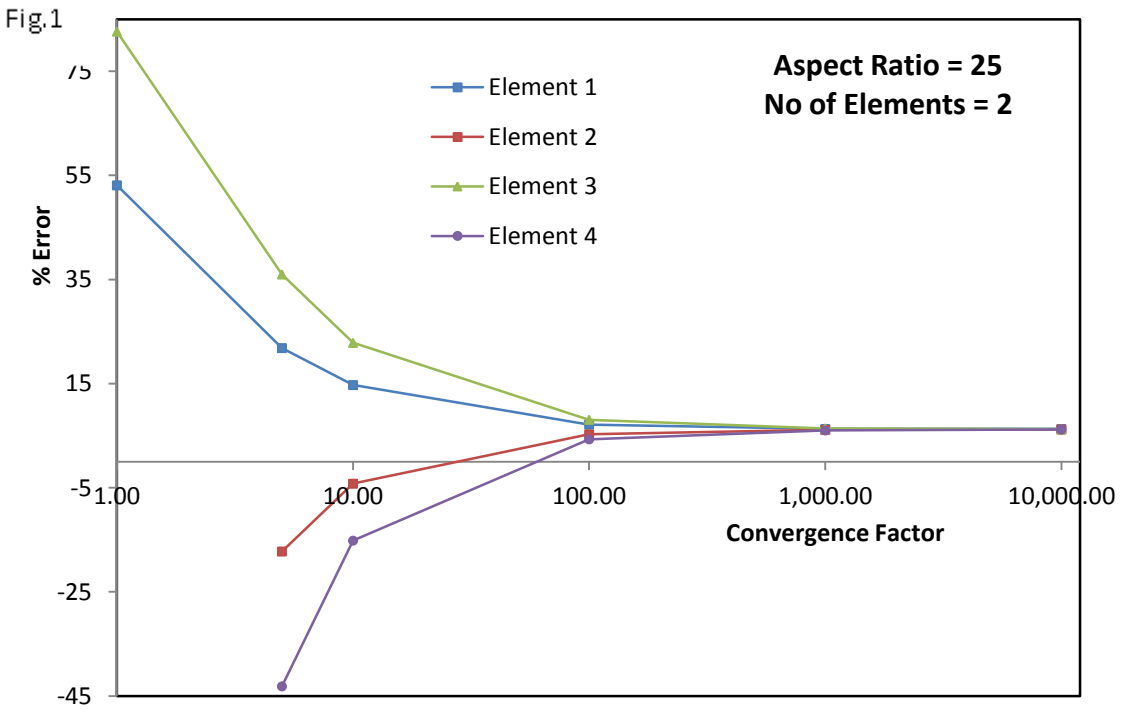


Fig.1

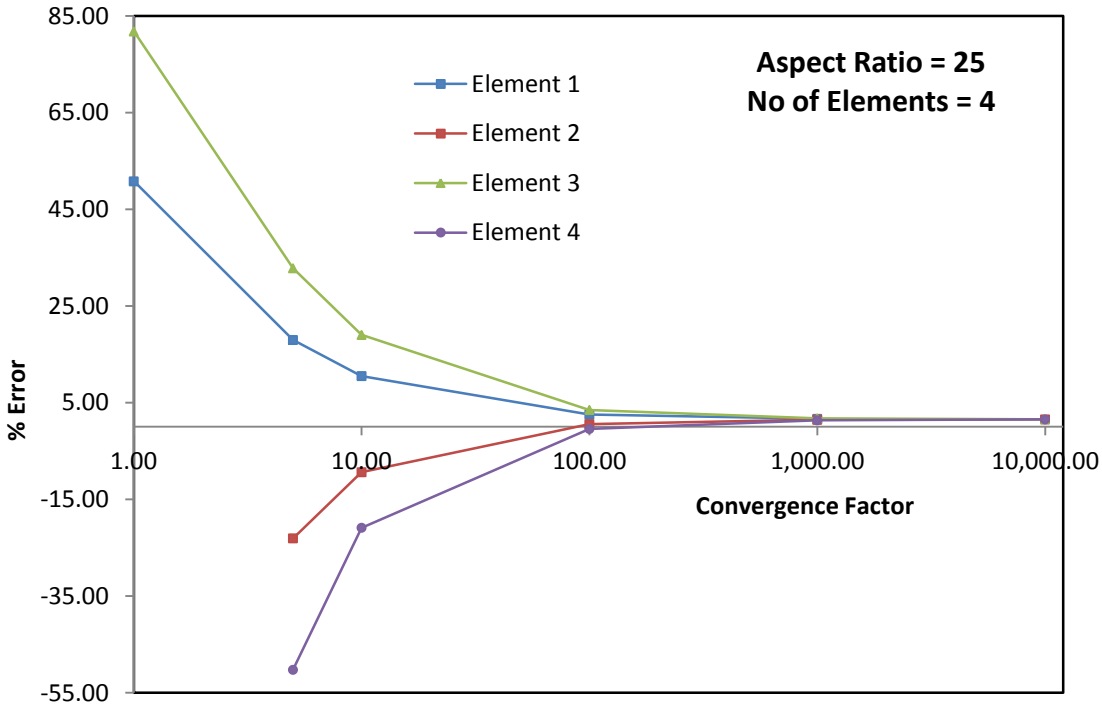


Fig.2

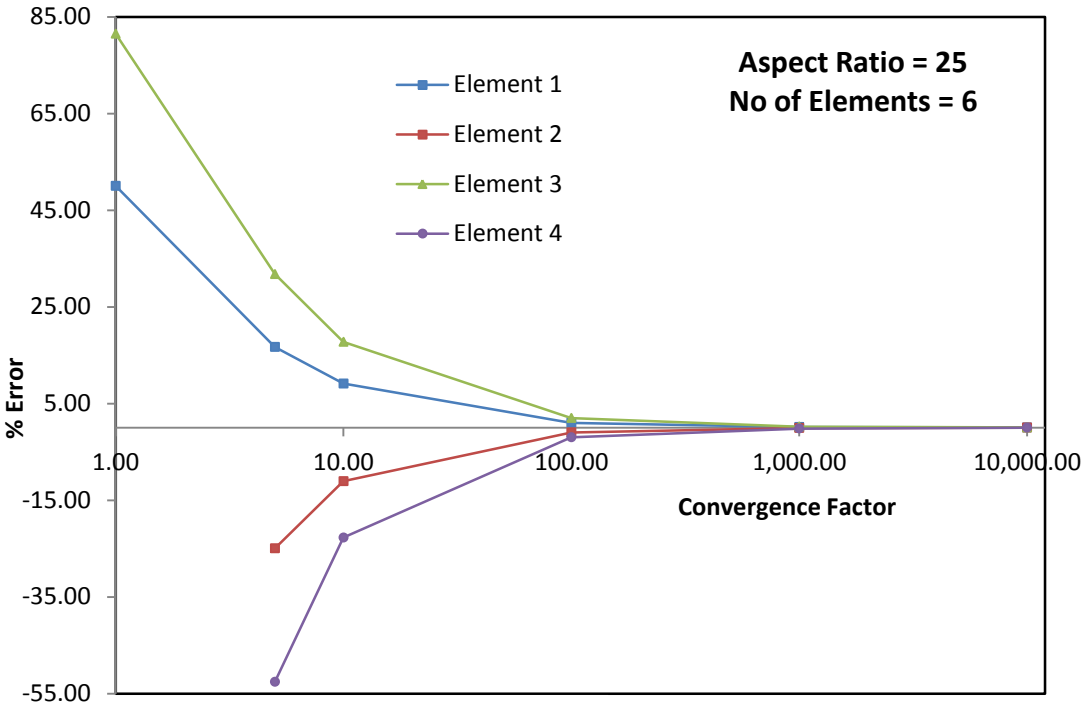

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 25
No of Elements = 6

Fig.3

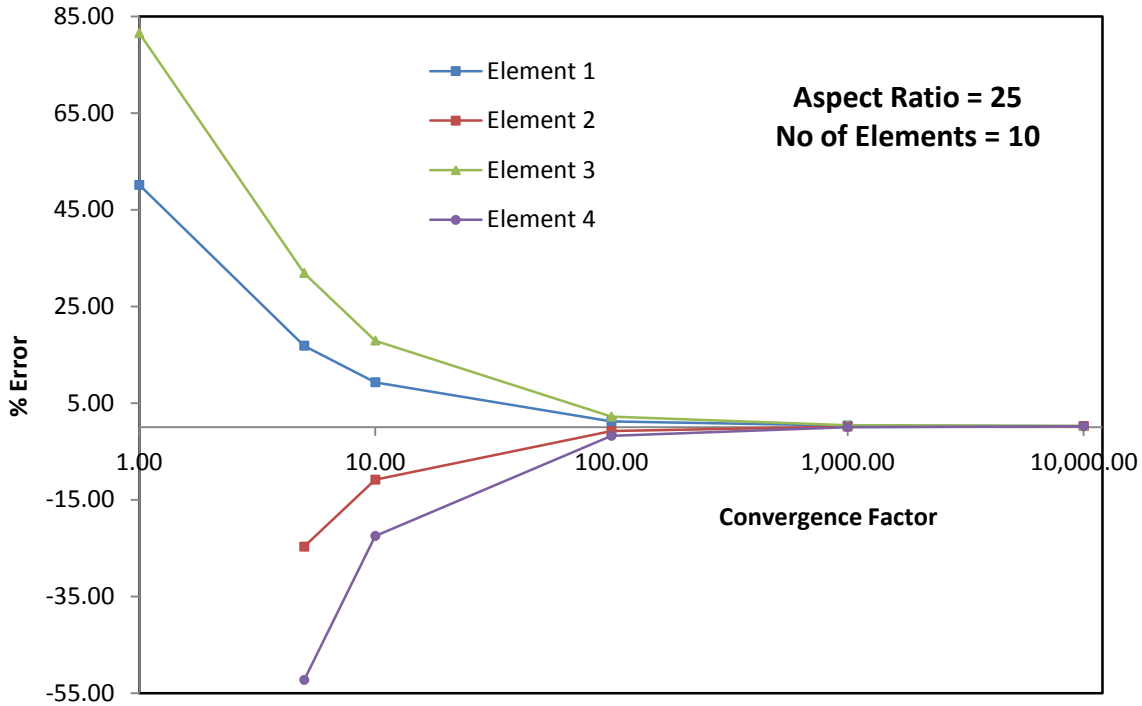

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 25
No of Elements = 10

Fig.4

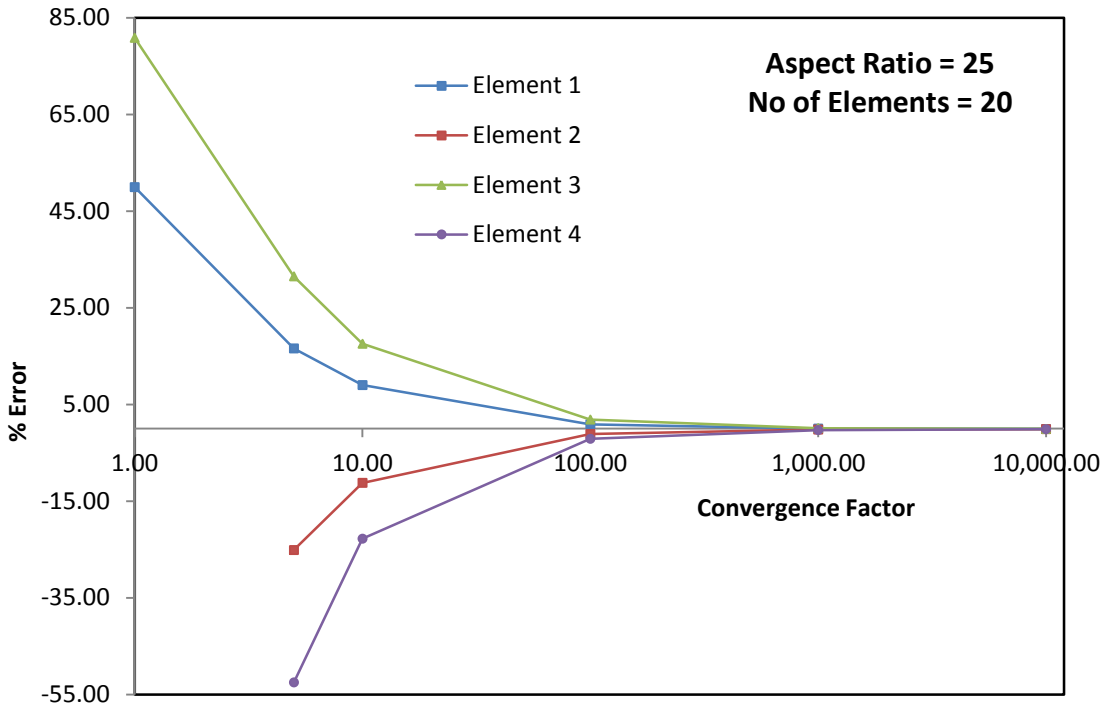


Fig.4

Cantilever beam with tip load for ASR =100.0
error in deflection at centre

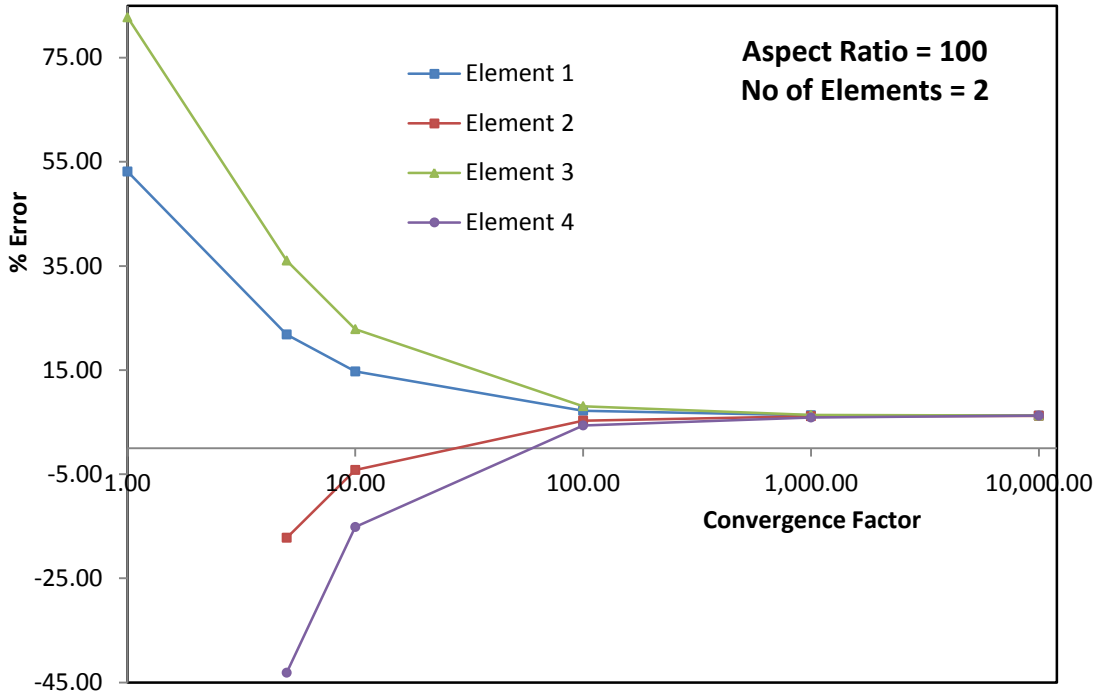


Fig.1

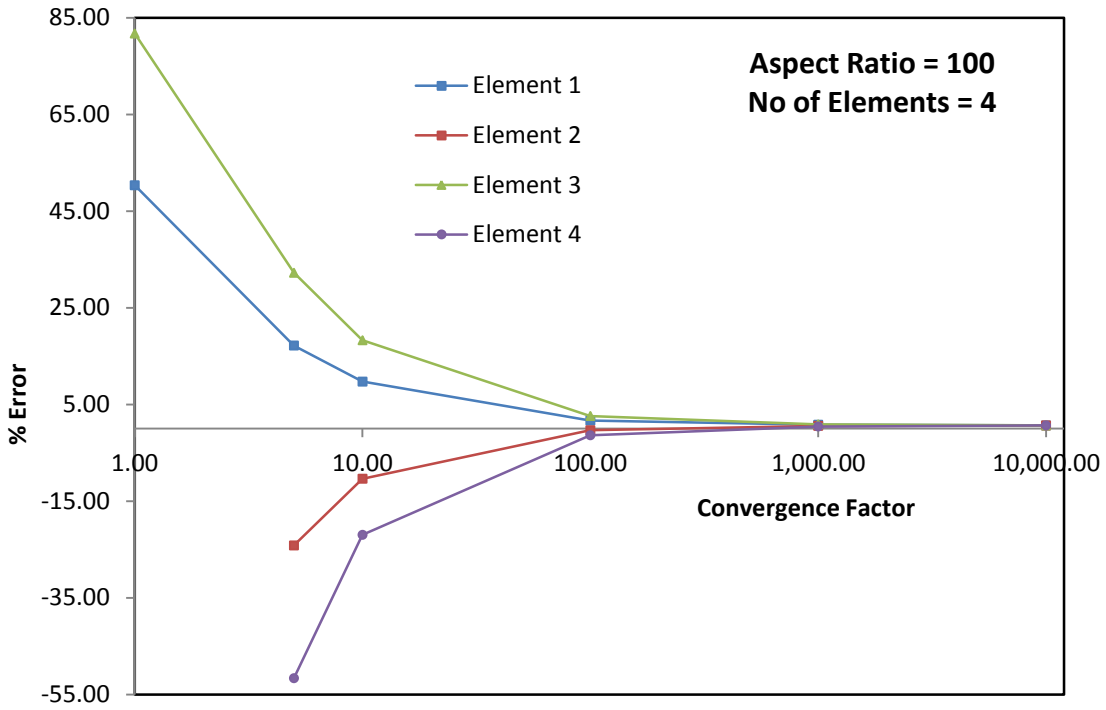

Aspect Ratio = 100
No of Elements = 4
Element 1
Element 2
Element 3
Element 4
% Error
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.2

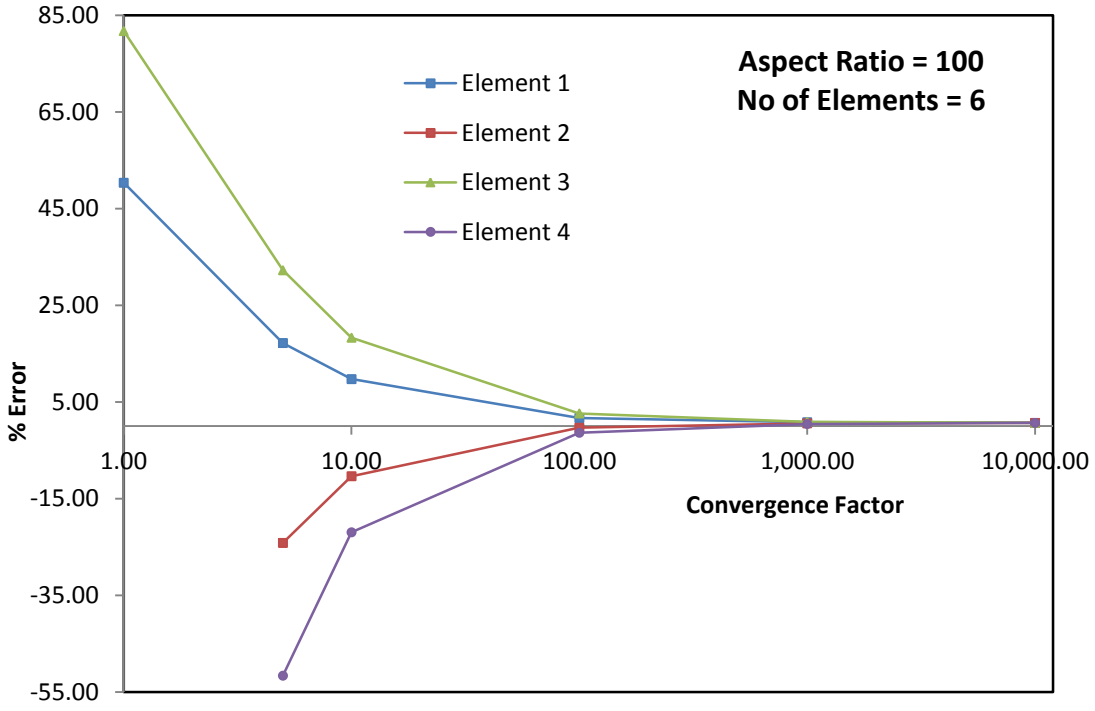

Aspect Ratio = 100
No of Elements = 6
Element 1
Element 2
Element 3
Element 4
% Error
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.3

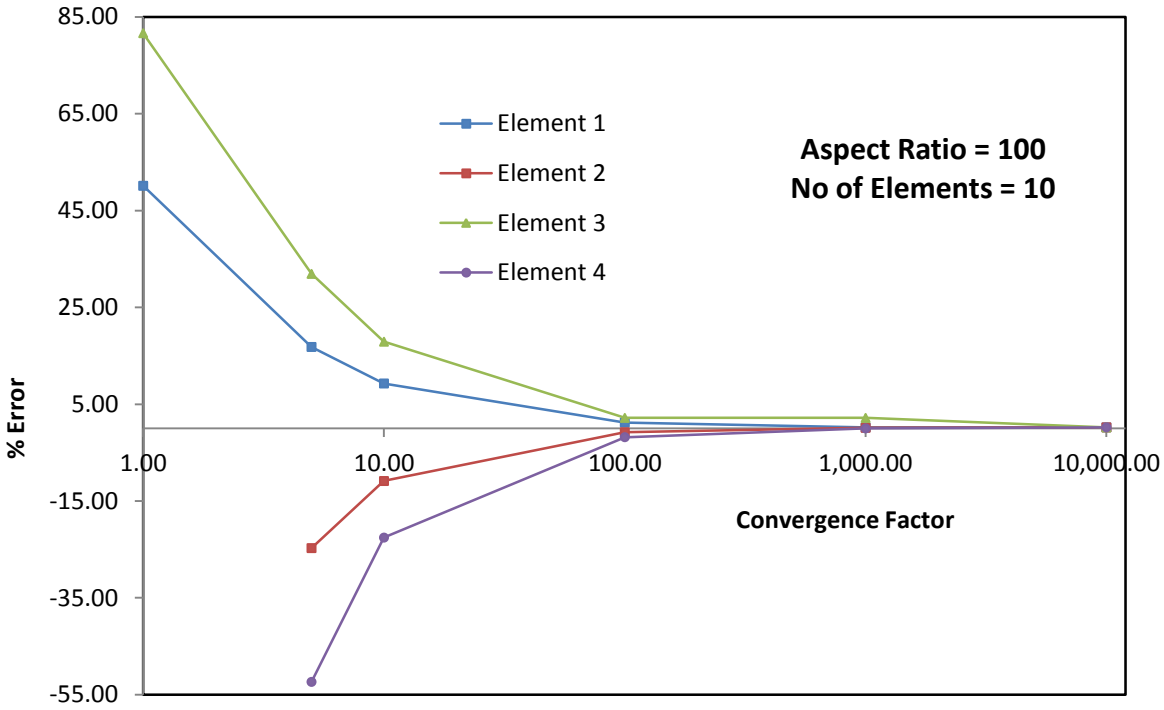

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 100
No of Elements = 10

Fig.4

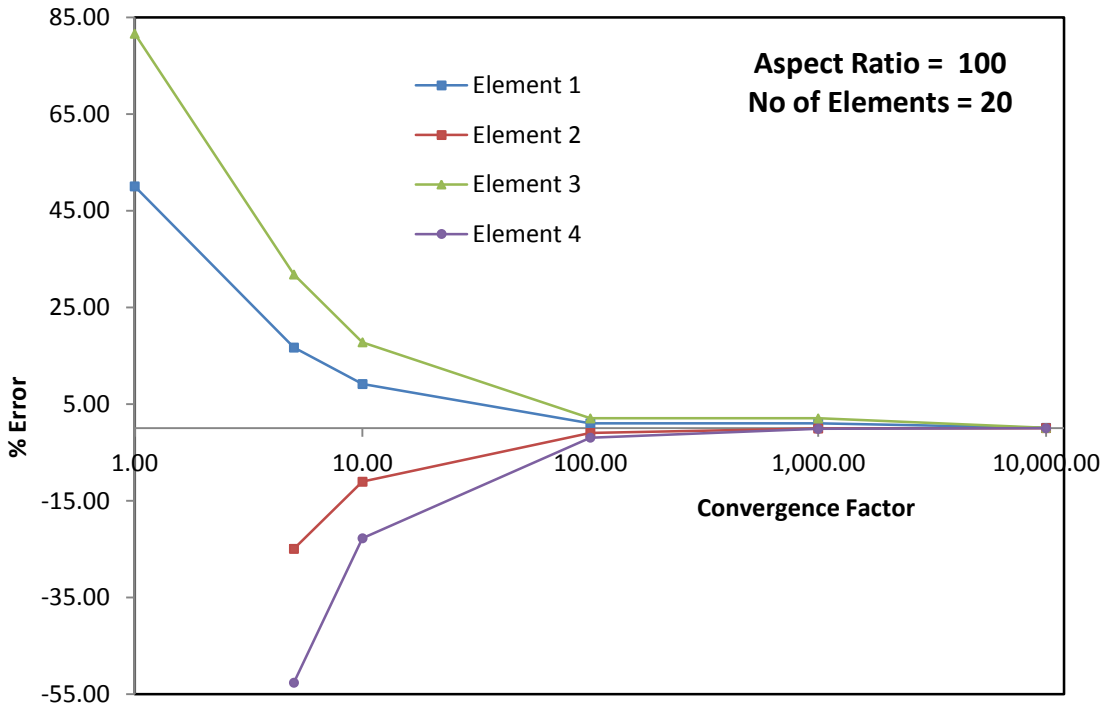

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 100
No of Elements = 20

Fig.5

Cantilever beam with tip load for ASR = 200.0
error in deflection at centre

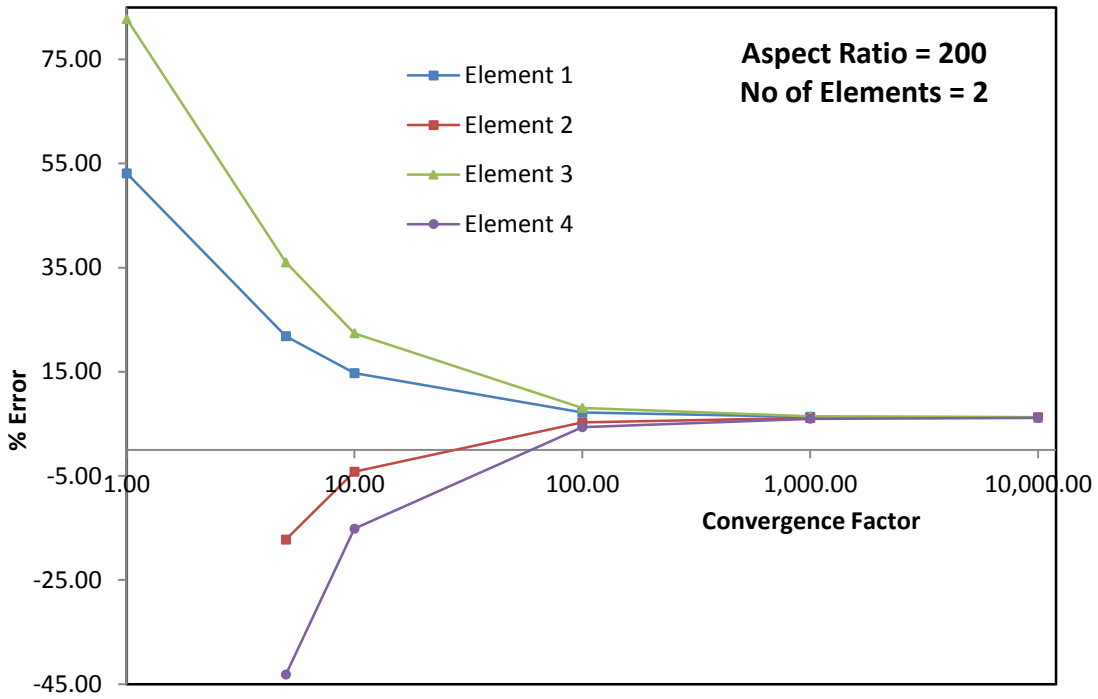


Fig.1

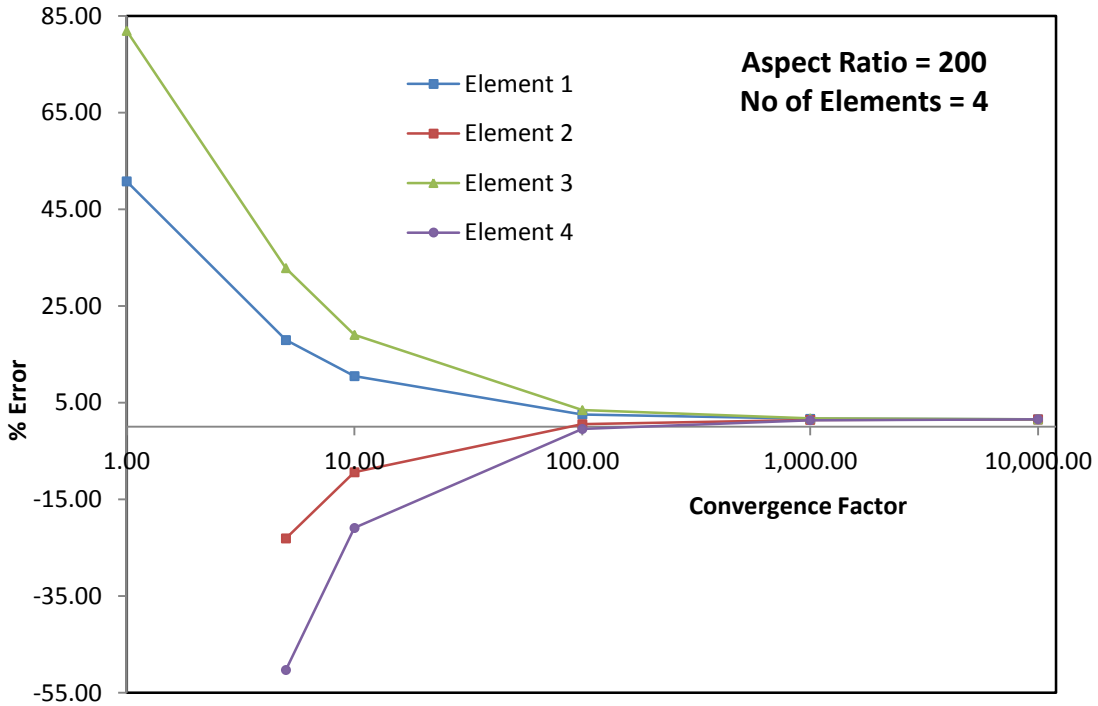


Fig.2

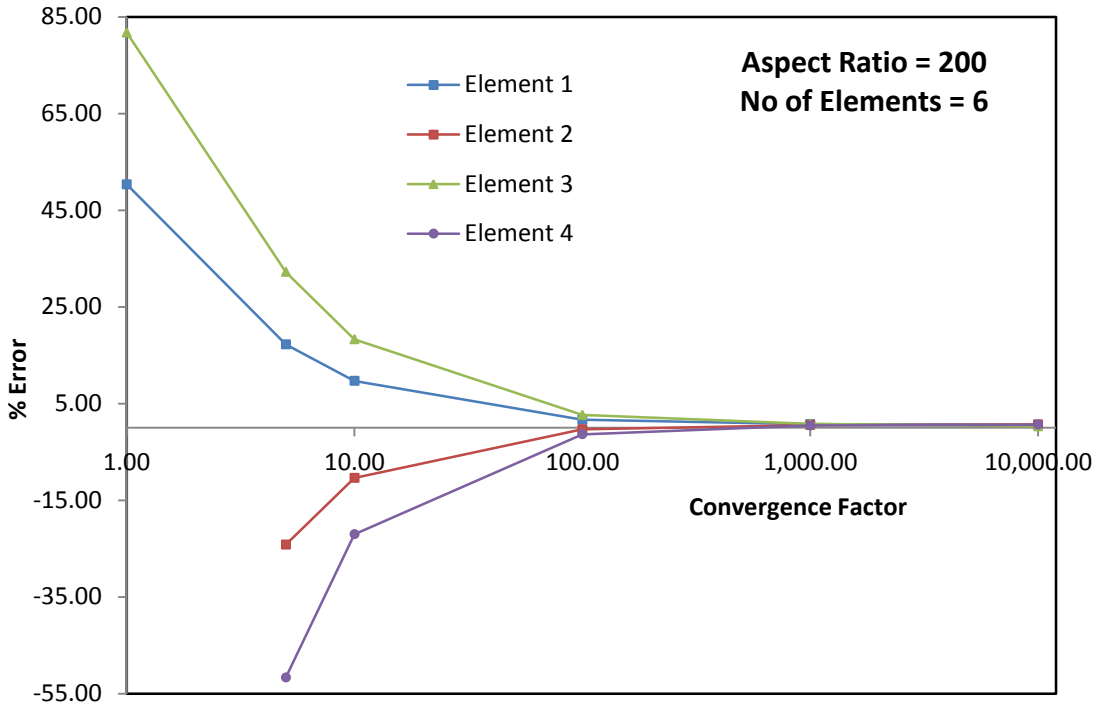

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 200
No of Elements = 6

Fig.3

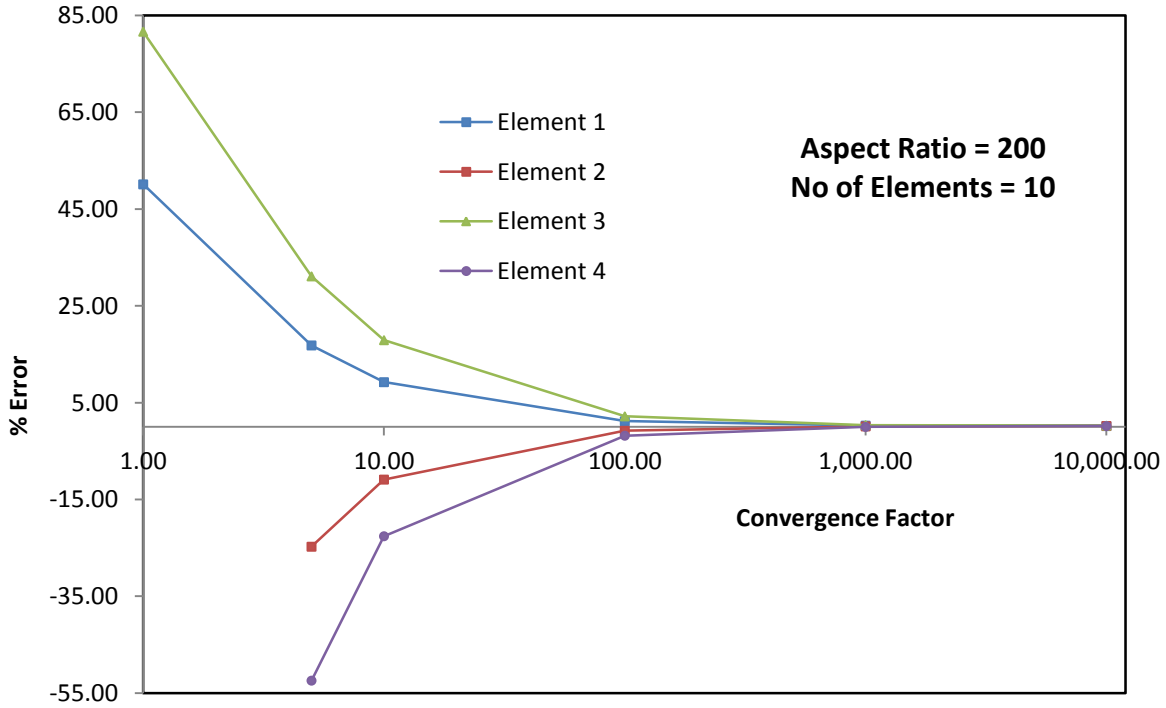

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 200
No of Elements = 10

Fig.4

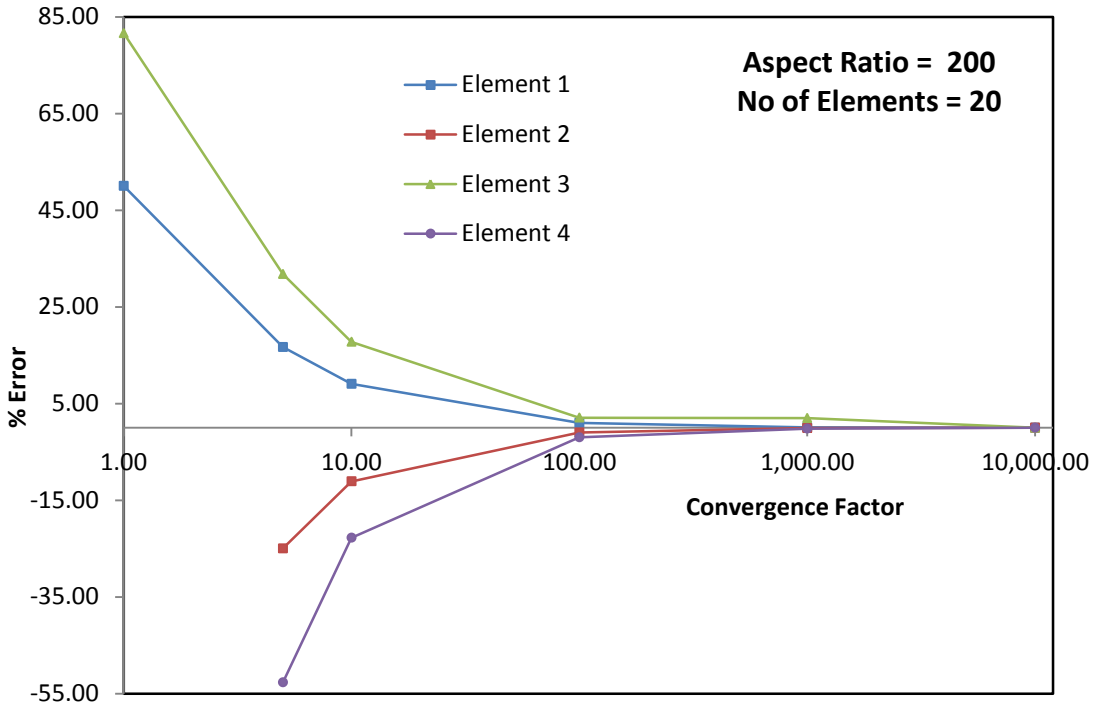


Fig.5

Cantilever beam with tip load for ASR =400.0
error in deflection at centre

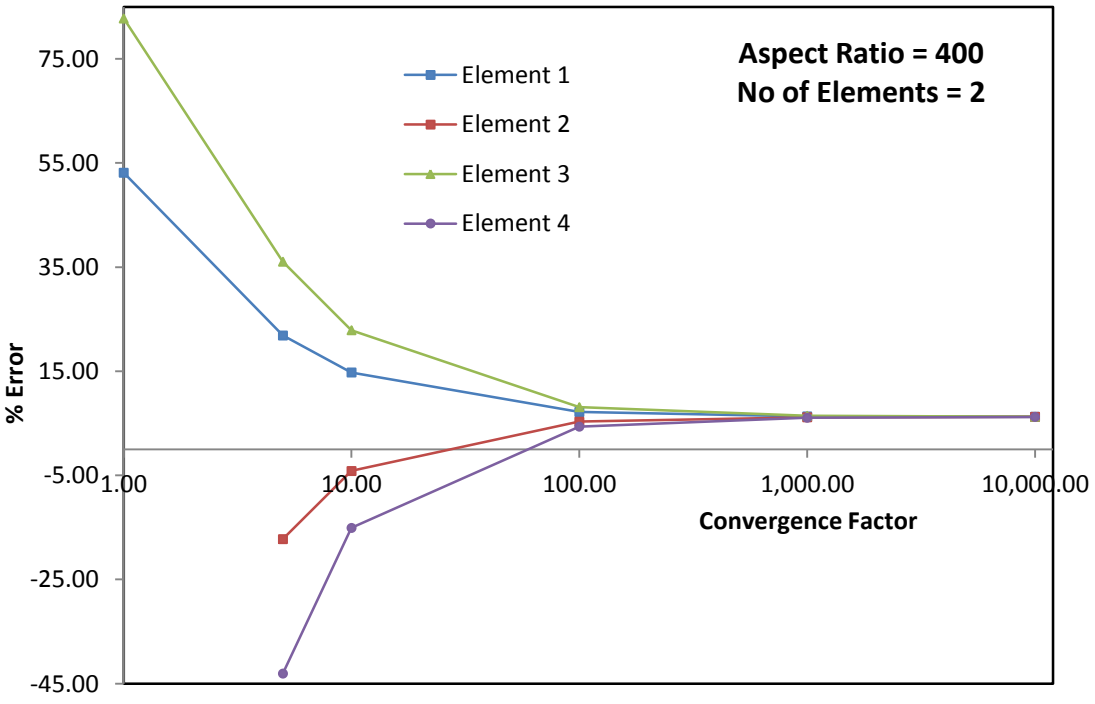


Fig.1

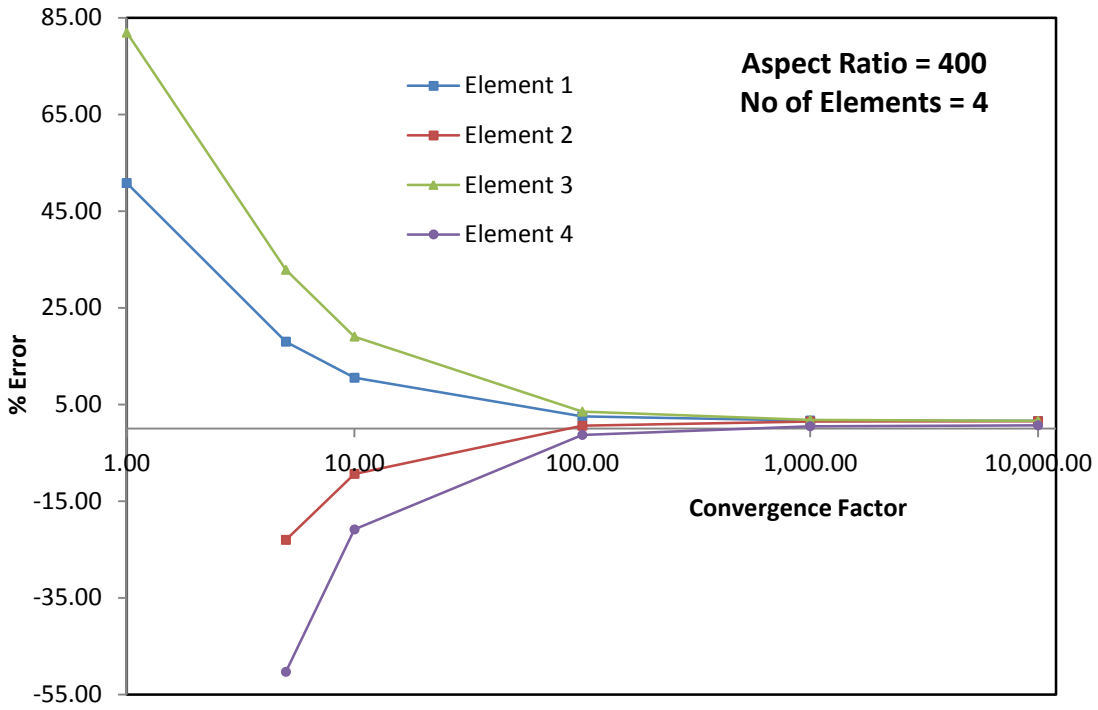

Aspect Ratio = 400
No of Elements = 4
Element 1
Element 2
Element 3
Element 4
% Error
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.2

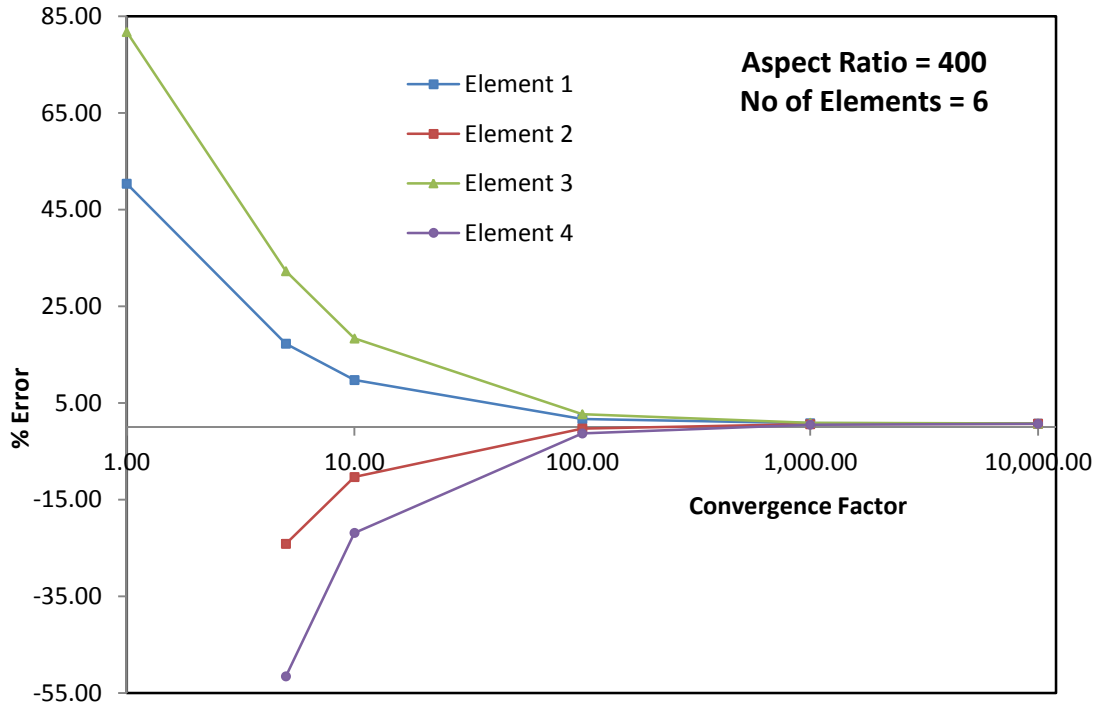

Aspect Ratio = 400
No of Elements = 6
Element 1
Element 2
Element 3
Element 4
% Error
85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.3

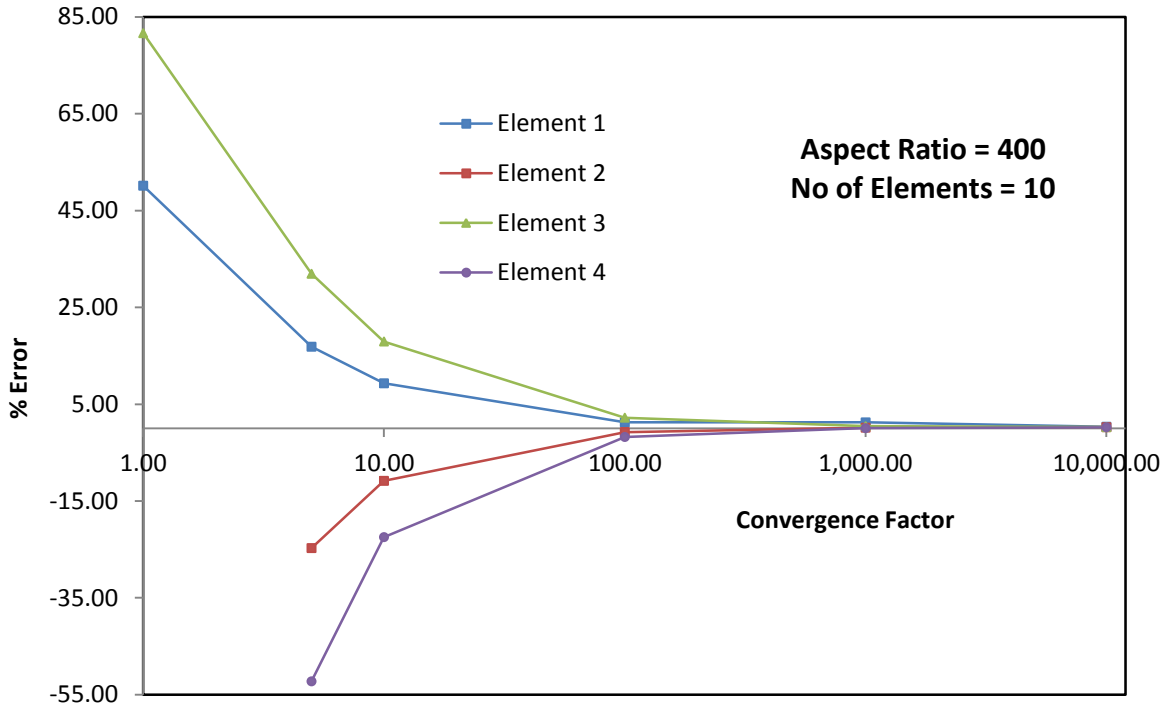

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 400
No of Elements = 10

Fig.4

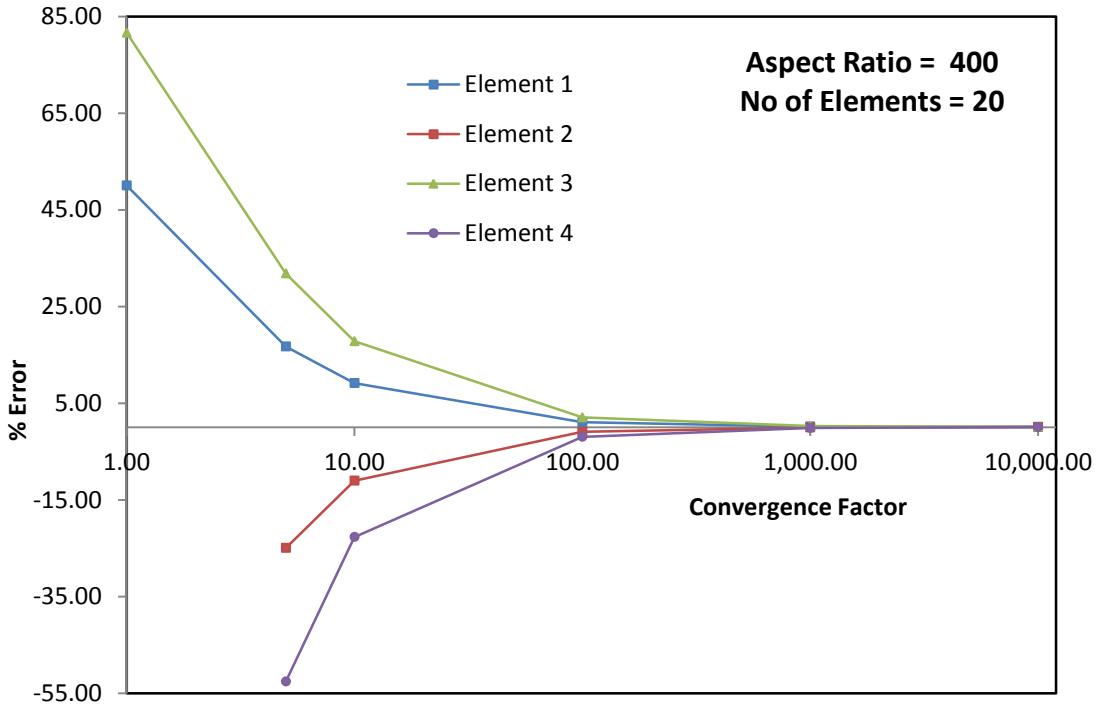

85.00
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 400
No of Elements = 20

Fig.5

Plate

Simply supported plate with UDL for ASR = 5.0

error in deflection at centre

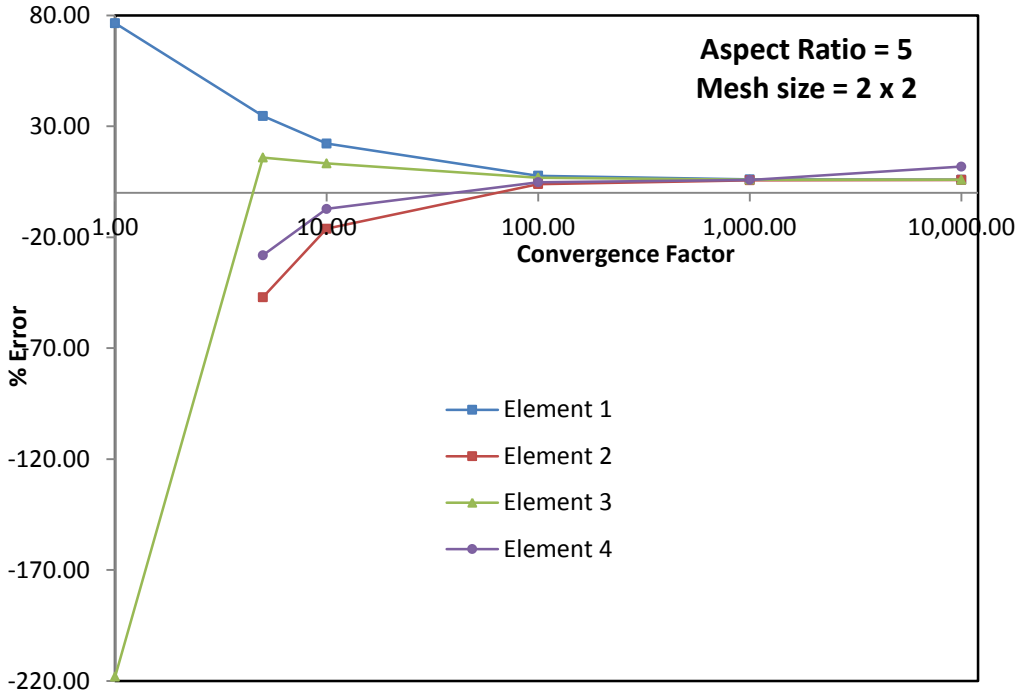


Fig.1

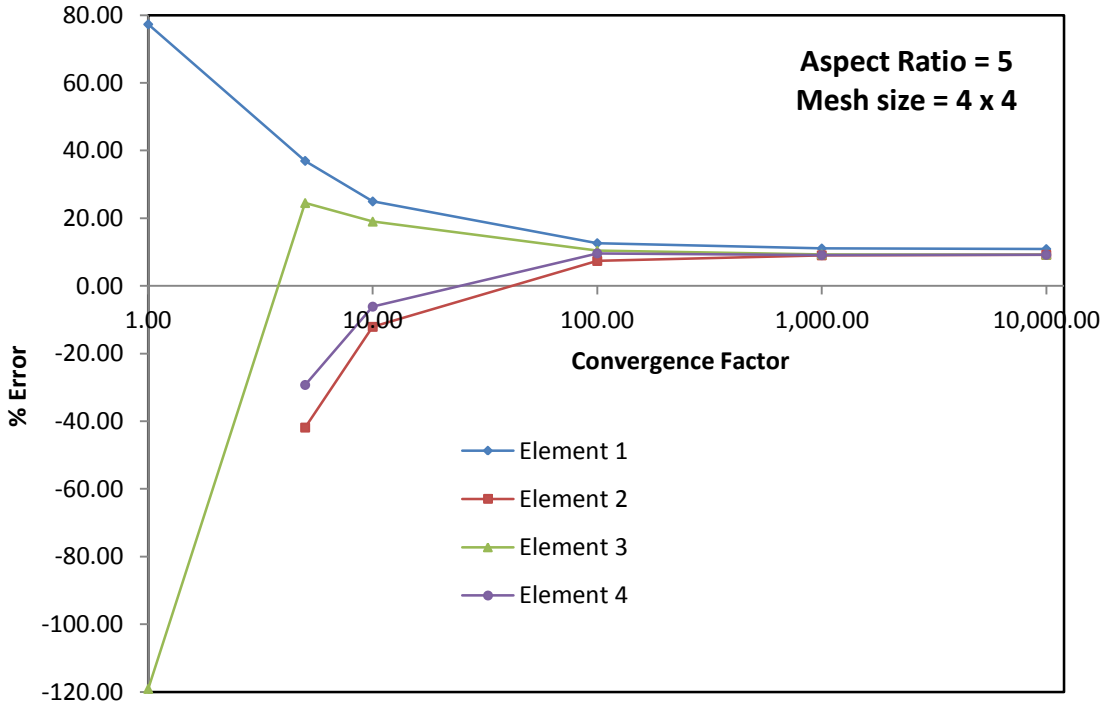


Fig.2

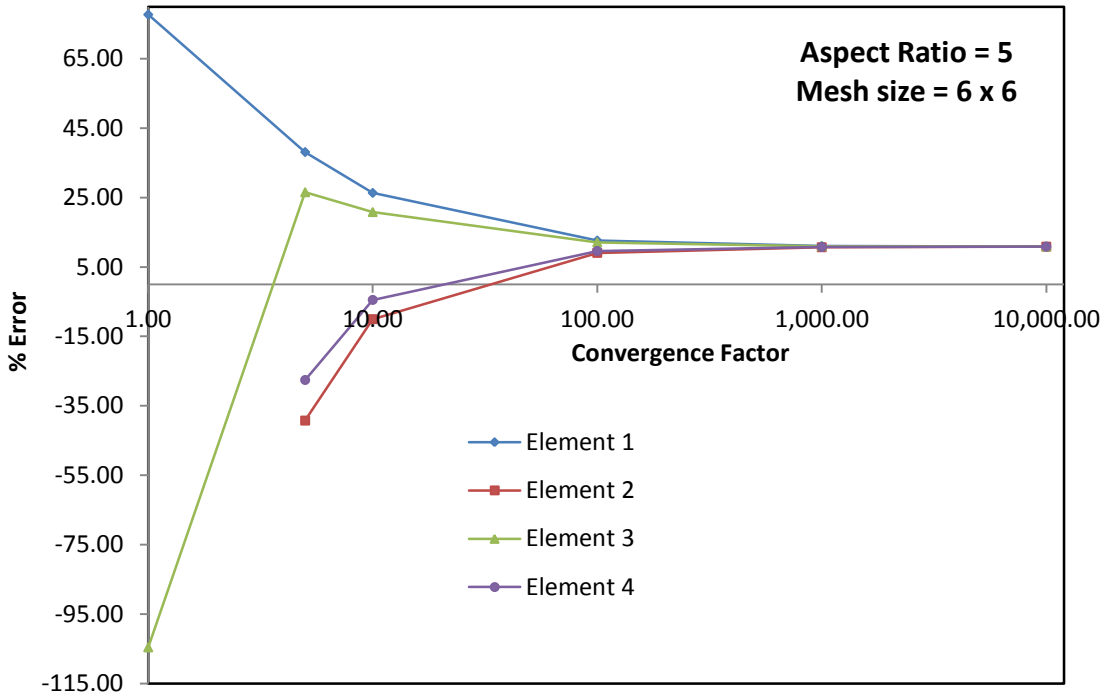

Aspect Ratio = 5
Mesh size = 6 x 6
% Error
65.00
45.00
25.00
5.00
-15.00
-35.00
-55.00
-75.00
-95.00
-115.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4

Fig.3

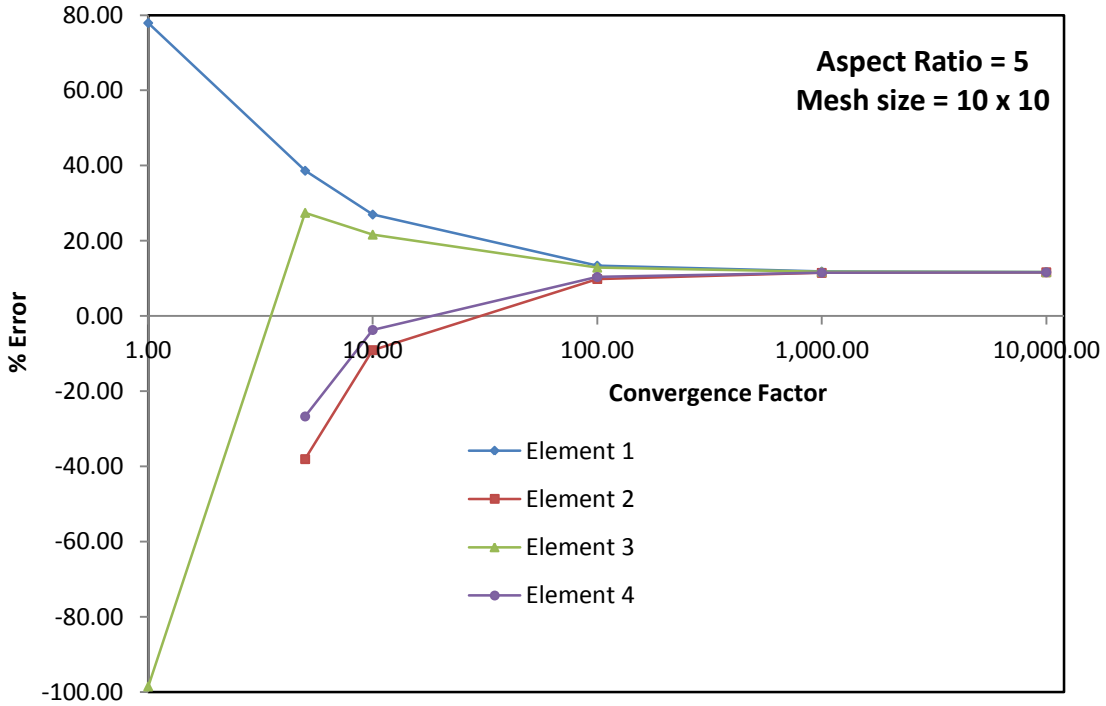

Aspect Ratio = 5
Mesh size = 10 x 10
% Error
80.00
60.00
40.00
20.00
0.00
-20.00
-40.00
-60.00
-80.00
-100.00
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4

Fig.4

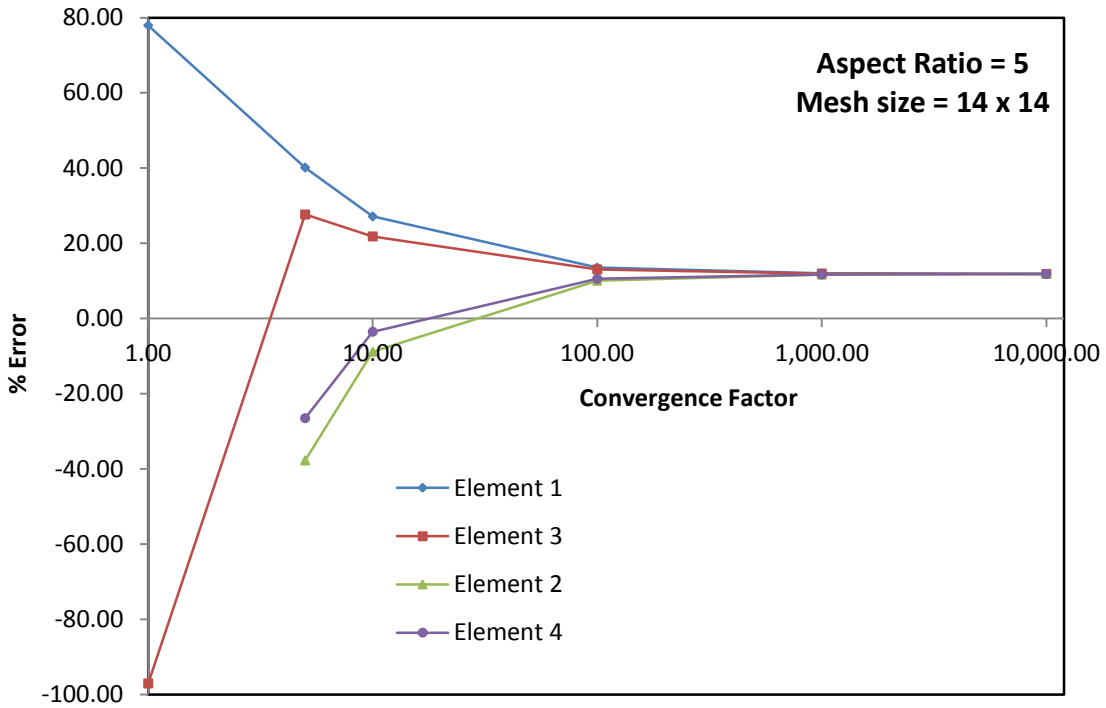


Fig.5

Simply supported plate with UDL for ASR = 7.14
error in deflection at centre

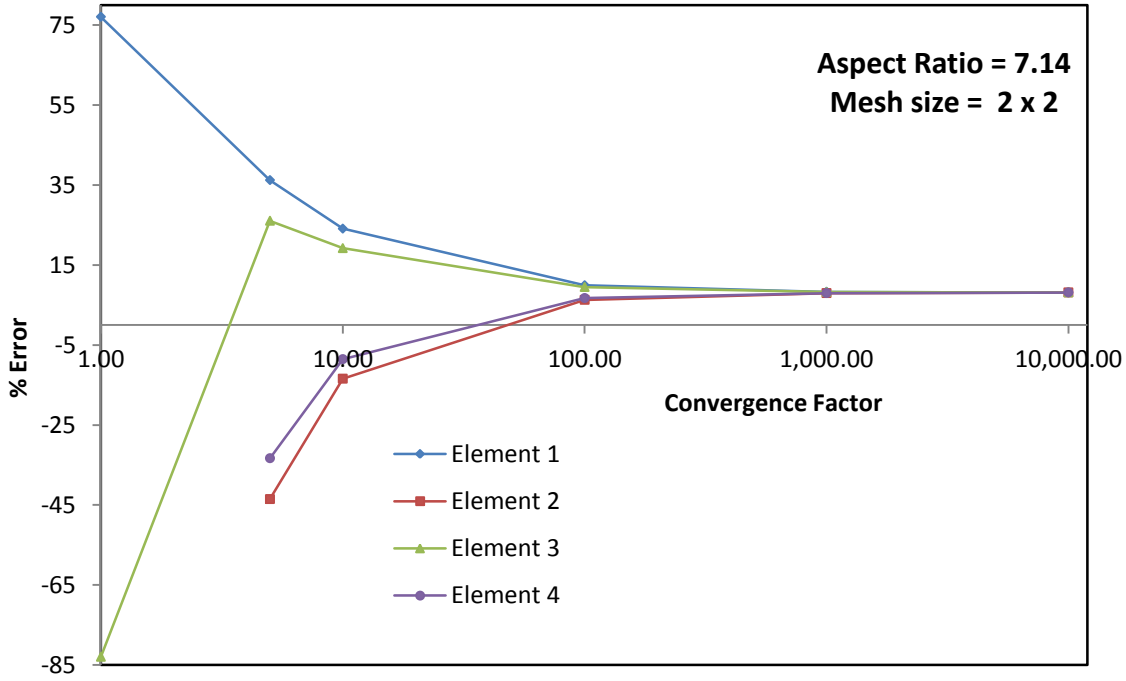


Fig.1

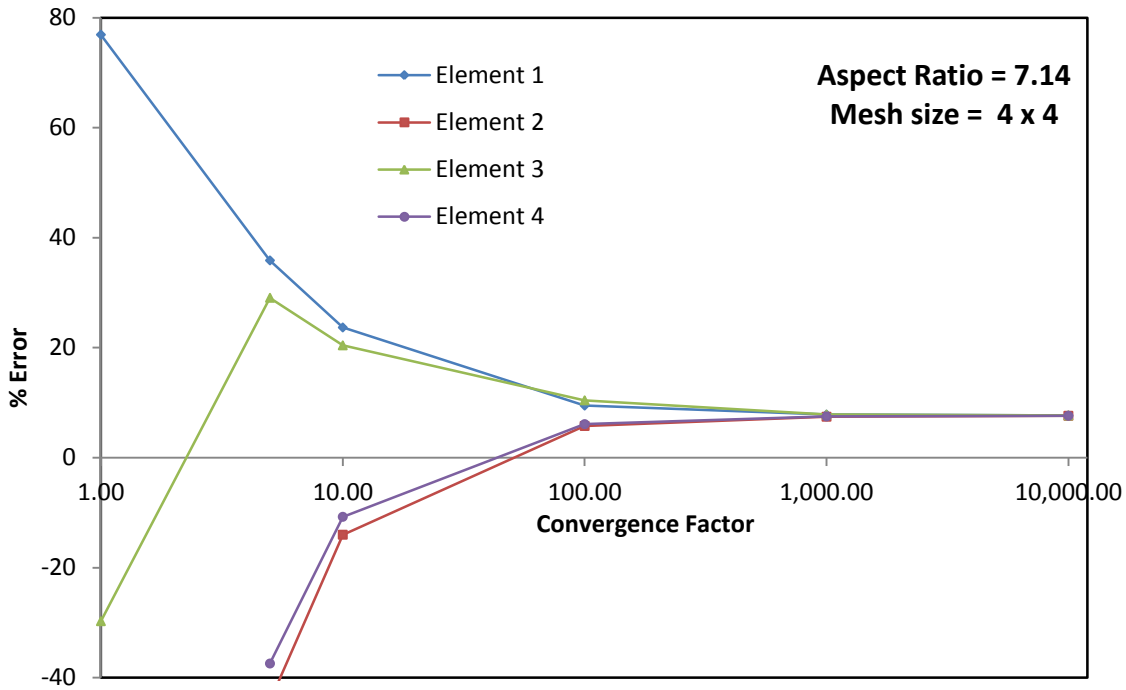

80
60
40
20
0
-20
-40
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 7.14
Mesh size = 4 x 4

Fig.2

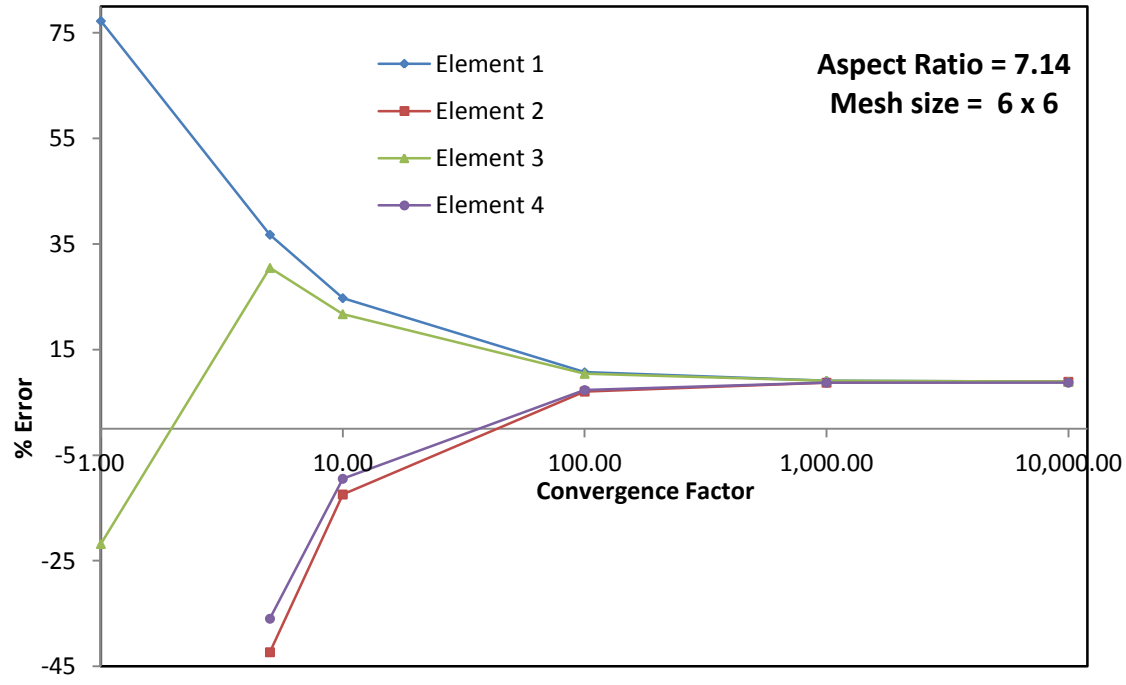

75
55
35
15
-5
-25
-45
% Error
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor
Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 7.14
Mesh size = 6 x 6

Fig.3

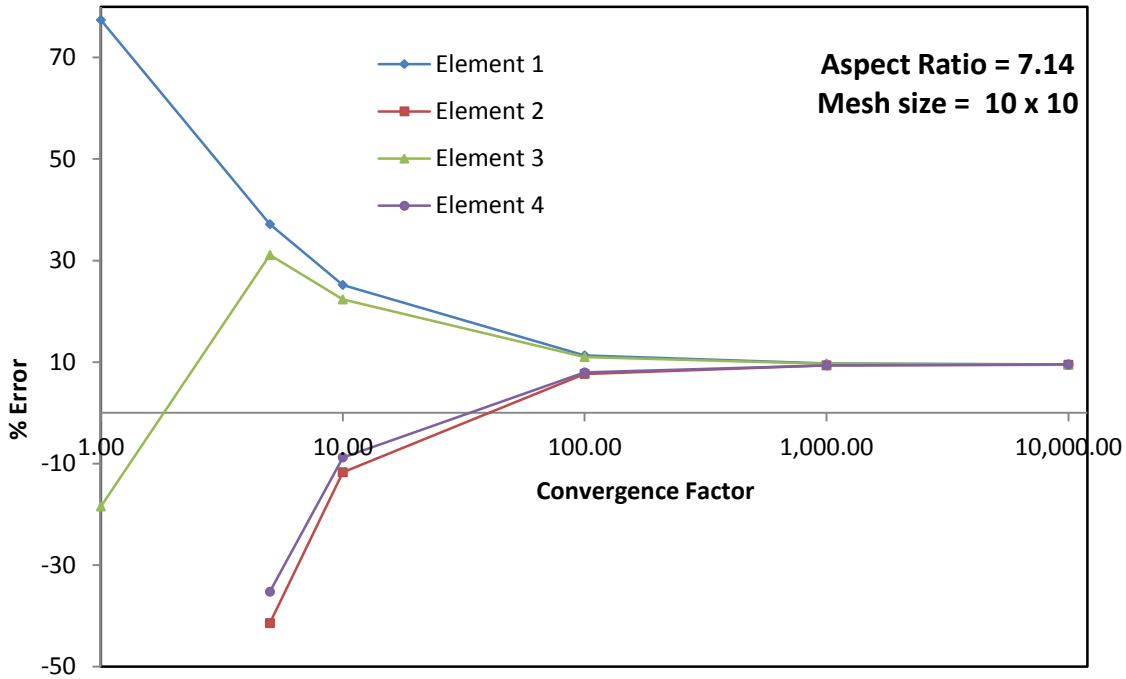

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 7.14
Mesh size = 10 x 10
% Error
Convergence Factor
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00

Fig.4

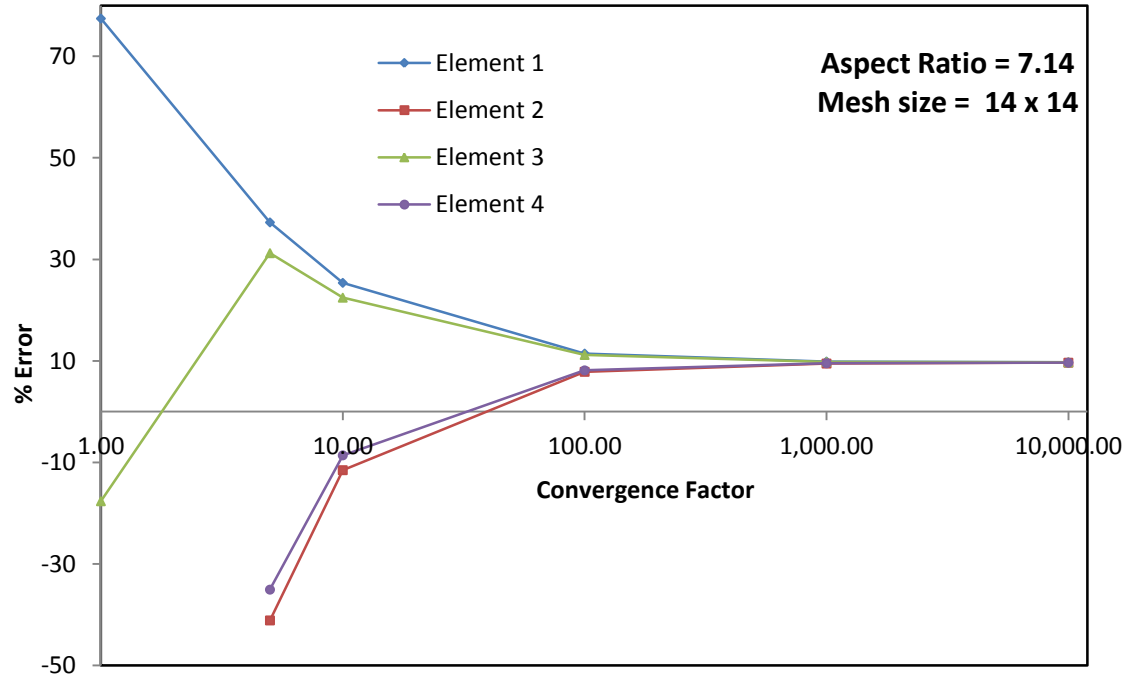

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 7.14
Mesh size = 14 x 14
% Error
Convergence Factor
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00

Fig.5

Simply supported plate with UDL for ASR =10.0
error in deflection at centre

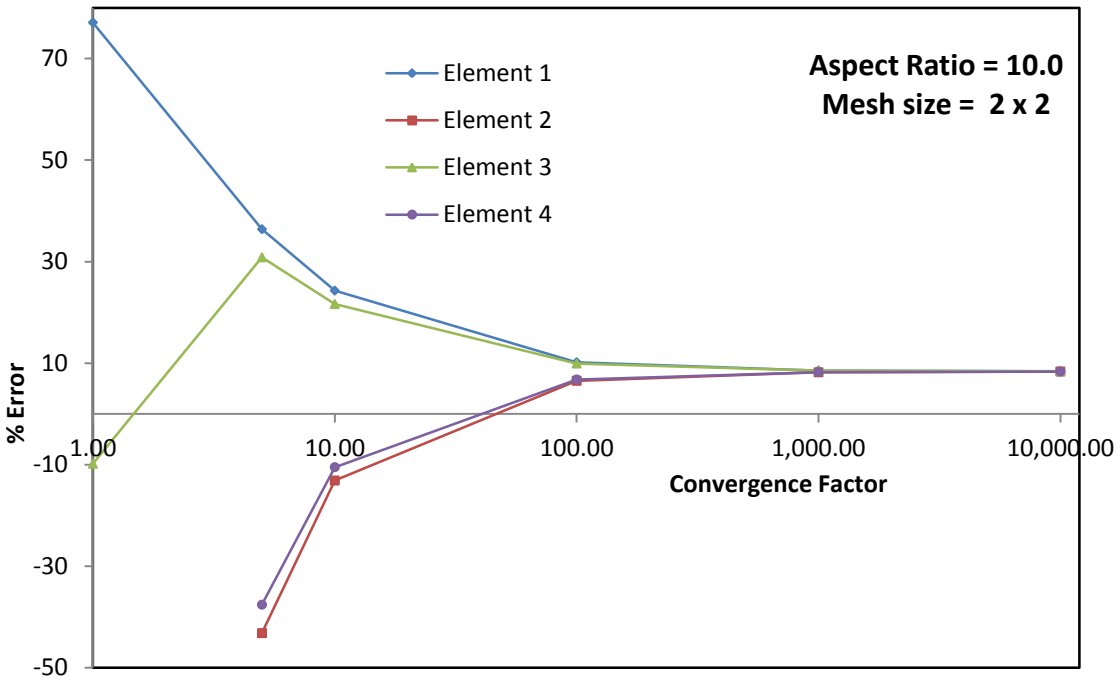


Fig.1

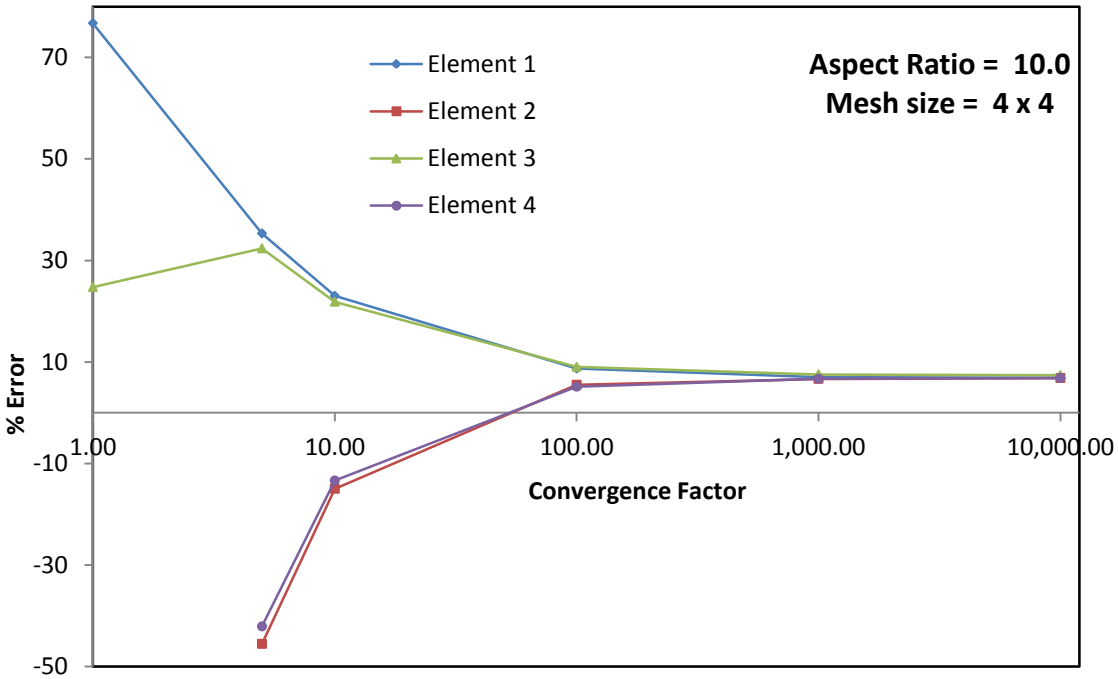


Fig.2

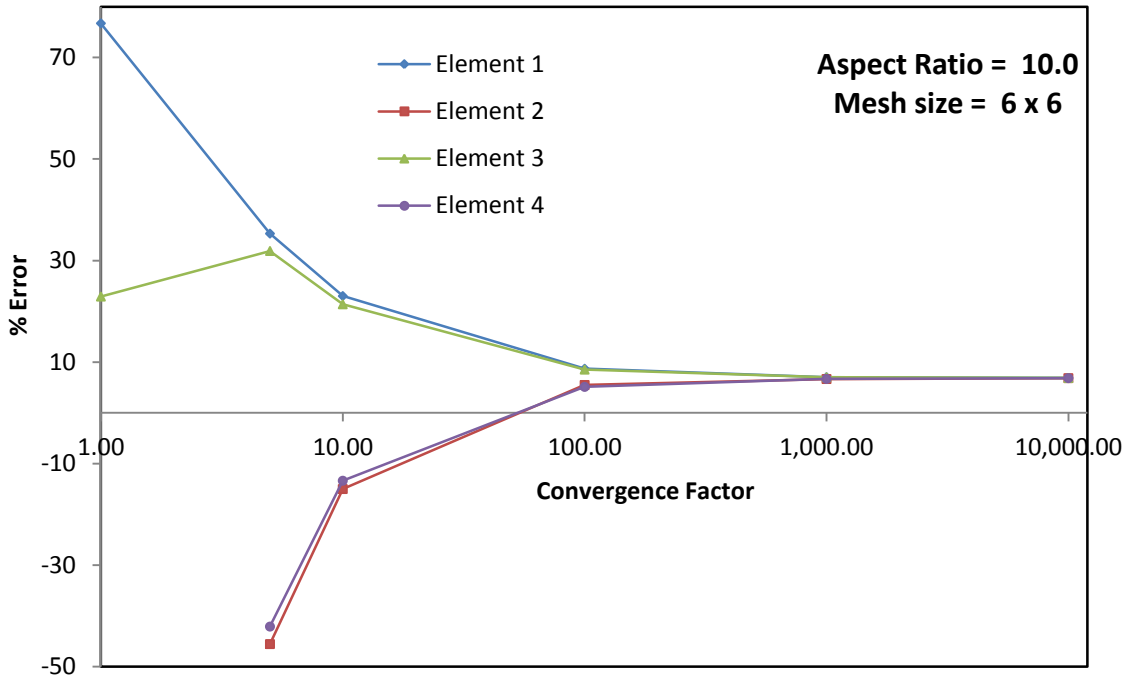

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 10.0
Mesh size = 6 x 6
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.3

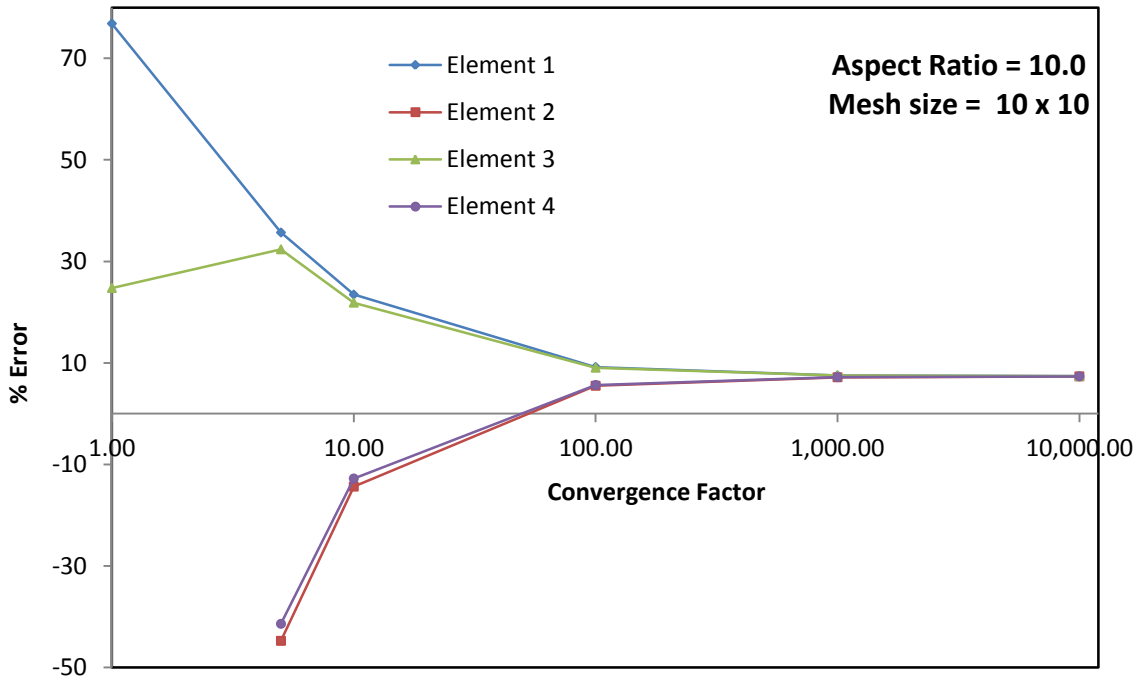

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 10.0
Mesh size = 10 x 10
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.4

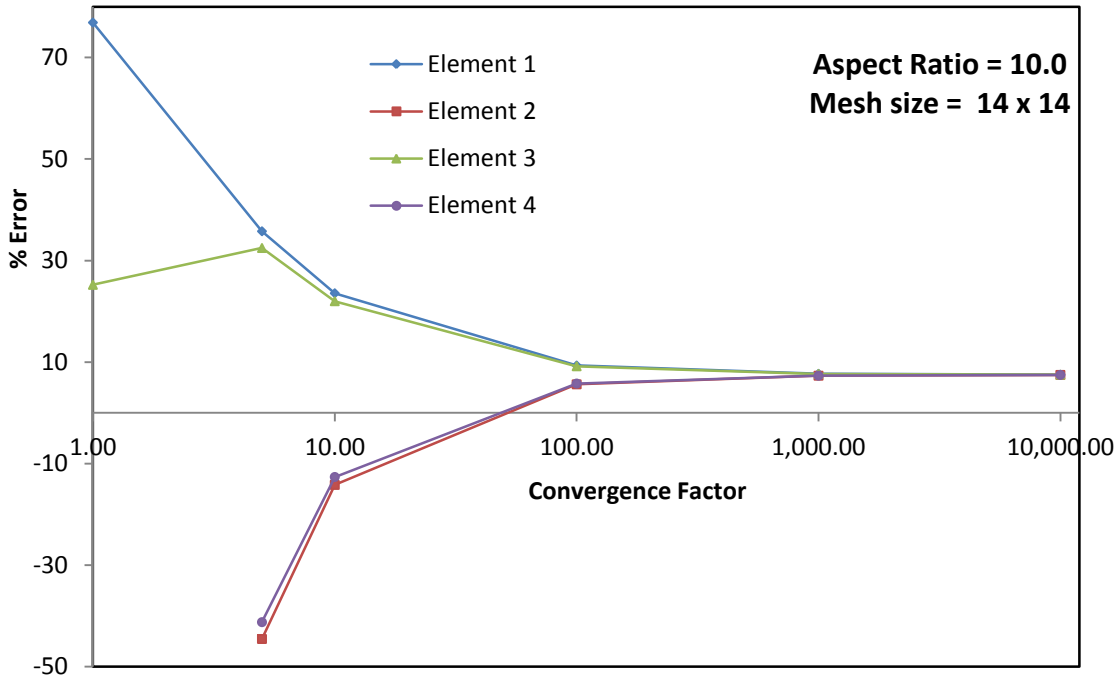


Fig.5

Simply supported plate with UDL for ASR =20.0
error in deflection at centre

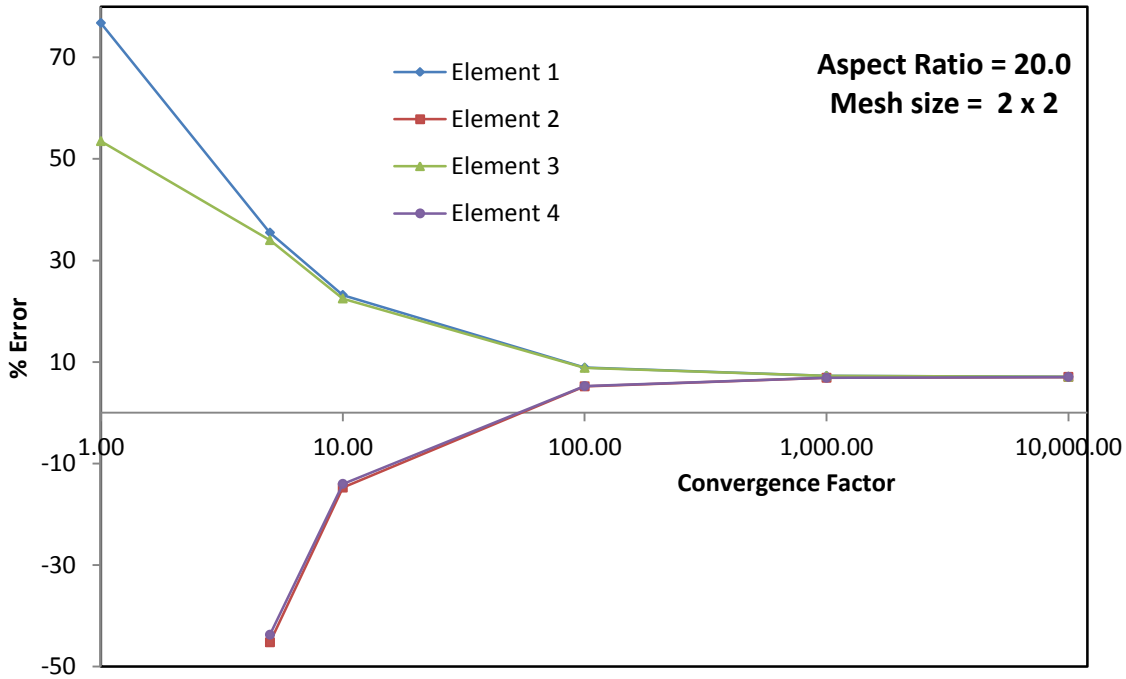


Fig.1

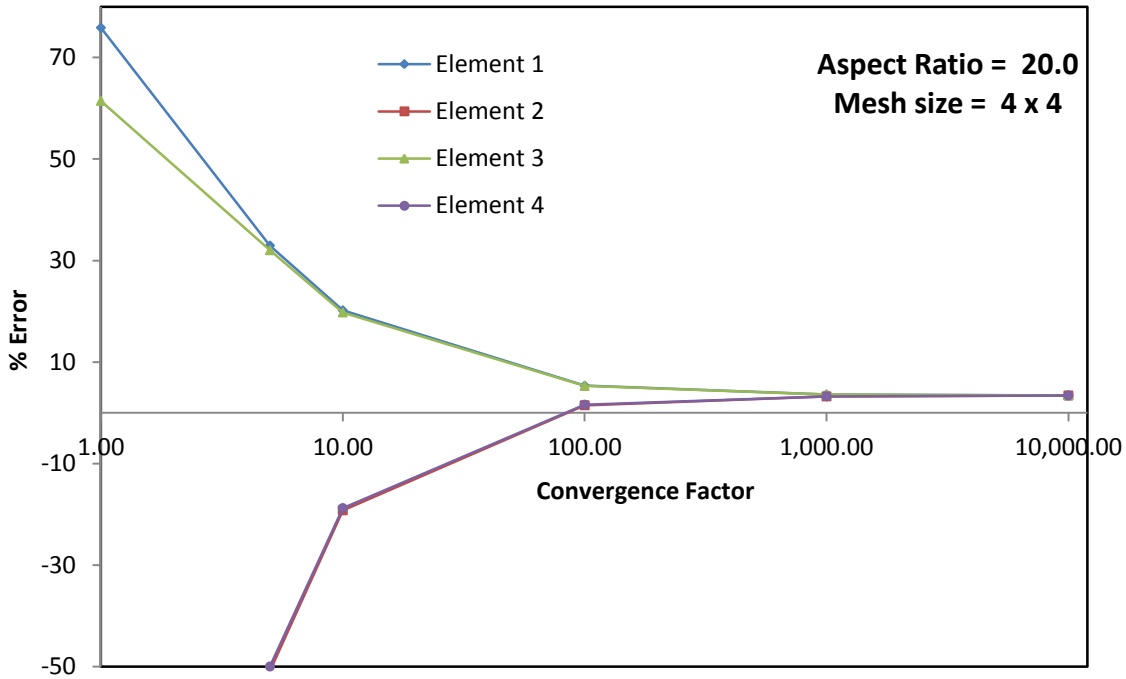

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 20.0
Mesh size = 4 x 4
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.2

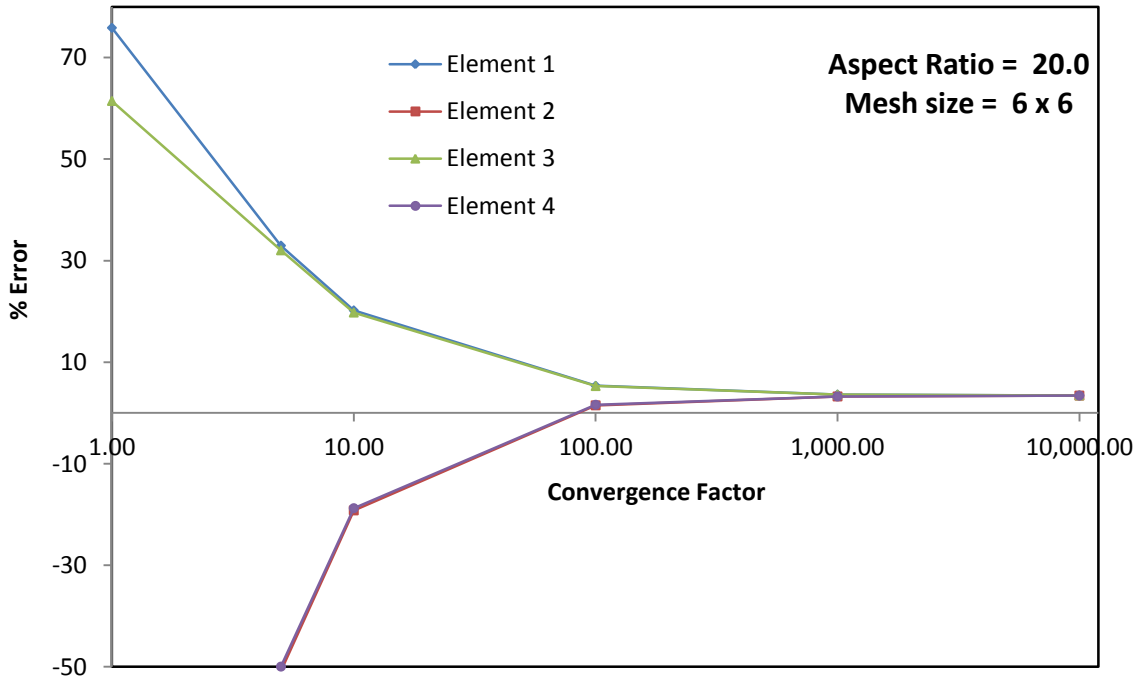

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 20.0
Mesh size = 6 x 6
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.3

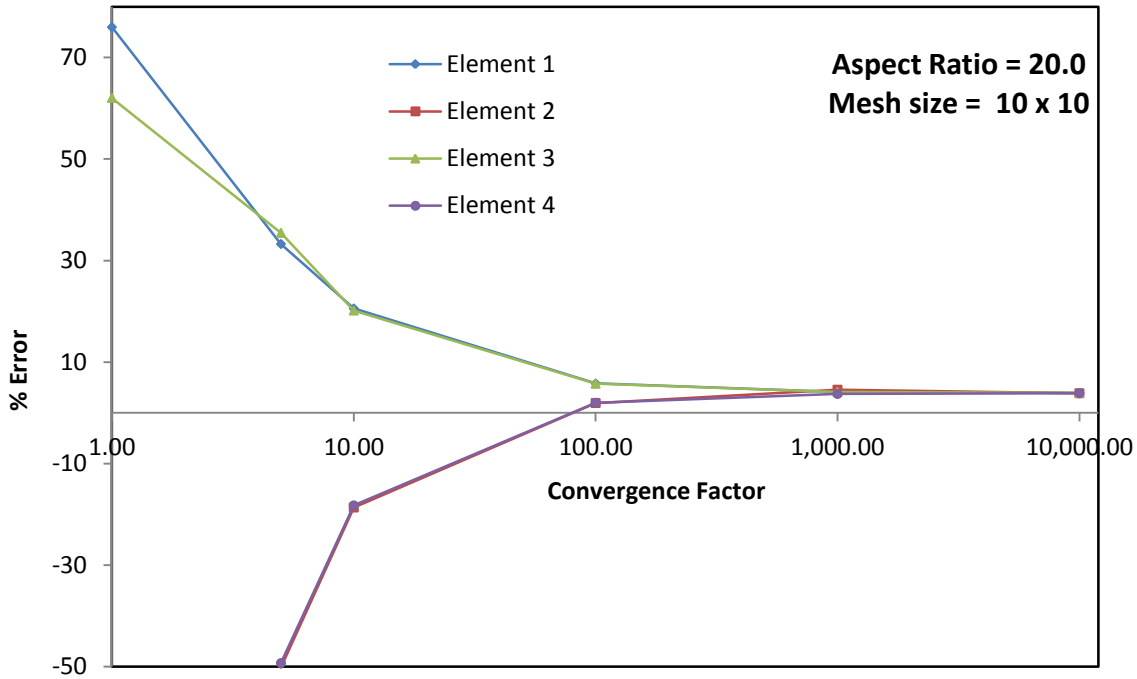

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 20.0
Mesh size = 10 x 10
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.4

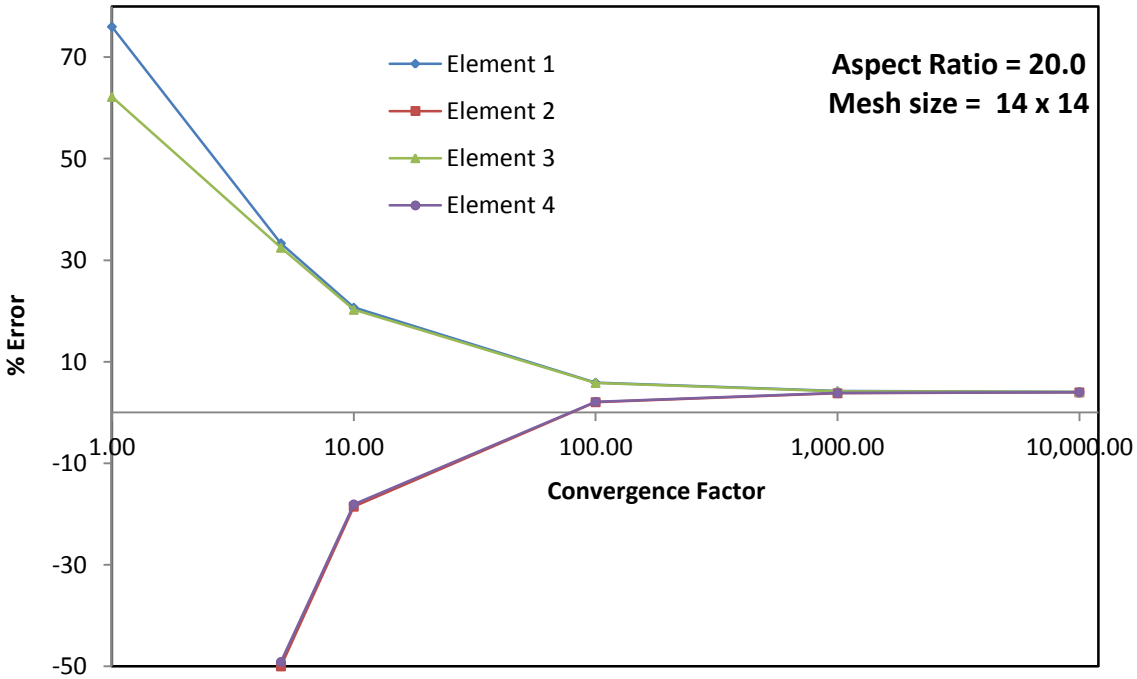
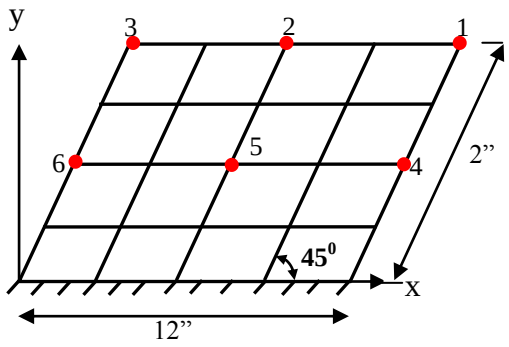

Element 1
Element 2
Element 3
Element 4
Aspect Ratio = 20.0
Mesh size = 14 x 14
% Error
70
50
30
10
-10
-30
-50
1.00
10.00
100.00
1,000.00
10,000.00
Convergence Factor

Fig.5

2. Rhombic Plate under UDL

E=10.5e5 psi
υ=0.3
h=0.125 in
q=0.25066 psi

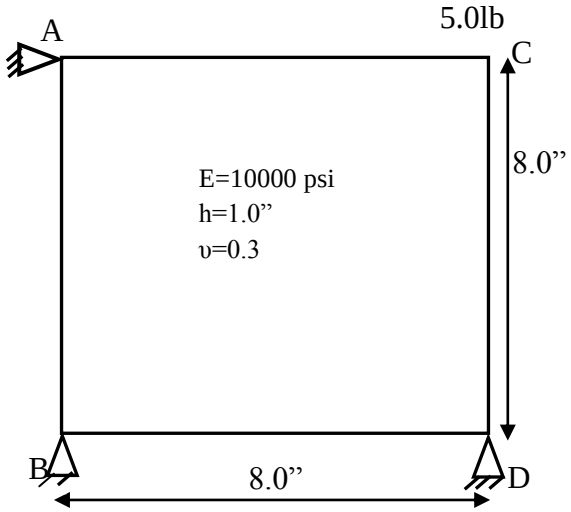


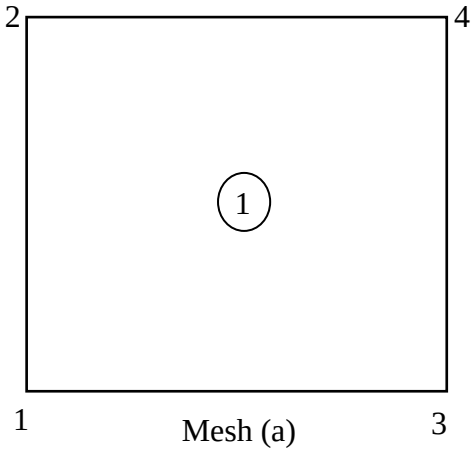
Element	Mesh	DOF	1	2	3	4	5	6
MR_FE_1[1]	4 x 4	75	**0.3132**	**0.2059**	**0.1144**	**0.1255**	**0.05239**	**0.02053**
MR_FE_2[1]	4 x 4	75	**0.3133**	**0.2010**	**0.1144**	**0.1256**	**0.05241**	**0.02054**
MR_FE_3[1]	4 x 4	75	**0.3132**	**0.2059**	**0.1144**	**0.1255**	**0.05239**	**0.02053**
MR_FE_4[1]	4 x 4	75	**0.3133**	**0.2060**	**0.1144**	**0.1256**	**0.05241**	**0.02054**
DKT[2]	4 x 4	75	**0.304**	**0.198**	**0.113**	**0.121**	**0.055**	**0.028**
HSM	4 x 4	75	0.264	0.173	0.100	0.095	0.043	0.021
ACM	8 x 6	189	0.296	0.198	0.114	0.114	0.052	0.020
HCT	8 x 6	189	0.281	0.188	0.111	0.111	0.049	0.018
Experimental Value			**0.297**	**0.204**	**0.121**	**0.129**	**0.056**	**0.022**

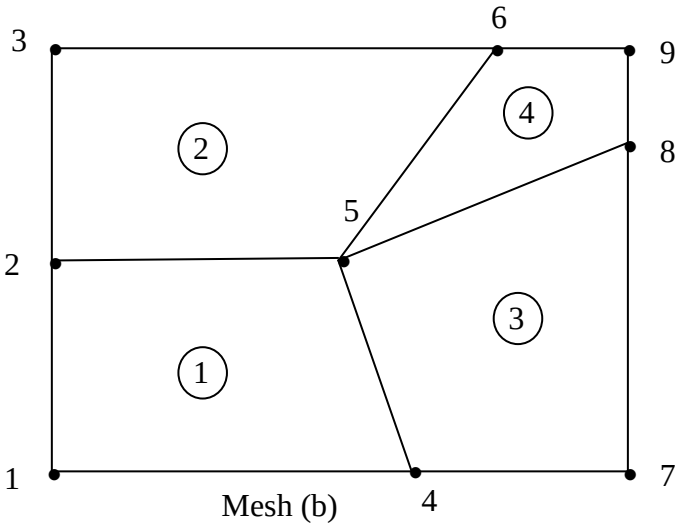
[1] **one point integration;** [2] Three point integration;

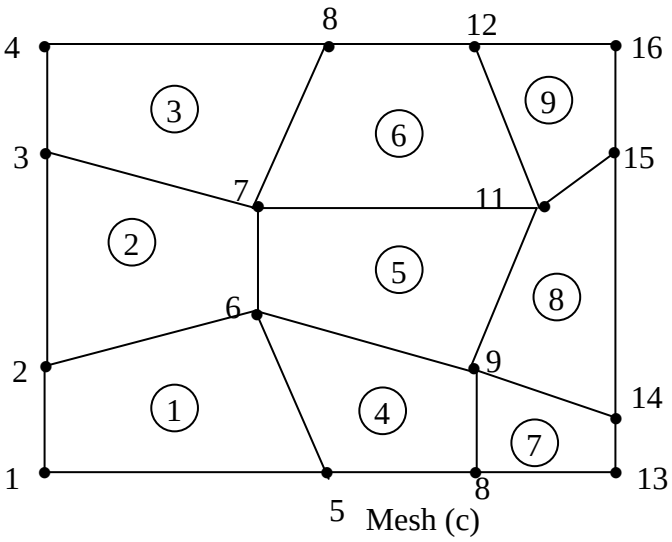
DKT – Discrete Krichhoff Theory element , Jean-Louis Batoz et al, A study of three-node triangular plate bending elements, international journal for numerical methods in engineering, Vol. 15, 1771-1812 (1980).

3. Twisting of a square plate









Mesh (b)

Node No	1	2	3	4	5	6	7	8	9
X-co	0	0	0	5	5	6.2	8	8	8
Y-co	0	4	8	0	4	8	0	6.2	8

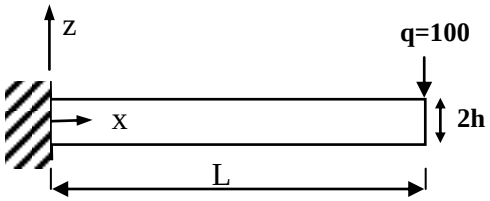
Mesh (c)

Node No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
X-co	0	0	0	0	4	3	3	4	6	6	7	6	8	8	8	8
Y-co	0	2	6	8	0	3	5	8	0	2	5	8	0	1	5	8

Element Type	DKT	HSM	ACM	HCT	Mesh	MR_FE_1	MR_FE_2	MR_FE_3	MR_FE_4
Point C*	0.24960	0.24960	.24972	0.25002	a	0.25740	0.25740	0.25742	0.25739
					b	0.18963	0.18963	0.18945	0.19218
					c	0.19229	0.19229	0.19218	0.19239
0.24960 (Exact thin plate solution)									

* Three point integration

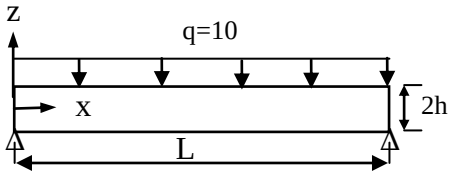
4. Cantilever Beam (Tip Load)



	L/2h=160/12						L/2h=80/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	1.8136	1.8136	32.2712 (6 elements with 1000 CF)	32.2391 (6 elements with 1000 CF)	32.2712 (6 elements with 1000 CF)	32.2712 (6 elements with 1000 CF)	0.7867	0.7867	4.1140 (6 elements with 1000 CF)	4.1100 (6 elements with 1000 CF	4.1258 (6 elements with 1000 CF)	4.1218 (6 elements with 1000 CF)
4	6.2233	6.2233					2.0073	2.0073				
8	15.8701	15.8701					3.2797	3.2797				
12	22.2602	22.2602					3.7158	3.7158				
20	28.0410	28.0410					3.9874	3.9874				
24	29.3511	29.3511					4.0381	4.0381				
30	30.5177	30.5177					4.0806	4.0806				
36	31.1911	31.1911					4.1040	4.1040				
Reddy *	32.823						4.1567					
Exact Solution	**32.7844**						**4.1317**					

	L/2h=40/12						L/2h=12/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_2**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_NFE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.2626	0.2626	0.5366 (6 elements with 1000 CF)	0.5362 (6 elements with 1000 CF)	0.5424 (6 elements with 1000 CF)	0.5419 (6 elements with 1000 CF)	0.02194	0.02194	0.02264 (6 elements with 1000 CF)	0.02264 (6 elements with 1000 CF)	0.02419 (6 elements with 1000 CF)	0.02418 (6 elements with 1000 CF)
4	0.4302	0.4302					0.02367	0.02367				
8	0.5118	0.5118					0.02414	0.02414				
12	0.5305	0.5305					0.02423	0.02423				
20	0.5406	0.5406					0.02428	0.02428				
24	0.5424	0.5424										
30	0.5439	0.5439										
36	0.5447	0.5447										
Reddy *	0.54588						0.02393					
Exact Solution	**0.5333**						**0.02052**					

5. Simply Supported Beam (Uniformly distributed load)



	L/2h=160/12						L/2h=80/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.8254	0.8254	20.326 (10 elements with 1000 CF)	20.306 (10 elements with 1000 CF)	20.374 (10 elements with 1000 CF)	20.354 (10 elements with 1000 CF)	0.1849	0.1849	1.3151 (10 elements with 1000 CF)	1.3139 (10 elements with 1000 CF)	1.3271 (10 elements with 1000 CF	1.3259 (10 elements with 1000 CF
4	3.6431	3.6431					0.6046	0.6046				
8	9.8570	9.8570					1.0482	1.0482				
12	13.9791	13.9791					1.2009	1.2009				
20	17.7108	17.7108					1.2960	1.2960				
24	18.5568	18.5568					1.3138	1.3138				
30	19.3102	19.3102					1.3287	1.3287				
36	19.7452	19.7452					1.3369	1.3369				
Reddy *	20.717						1.3486					
Exact Solution	**20.6892**						**1.3408**					

	L/2h=40/12						L/2h=12/12					
N	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**	**HFE_1**	**HFE_2**	**Timo_FE_1**	**Timo_FE_3**	**FE_NSF_1**	**FE_NSF_3**
2	0.03475	0.03475	0.09339 (10 elements with 1000 CF)	0.09333 (10 elements with 1000 CF)	0.09634 (10 elements with 1000 CF)	0.09628 (10 elements with 1000 CF)	0.0016486	0.0016486	0.001979 (10 elements with 1000 CF)	0.001980 (10 elements with 1000 CF)	0.002230 (10 elements with 1000 CF)	0.002210 (10 elements with 1000 CF)
4	0.07202	0.07202					0.0021069	0.0021069				
8	0.09064	0.09064					0.0022298	0.0022298				
12	0.09491	0.09491					0.0022529	0.0022529				
20	0.09723	0.09723					0.0022649	0.0022649				
24	0.09764	0.09764					----------	----------				
30	---------	---------					----------	----------				
36	---------	---------					----------	----------				
Reddy *	0.09770						0.002220					
Exact Solution	**0.09576**						**0.002082**					