A comment on "A dimensionless measure for adhesion and effects of the range of adhesion in contacts of nominally flat surfaces" by M. H. Muser

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Muser attempts in a recent paper (Muser, 2016) to introduce single dimensionless parameters to razionalize the complex problem of the contact of rough multiscale surfaces in the presence of adhesion, and checks his parameters against some numerical simulations with a quite large number of grid points for today's computational capabilities ($\sim 10^6$).

1) The first generalization relates to so called "Tabor parameter". The original Tabor parameter (Tabor, 1977) which controls the validity of the DMT ("long-range" adhesion) vs JKR regime ("short-range" adhesion) solution for the sphere, is

$$\mu_{sphere} = \left(\frac{R\Delta\gamma^2}{E^{*2}\Delta r^3}\right)^{1/3} \tag{1}$$

where R is the sphere radius, $\Delta\gamma$ is work of adhesion, Δr is the range of attraction of adhesive forces, and E^* the plane strain elastic modulus. Similarly to Persson and Scaraggi (2014), Muser merely generalizes the Tabor parameter, by considering the relevant "local" mean radius $R_c \simeq 2/h''_{rms}$ which largely depends on the truncating wavevector of roughness q_1, and denominates this Tabor parameter μ_T. But while the original Tabor parameter is unambigously defined for the sphere, for the case of multiscale surfaces, we need to return to the "real" meaning of it, to understand why various possibilities exist for its generalization, and the dimensional analysis Muser suggests is just one of many possible alternatives. In essense, the Tabor parameter defines the limit of validity of the Linear Elastic Fracture Mechanics, like in the 'small-scale yielding' criterion. Since the elastic tensile stresses near the boundary of the contact region are given by $\sigma(x) = \frac{K_I}{\sqrt{2\pi x}}$, where x is the distance from the boundary, the width of the region where the stress is higher than the theoretical strength σ_0 is $s_0 = \frac{E^*\Delta\gamma}{\pi\sigma_0^2}$, and we prescribe for JKR to be a good approximation that $s_0 \ll a_{\min}$, where $a_{\min}$ is strictly for a rigorous application of dimensional analysis, the (*) *smallest length scale associated with the geometry of the problem.* This could be for example (i) the smallest width of the contact region (and this leads univoquely to the original Tabor parameter in the sphere problem, given the dependence on R), or of the separation region, when the contact is almost complete (see Ciavarella, et al. 2019). So, in truth, the "generalized Tabor parameter" is load-dependent, and this limits the generalization μ_T of (Muser,

2016) to the small-load limit, as it should be clearly recognized, or anyway when condition (*) is satisfied – which may not be trivial to check.

2) The second attempt of a dimensionless general quantity is the "dimensionless surface energy", defined as

$$\Delta\gamma_{rss} = \frac{\Delta\gamma}{E^*} \frac{\tanh(\mu_T)}{(h'_{rms})^3} \tag{2}$$

where h'_{rms} is the root mean-square gradient of the surface, tanh is introduced as an empirical fitting between the "correct" asymptotics in the two limits of small and large Tabor coefficients μ_T. Unfortunately, this second attempts sums the uncertainties of the generalization of Tabor parameter, with further ones, like assuming we are in "unsticky range" and that the area increases due to adhesion by a linear proportion of the load, and probably other, hidden, ones. The positive check with some numerical results suffers from the fact that the numerical results of Muser correspond to limited range in the spectrum of roughness (spanning less than 3 decades, so only reaching from nm to μm wavelength size, while clearly most real surfaces will necessarily have roughness on scales much larger than this), for which Muser is not to blame since this is limited by present computational capabilities (similarly narrow spectra are considered by Pastewka & Robbins, 2014).

A rigorous physicist would immediately try a possible extrapolation to $\Delta\gamma_{rss}$

when the spectrum becomes infinitely large: defining "magnification" $\zeta = q_1/q_0$, where q_0 is some reference short wavevector truncation, and q_1 is the large wavevector truncation, and assuming a power law PSD spectrum for simplicity as it is common, $h'_{rms} \sim \zeta^{1-H}$, while $h''_{rms} \sim \zeta^{2-H}$, where H is Hurst exponent which is $H = 3 - D$ where D is fractal dimension of the surface) and hence $\tanh(\mu_T) \sim (h''_{rms})^{-1/3}$ so that

$$\Delta\gamma_{rrs} \sim \zeta^{(2-H)\frac{2}{3}-3+3H} \sim \zeta^{(7H-5)/3}, \qquad \zeta \to \infty \tag{3}$$

which goes to zero if $(7H-5)/3 < 0$ or $H < 5/7 = 0.714$. However, we know from alternative semi-analytical investigations which permit to explore wide range of PSD spectra (Joe et al., 2018), that surfaces may be sticky or non-sticky even in this limit but independently on H, as the contact solution converges and doesn't depend on the "local" quantities h''_{rms}, h'_{rms}, that the reference to "DMT" or "JKR" regimes is misleading since the "generalized Tabor parameter" goes to zero but the problem may remain not defined by a DMT analysis, and the main parameters ruling the problems are the macroscopic quantities rms heights h_{rms} and the short wavevector truncation q_0 : hence, this "dimensionless surface energy" quantity is *meaningless*.

Notice incidentally that similar apparent contradictions occur in the Pastewka-Robbins's suggested criterion for stickiness based on numerical observations (see Ciavarella, et al. 2019), which can be cast in the form

$$\zeta^{(1-5H/3)} < C \ , \tag{4}$$

where C is a positive constant, which in the limit $\zeta \to \infty$, would seems to suggest similar results than Muser (2016), except for $H < 0.6$ (and the difference in the limit H may be due to different ways of extrapolating numerical results or to different precisions). Viceversa, Violano *et al.* (2019) suggest that for low fractal dimension ($D \simeq 2.2$) rough hard surfaces stick for

$$\frac{h_{rms}}{\Delta r} < \left(\frac{9}{4} \frac{\sigma_0 / E^*}{\Delta r q_0} \right)^{3/5} \tag{5}$$

which naturally and simply corresponds to well known empirical Dalhquist criterion (Dalhquist, 1969a, 1969b), which for 50 and more years has simply postulated that effective adhesives should have elastic modulus lower than about 1 MPa, regardless of the local h''_{rms}, h'_{rms} values of roughness, which would anyway be extremely difficult to measure in real life. Similar results were obtained in a much simpler theory (Ciavarella, 2018, Ciavarella & Papangelo, 2019) which incidentally fit Pastewka-Robbins' (2014) results regardless of the local h''_{rms}, h'_{rms} which seem so crucial in the Muser (2016) (and also Pastewka-Robbins' 2014) numerical interpolations.

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