Energy function

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Fung Type function
Elastic part of energy,

$$\Omega_e = \alpha/\beta(e^{(\beta(I_1-3)^2)} - 1) + k_1/k_2(e^{(k_2(I_4-1)^2)} - 1) \tag{1}$$

where, α,β, k_1 and k_2 are parametric constant. I_1 is strain invariant and I_4 is pseudo-strain invariant.
Viscous part of Energy,

$$\Omega_v = \mu_v/2(\mathbf{C} : \mathbf{C_v^{-1}} - 3) \tag{2}$$

Time evolution of the internal variable $\mathbf{C_v} = \mathbf{F_v^t F_v}$ is given by

$$\frac{d\mathbf{C}_v}{dt} = 1/T_v(\mathbf{C} - \mathbf{1/3}(\mathbf{C} : \mathbf{C_v^{-1}})\mathbf{C_v}) \tag{3}$$

First Piola-Kirchoff stress,

$$\mathbf{T} = \frac{\partial \Omega}{\partial \mathbf{F}} - p\mathbf{F}^{-1} \tag{4}$$

Now,

$$\Omega = \Omega_e + \Omega_v \tag{5}$$

$$\frac{\partial \Omega}{\partial \mathbf{F}} = \frac{\partial \Omega_e}{\partial \mathbf{F}} + \frac{\partial \Omega_v}{\partial \mathbf{F}} \tag{6}$$

Let us calculate some preliminary partial derivatives

$$\frac{\partial I_1}{\partial \mathbf{F}} = \frac{\partial tr(\mathbf{C})}{\partial \mathbf{F}} = (\frac{(\partial \mathbf{C} : \mathbf{I})}{\partial \mathbf{C}})\frac{\partial \mathbf{C}}{\partial \mathbf{F}} = 2\mathbf{F^t} \tag{7}$$

$$\frac{\partial I_4}{\partial \mathbf{F}} = (\frac{(\partial \mathbf{C} : \mathbf{M})}{\partial \mathbf{C}})\frac{\partial \mathbf{C}}{\partial \mathbf{F}} = 2\mathbf{MF^t} \tag{8}$$

$$\frac{\partial \mathbf{C} : \mathbf{C_v^{-1}}}{\partial \mathbf{F}} = \frac{\partial \mathbf{C} : \mathbf{C_v^{-1}}}{\partial \mathbf{C}}\frac{\partial \mathbf{C}}{\partial \mathbf{F}} = 2\mathbf{C_v^{-1}F^t} \tag{9}$$

- I want to correct my eq. 9 for which I need $\frac{\partial \mathbf{C}_v}{\partial \mathbf{C}}$

Now, using chain rule and these partial derivatives we can find following:

$$\frac{\partial \Omega_e}{\partial \mathbf{F}} = \alpha/\beta(e^{(\beta(I_1-3)^2)})*2*\beta(I_1-3)*2*\mathbf{F^t} + k_1/k_2(e^{k_2(I_4-1)^2})*k_2*2*(I_4-1)*2\mathbf{MF^t} \tag{10}$$

$$\frac{\partial \Omega_e}{\partial \mathbf{F}} = 4\alpha e^{\beta(I_1-3)^2}(I_1-3)\mathbf{F^t} + 4k_1 e^{k_2(I_4-1)^2}(I_4-1)\mathbf{MF^t} \tag{11}$$

$$\frac{\partial \Omega_v}{\partial \mathbf{F}} = \mu_v/2 * (2\mathbf{C_v^{-1}F^t}) = \mu_v\mathbf{C_v^{-1}F^t} \tag{12}$$

Putting all the equation together from above we can get:

$$\mathbf{T} = 4\alpha e^{\beta(I_1-3)^2}(I_1-3)\mathbf{F^t} + 4k_1 e^{k_2(I_4-1)^2}(I_4-1)\mathbf{MF^t} + \mu_v\mathbf{C_v^{-1}F^t} - p\mathbf{F^{-1}} \tag{13}$$

where $\mathbf{C}$ is Cauchy strain tensor, $\mathbf{C}_v$ is Cauchy viscous strain tensor,$\mathbf{F}$ is deformation gradient