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Interdisciplinare del DMMM
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Fractals in tribology

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“I have spent half my life having hard time to get my idea published, and the other half having hard time with people copying my ideas”

«I am having hard time completing my 8 volumes authobiography….»

BB. Mandelbrot, personal communications after ICF11 dinner, Torino 2005

BB Mandelbrot (1924-2010)

- Papers in geometry, finance, physics, image creation and compression, turbulence, fracture, hydraulics, medicine and many more. Which SSD is he in? Would he get one or more ASN?
- has today in GS 114808 citations and h-index = 92
- The oldest prof to get tenure @Yale (Economics), after long IBM career
- Possible ground for collaboration…

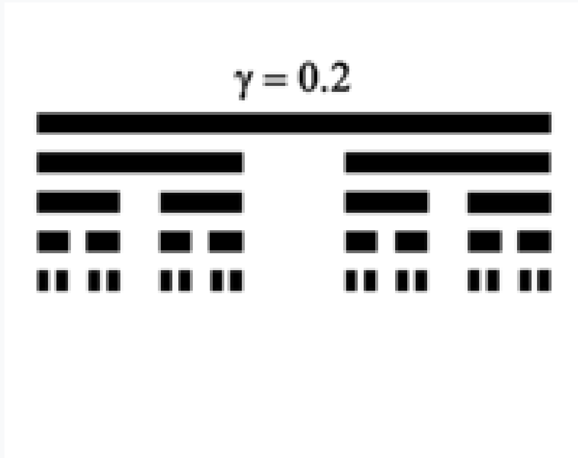
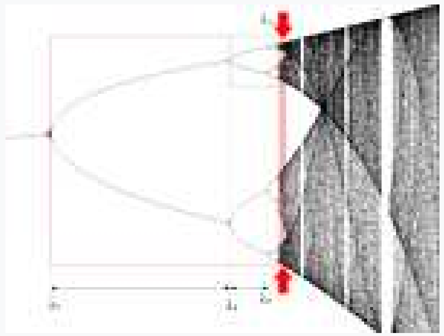
Fractals

Frost crystals occurring naturally on cold glass form fractal patterns

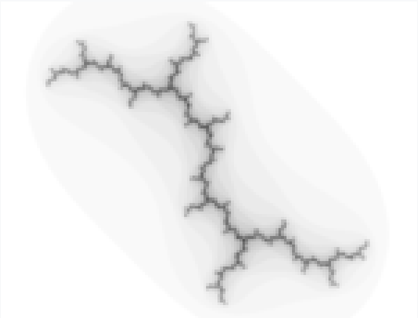
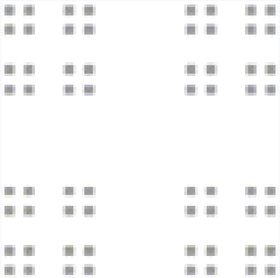
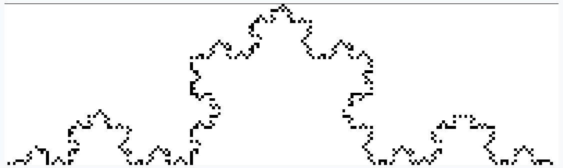
In the most basic sense, fractals are objects that display self-similarity over a wide (??) range of scales

A fractal is formed when pulling apart two glue-covered acrylic sheets

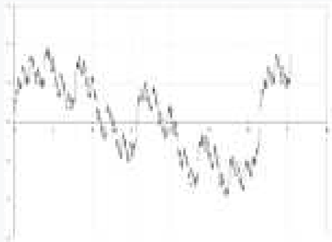
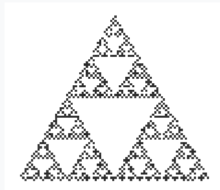
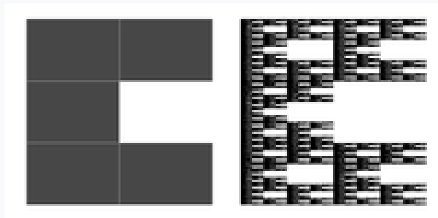
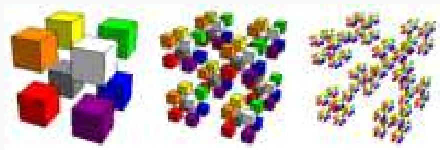
0<D<1

0<D<1	**Generalized Cantor set**	
0.538	**[Feigenbaum attractor]**	

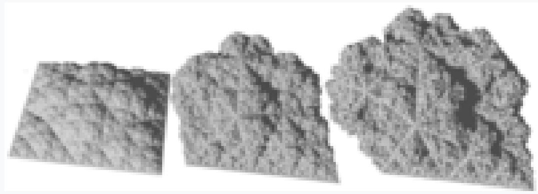
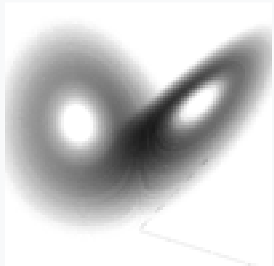
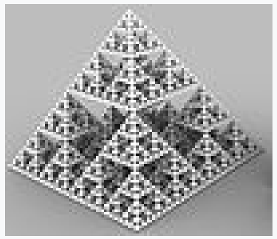
1<D<1.3

1.2	**Dendrite Julia set**	
1.2619	**2D Cantor dust**	
1.2619	**Koch curve**	

1.5<D<2

1.5000	a Weierstrass function	
1.5849	Sierpinski triangle	
1.8272	A self-affine fractal set	
1.8928	3D Cantor dust	

2<D<2.5

2<D<2.3	Pyramid surface	
2.06 ±0.01	Lorenz attractor	
2.3219	Fractal pyramid	

nature

Letter | Published: 19 April 1984

Fractal character of fracture surfaces of metals

Benoit B. Mandelbrot, Dann. E. Passoja & Alvin J. Paullay

Nature **308**, 721–722 (19 April 1984) | Download Citation

Abstract

When a piece of metal is fractured either by tensile or impact loading (pulling or hitting), the fracture surface that is formed is rough and irregular. Its shape is affected by the metal's microstructure (such as grains, inclusions and precipitates, whose characteristic length is large relative to the atomic scale), as well as by 'macrostructural' influences (such as the size, the shape of the specimen, and the notch from which the fracture begins). However, repeated observation at various magnifications also reveals a variety of additional structures that fall between the 'micro' and the 'macro' and have not yet been described satisfactorily in a systematic manner. The experiments reported here reveal the existence of broad and clearly distinct zone of intermediate scales in which the structure is modelled very well by a fractal surface. A new method, slit island analysis, is introduced to estimate the basic quantity called the fractal dimension, *D*. The estimate is shown to agree with the value obtained by fracture profile analysis, a spectral method. Finally, D is shown to be a measure of toughness in metals.

Many earlier oversimplified conclusions by Mandelbrot have not resisted the test of time.
Fracture mechanics essentially dissipates energy both on a surface and on a volume (plastic deformation), so one can artificially say that it dissipates over a fractal surface and then the fractal dimension needs to change (multifractal…)…

Never heard again of D as measure of toughness in metals…

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Science, Vol 279, Issue 5347, 39-40 , 2 January 1998
[DOI: 10.1126/science.279.5347.39]

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APPLIED MATHEMATICS:

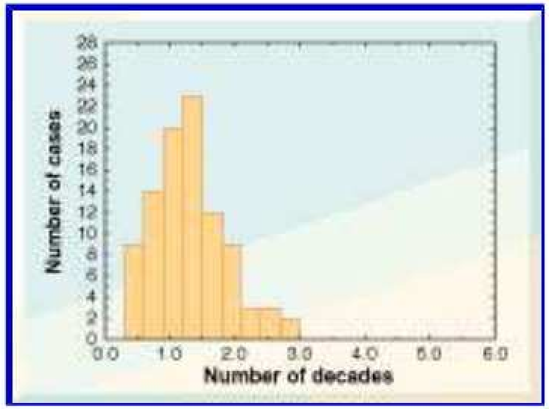
Is the Geometry of Nature Fractal?

David Avnir

Ofer Biham

Daniel Lidar

Ofer Malcai

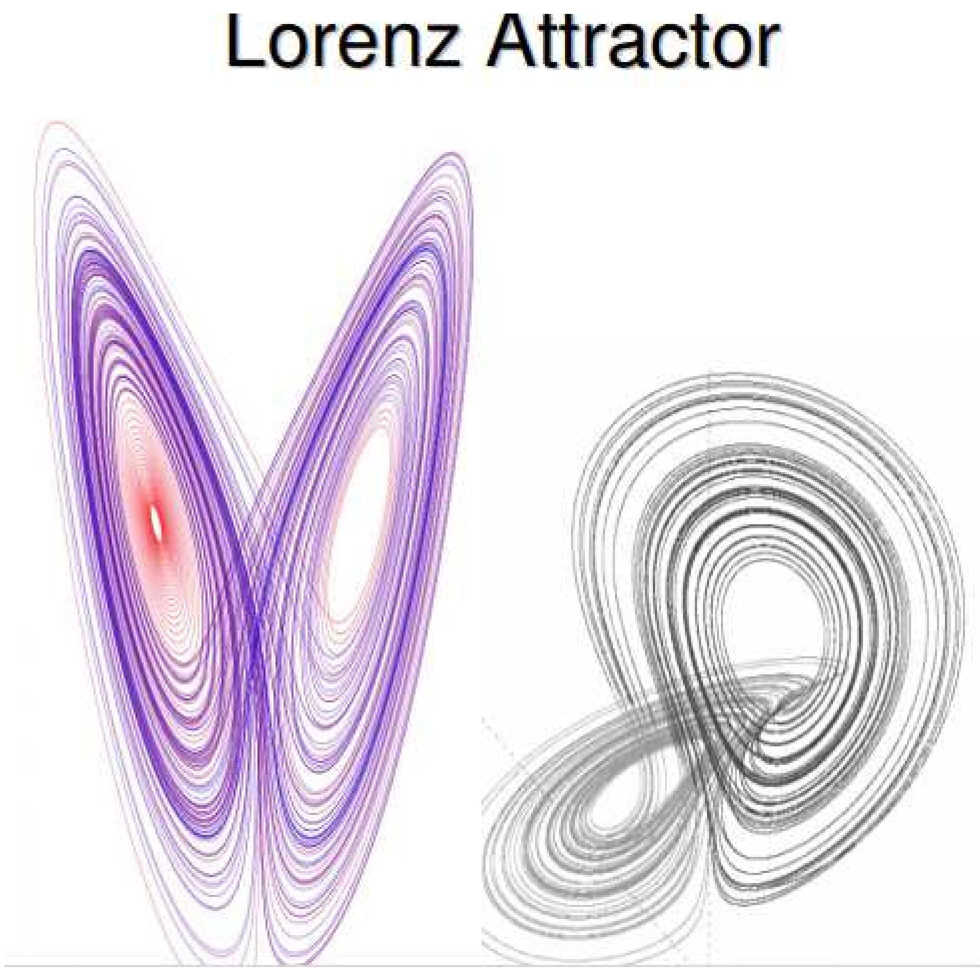


Limited scaling range. The number of decades (factors of 10) spanned by experimentally
led to the labeling of the studied systems as fractal (*4*).

There was also a bitter debate at the end of 1990's about too much literature claiming «fractal» scaling when in fact geometry had at most 1 – 2 decades of selfsimilarity

Fractals & chaos

- Can be used in non linear dynamics to classify the dimension of strange attractors and hence the «degree» of chaos



Nothing new under the sun?

- Pure mathematicians tend to dismiss Mandelbrot as a mere salesman.
- Mandelbrot claims that even if the objects he brings forth have been known to pure mathematicians, they tended to be disgusted by them as mere pathological monsters, and it is he who showed *how natural and useful they really are for the study of nature*.

Turbulence

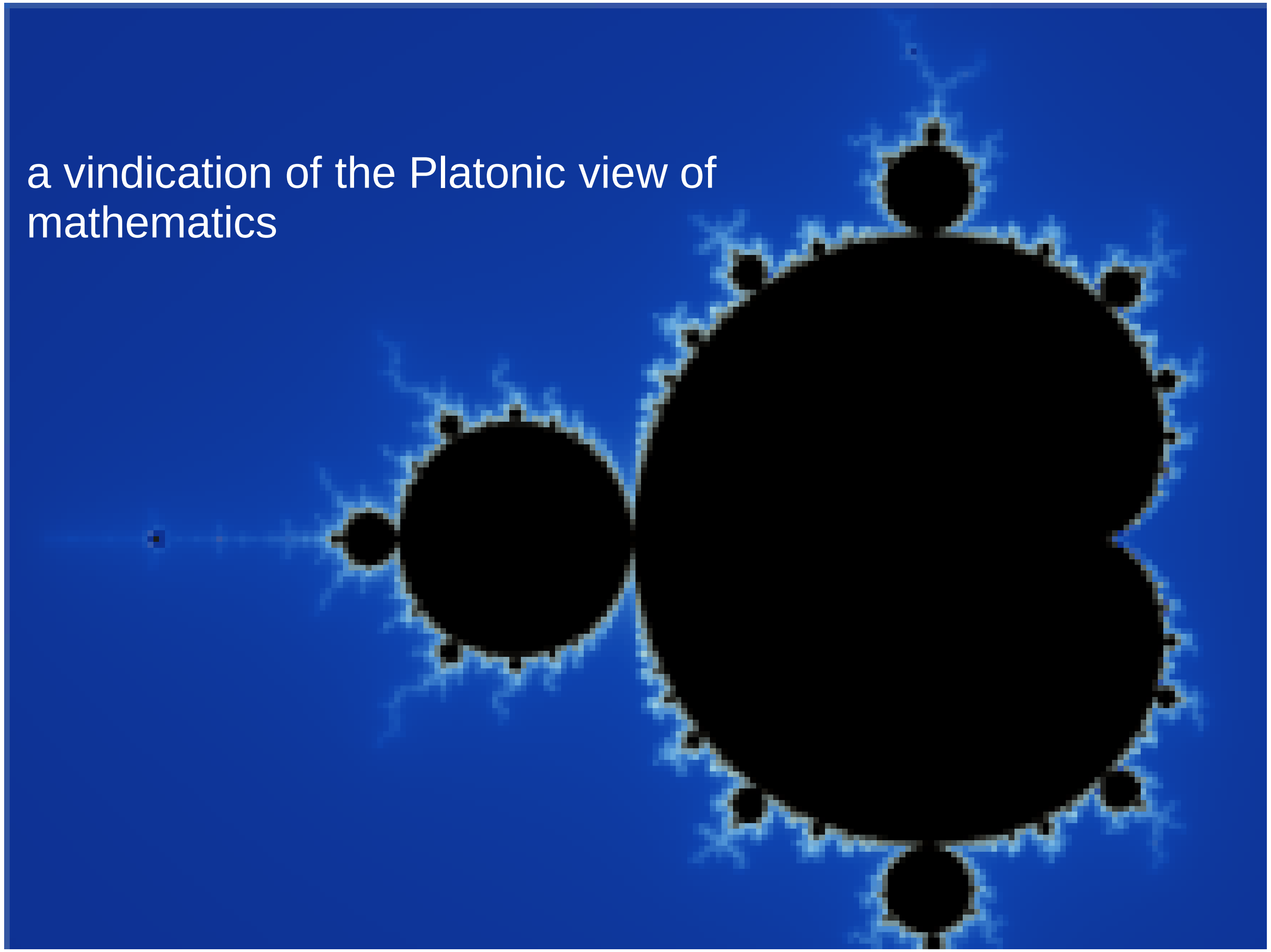
Mandelbrot (1982) ‘turbulence involves many fractal facets’ – claims geometric aspects of turbulence have been ignored

But his own investigations of 1974,1975 ‘they involve suggestions with few hard results as yet.’ (Mandelbrot 1982)

Finance

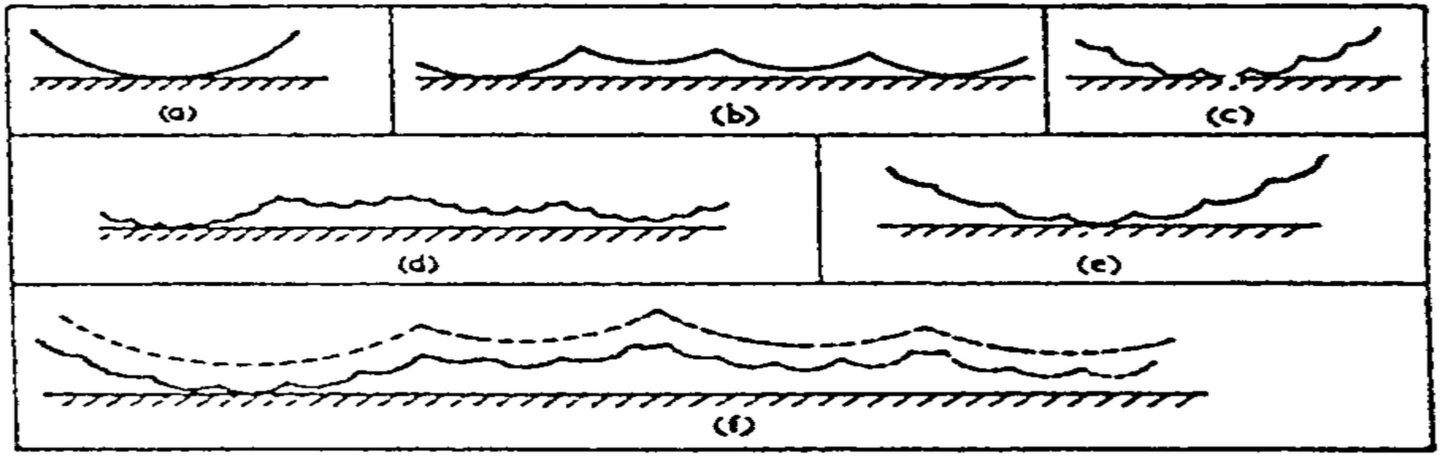
- another example is the modeling of commodity prices, which he claimed did not follow the standard Brownian motion with Gaussian distributions, but hyperbolic ones and not independently but showing some traces of memory. In particular this led to models with much bigger fluctuations, more in accordance with observations.

Mandelbrot set $z_{n+1}=z_n$^2+c



Application to tribology

- Archard 1957 was a fractal ante lteram (**magnification-dependent solution**)



- Fractal dimension was introduced in tribology by Majumbdar and Bhushan in 1990, but the contact area was arbitrarily defined as non fractal by a geometrical intersection of the rough surface with a plane, leading to a power law distribution of contact spot diameters (Korcak's law).

First rigorous contact theory

THE ROYAL SOCIETY

Linear elastic contact of the Weierstrass profile†

By M. Ciavarella[1]‡, G. Demelio[2], J. R. Barber[3] and Yong Hoon Jang[3]

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[2] *Dipartimento di Progettazione e Produzione Industriale, Politecnico di Bari, Viale Japigia 182, 70126 Bari, Italy*
[3] *Department of Mechanical Engineering and Applied Mechanics, University of Michigan, Ann Arbor, MI 48109-2125, USA*

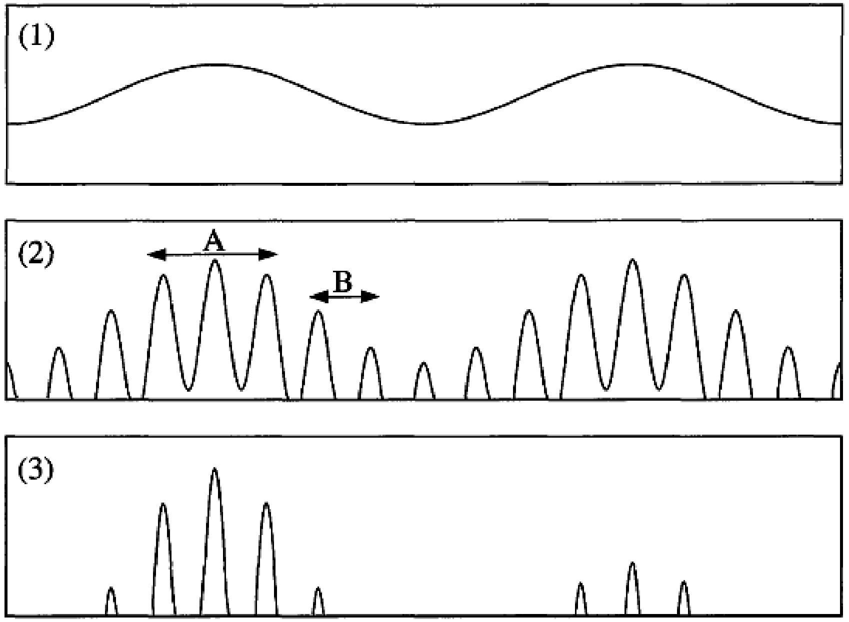
Received 12 January 1999; revised 18 May 1999; accepted 2 June 1999

A contact problem is considered in which an elastic half-plane is pressed against a rigid fractally rough surface, whose profile is defined by a Weierstrass series. It is shown that no applied mean pressure is sufficiently large to ensure full contact and indeed there are not even any contact areas of finite dimension—the contact area consists of a set of fractal character for all values of the geometric and loading parameters.

The Weierstrass function:

$$z(x) = g_0 \sum_{n=0}^{\infty} \gamma^{(D-2)n} \cos(2\pi\gamma^n x/\lambda_0).$$

Weierstrass model



Magnification dependent solution

Figure 2. Evolution of the contact pressure distribution for $\gamma^{D-1} < 2$. A full contact region (1) evolves to (2) at the next scale. With one further reduction of scale, regions (2A) evolve once again to (2), while regions (2B) evolve to (3).

The Archard load redistribution process showed that what looks full contact eventually is split at smaller scales into partial contact, and so on. For some combination of parameters (in particular, for a nearly continuous spectrum), there is convergence towards full contact: despite the contact area tends to zero, the actual areas of contact are in full contact. This has profound implications as later on we cannot simplify the full Persson's theory.

Weiestrass model

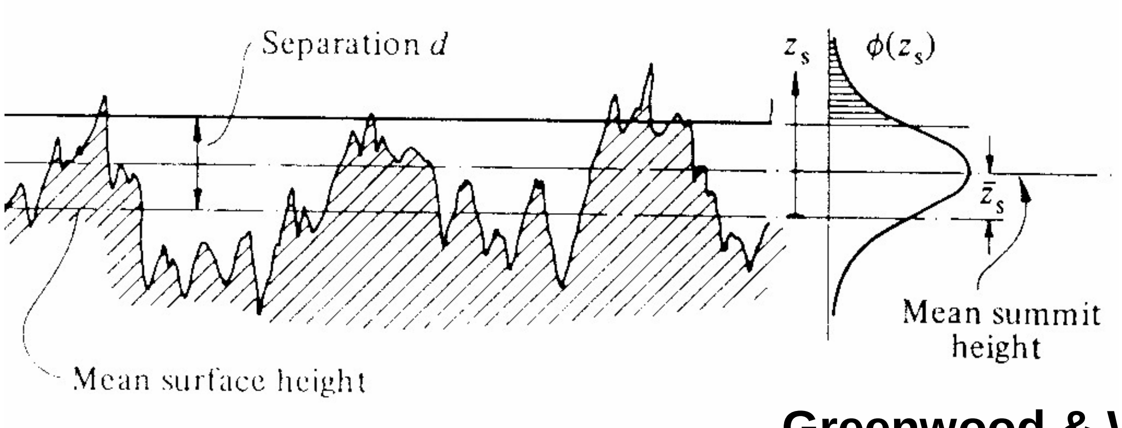
- Ciavarella Demelio Barber & Jang in 2000 showed instead contact area is itself a fractal of dimension less than a surface
- the elastic model for the tribological problem is essentially *ill-posed* as in the limit that the load P is shared on an infinite number of points, where the local pressure is infinite! $P=0 \times \infty$
- Bowden-Tabor old plastic model is more reasonable, but is forgotten……….

Persson 2001

- Persson (2001) introduced a very elegant theory for Gaussian random (fractal) surfaces which has evolved over the years to this day, permitting to solve the elastic contact problem in great details.
- It tries to solve the problem as a function of «magnification», and does not worry that the fractal limit is bizzarre.
- How to fix «magnification» then?

Persson's force vs separation solution

- Curiously, we have seen a long debate and "contact challenges" between "asperity models (GW, 1966)" and "Persson theory (2001)" mainly about the correct expression of contact area.



Greenwood & Williamson1966

- Most interesting and relevant part of Persson's theory (2007, 2008) is perhaps that force vs mean separation converges in the fractal limit, and so does electrical resistance or elastic stiffness, which depend on macroscopic quantities not on tail of the PSD spectrum

Predicting Friction?

- Almost all models predict linearity of real contact area with load, which is then used as argument to justify Amonton's law --- this implies that there is a constant shear strength, which seems to suggest a plastic deformation. The argument is really circular!

- We have no elastic contact model today that can predict the friction coefficient, based on the rough surface details

- Perhaps some effects of roughness on friction in viscoelastic materials are understood qualitatively, but extremely sensitive to the so-called large wavevector cutoff, which remains rather arbitrary (Persson suggests to truncate to h'rms=1.3).

- Ciavarella, M., 2018, Journal of Tribology, 140(1), 011403.

Adhesion

- Adhesion started with Bradley 1932 for rigid sphere, to JKR 1971 for elastic sphere, starting a long discussion for the case of a single sphere with the DMT «semi-rigid» solution, still ongoing
- David Tabor clarified in 1979 the controversy between the Cambridge School with the energetic approach (JKR) and the russian school of DMT (semirigid superposition of repulsive solution with forces in the gaps), introducing a parameter μ which is the ratio between length scale of the singular «fracture mechanics» field, and the contact area or sphere radius

Lennard-Jones

$3^{1/6}\varepsilon$ and is $\sigma_0 = 16\Delta\gamma/9\sqrt{3}\varepsilon$.

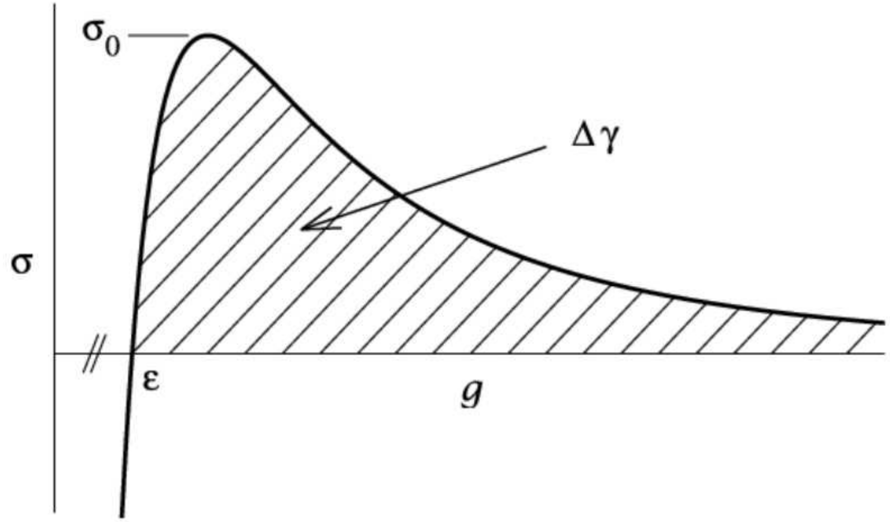


Figure 1: The Lennard-Jones traction law between two half spaces. The interface energy $\Delta\gamma$ corresponds to the shaded area.

Van der Waals forces are quite strong and they should lead to theoretical strength for a perfect crystal (adhesion paradox of Kendall's "sticky Universe"). This doesn't happen for inevitable roughness

From DMT to JKR

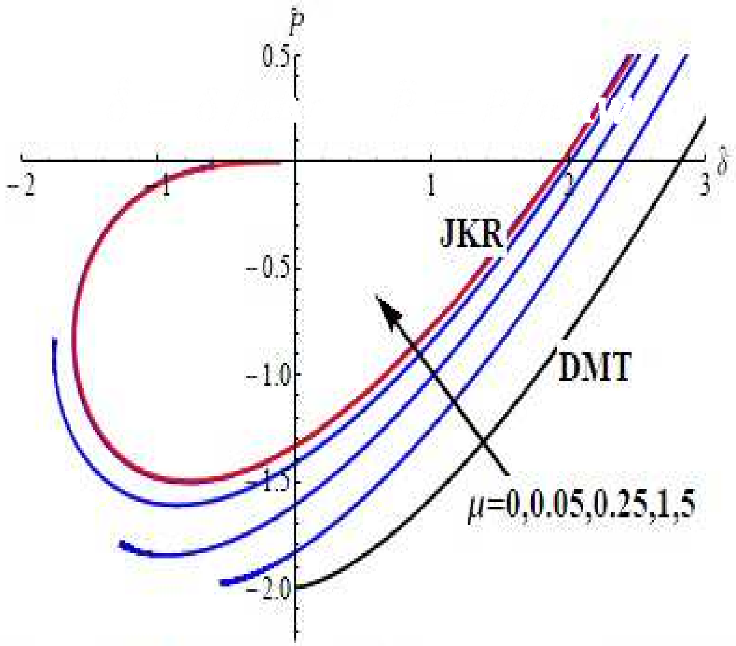


Fig.2 - Solutions of JKR, DMT and Maugis intermediate Tabor parameter range $\mu = 0, 0.05, 0.25, 1, 5$. JKR is obtained very closely at positive indentations for $\mu \simeq 1$

The DMT limit becomes non hysteretic, and is captured well with the Maugis solution simplifying the force gap relationship

The original DMT solution is not good unless Tabor parameter is extremely close to 0

Generalization of JKR approach

$$P_2 = P_1 - (\Delta_1 - \Delta_2)\left(\frac{\partial P}{\partial \Delta}\right)_{\Delta_1} \quad (4)$$

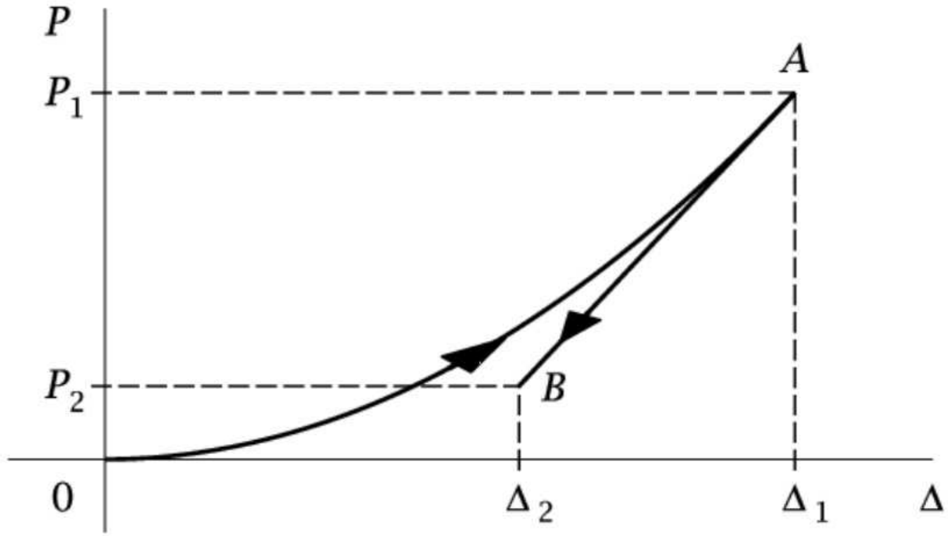


Figure 3: Two-step loading scenario. (i) "repulsive" loading without adhesive forces until a given contact area is reached (point A in the figure); (ii) Unloading at constant total contact area up point B.

JKR original energy calculation can be generalized approximately (Ciavarella, JMPS, 2018), but does not work in the fractal limit

Adhesion – FT, PR, PS theories

- Much less is known for rough surfaces:
- Fuller and Tabor 1975 using the GW asperity model and JKR theory seemed to fit results for rubber spheres in contact with rough perspex plates: adhesion was destroyed for very low amplitude of roughness
- DMT approaches: Pastewka and Robbins PNAS 2014, and Persson and Scaraggi J Chem Phys 2014.

Adhesion Paradoxes in the fractal limit

- Fuller and Tabor suggest stickiness is always zero (regardless of any other feature)!
- Pastewka and Robbins suggest stickiness is always infinite for all surfaces having D>2.4 (which include most of the natural and even man made surfaces)!

Bearing Area Model For Adhesion - 1

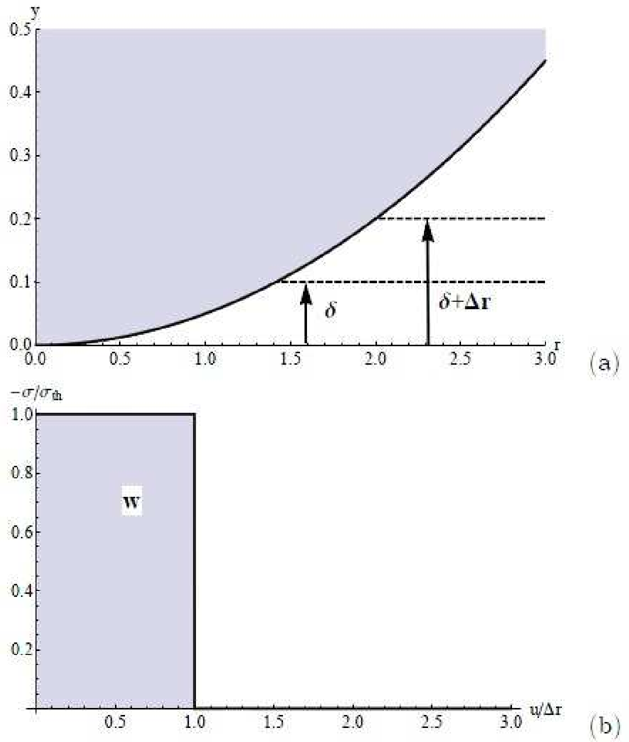


Fig.1 - (a) A parabolic elastic body in adhesive contact with a rigid plane;
(b) Maugis forces of attraction

The main idea of «BAM» model is purely geometrical.
1)the entire DMT solution for the sphere in the form reported by Maugis (not given by DMT), is obtained also by considering a Maugis constant force for separations up to a characteristic distance dr and force of attraction the product of theoret strength and overlap area A(d+dr)-A(d).
2)This is superposed to Hertz theory for the repulsive force F(d)

Bearing Area Model For Adhesion - 2

for not too large σ_{rep} (Persson 2007, eqt.20)[4]

$$\frac{\sigma_{rep}}{E^*} \simeq \frac{0.5 \times 0.75}{2} q_0 h_{rms} \exp\left(\frac{-u}{\gamma h_{rms}}\right)$$

Repulsive stress

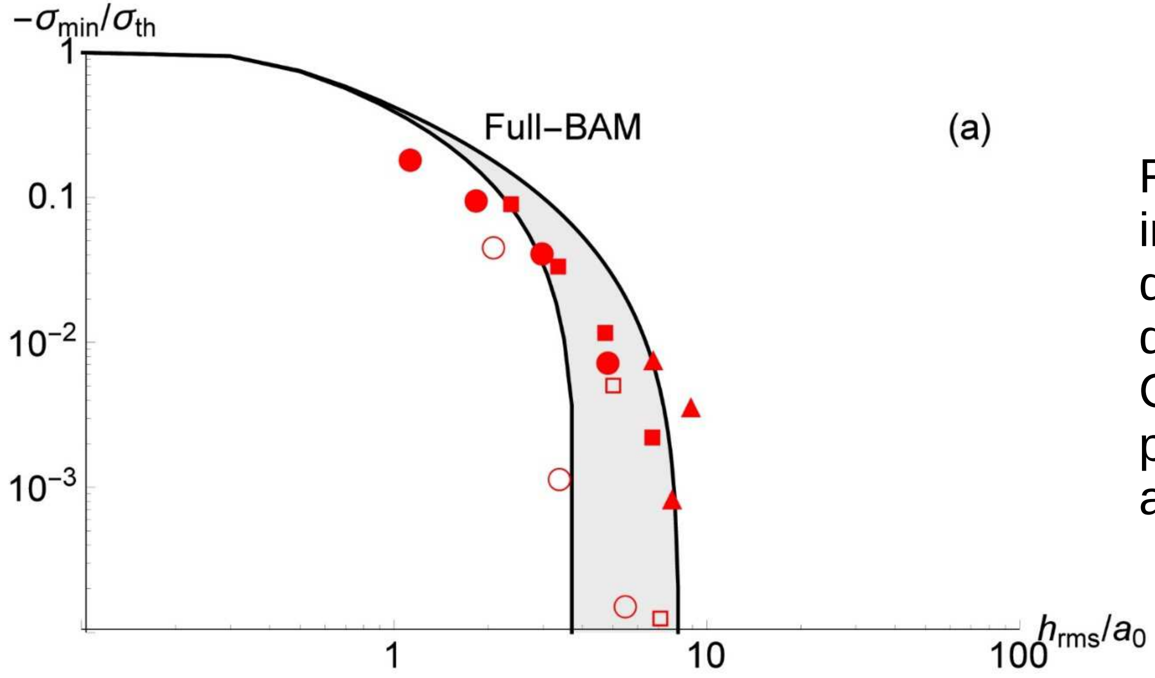
where $\gamma \simeq 0.4$, q_0 is the small wavevector in the self-affine process, and h_{rms} is the rms amplitude of roughness.

Therefore, using a simple Maugis model for adhesion, and the bearing area estimate for the area of attraction for a rough contact, we compute the difference of the bearing area at separation $u_{att} = u - \Delta r$, and u, and the attractive pressure can be estimated in a single line as

$$\frac{\sigma_{att}}{\sigma_{th}} = -\frac{1}{2k}\left[Erfc\left(\frac{u_{att}}{\sqrt{2}h_{rms}}\right) - Erfc\left(\frac{u}{\sqrt{2}h_{rms}}\right)\right]$$

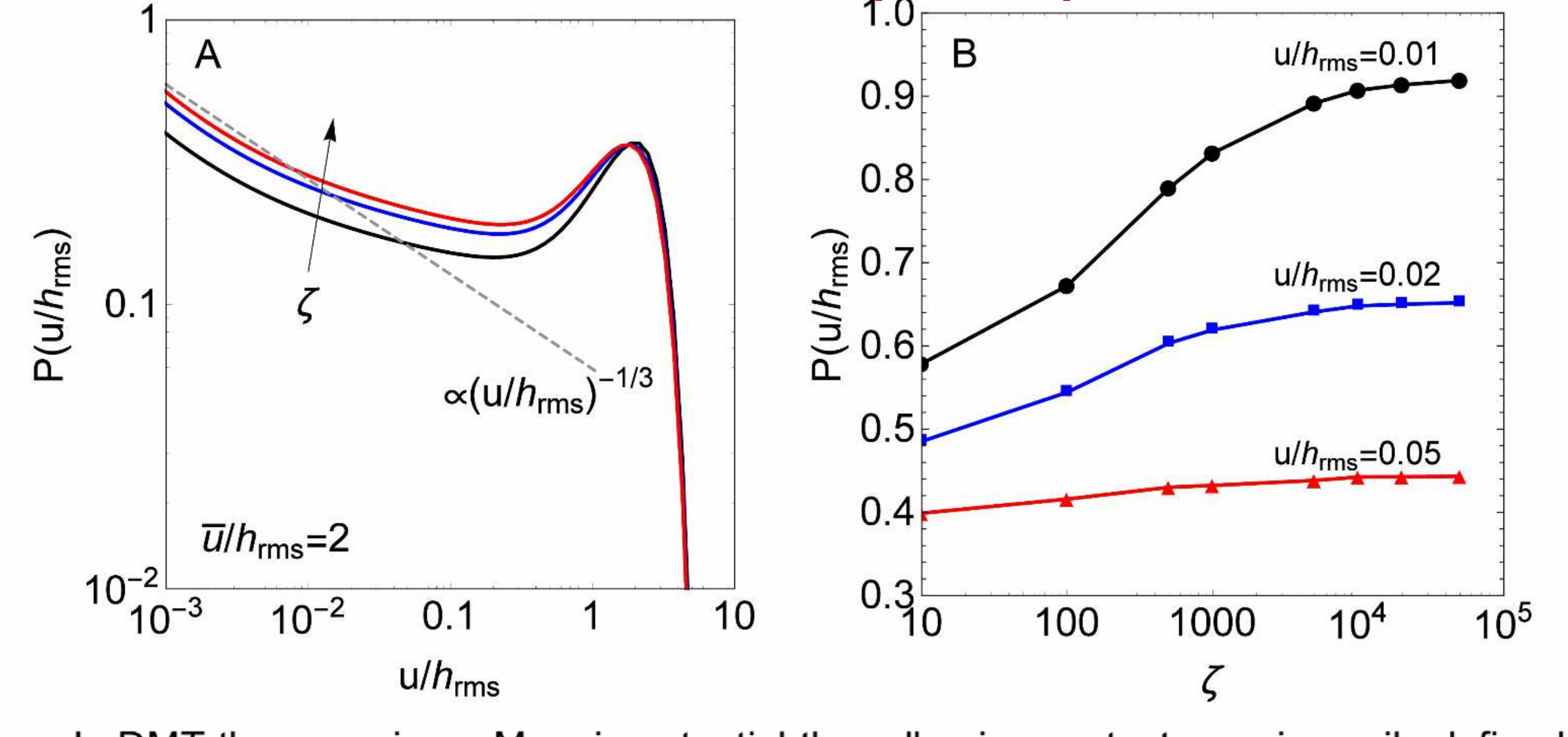
Attractive stress

Bearing Area Model For Adhesion - 3



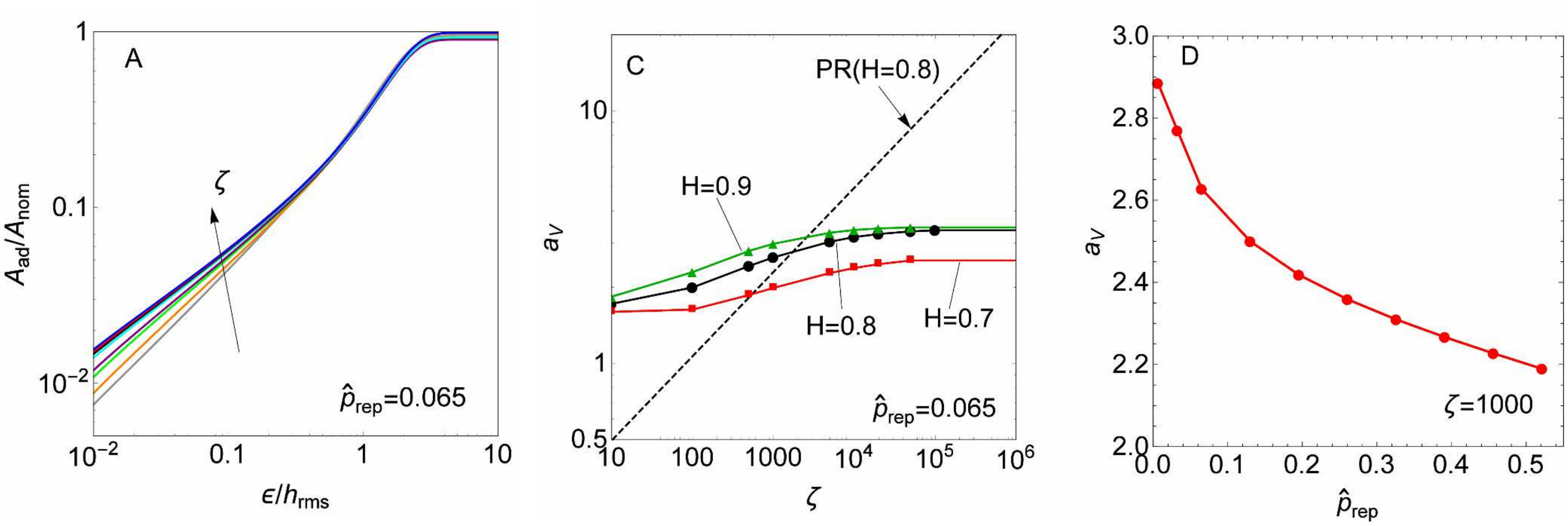
Full-BAM refers to the version including high fractal dimensions where there is dependence also on slopes. Comparison with PR data for pull-off shows reasonable agreement.

Stickiness of multiscale randomly rough surfaces (2018)



In DMT theory, using a Maugis potential the adhesive contact area is easily defined integrating the probability distribution of gaps P(u) from 0 to the Maugis range of attraction. We showed that P(u) converges with the magnification ζ, thus any criterion on stickiness within the DMT assumptions cannot depend on the PSD large wavevector components.

On stickiness of multiscale randomly rough surfaces



The attractive contact area can be written as

$$\frac{A_{ad}}{A_{nom}} = \frac{3}{2}\alpha_V \hat{p}_{rep} \left(\frac{\epsilon}{h_{rms}}\right)^{2/3}$$

where $\hat{p}_{rep}$ is the dimensionless average repulsive pressure, ϵ is the Maugis range, h_{rms} is the surface rms and α_V is a coefficient that can be obtained from the $\frac{A_{ad}}{A_{nom}}$ vs $\left(\frac{\epsilon}{h_{rms}}\right)^{2/3}$ curve

On stickiness of multiscale randomly rough surfaces

The external pressure is the sum between the repulsive and the adhesive contribution

$$\frac{p_{ext}}{E^*} = \frac{p_{rep}}{E^*} - \frac{\sigma_0}{E^*}\frac{A_{ad}}{A_{nom}} = \frac{A_{rep}}{A_{nom}}\frac{\sqrt{2m_2}}{2}\left[1 - \frac{l_a}{\epsilon}\frac{3}{2}\frac{a_V}{q_0 h_{rms}}\left(\frac{\epsilon}{h_{rms}}\right)^{2/3}\right]$$

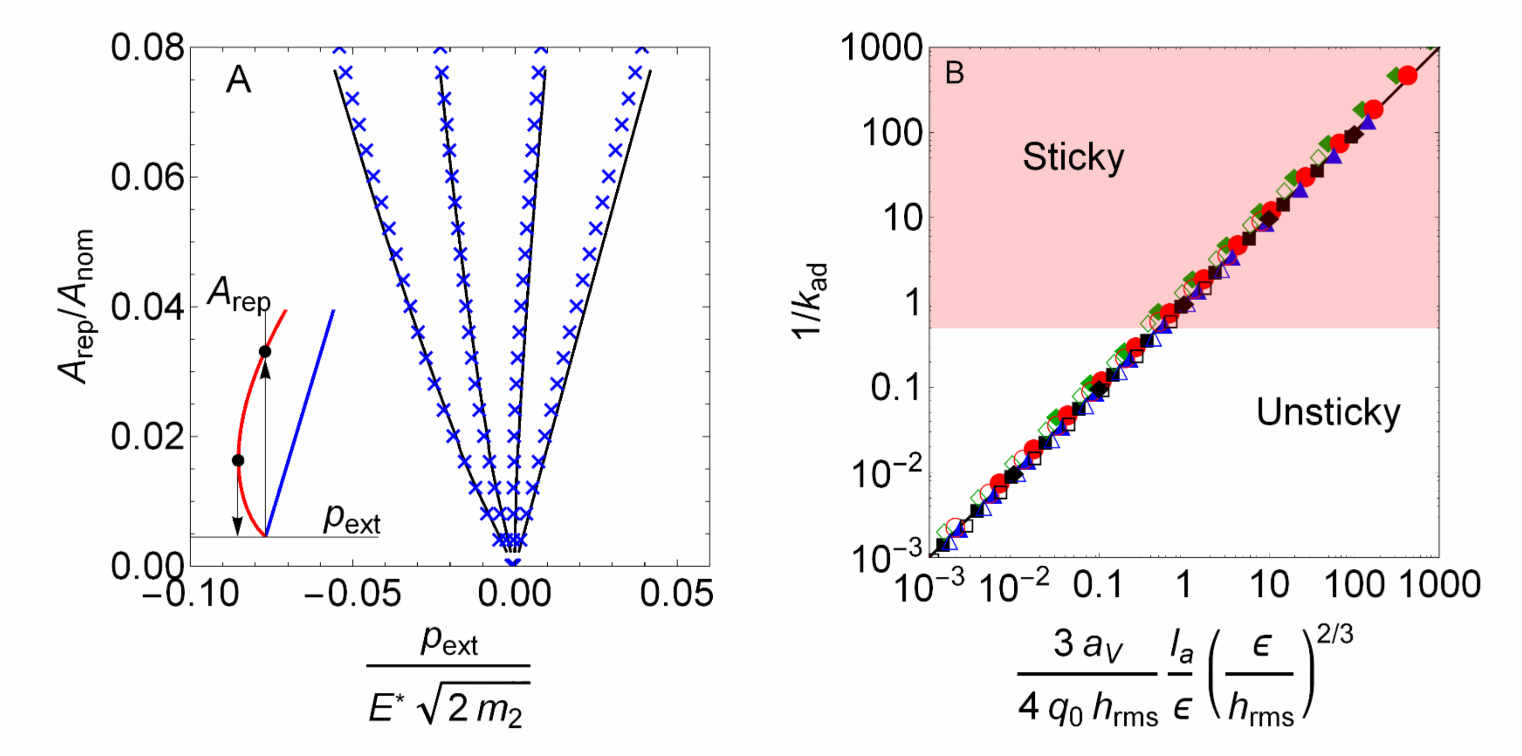
Stickiness is obtained when the slope becomes negative $\frac{1}{\kappa} = \frac{p_{ext}/\left(E^*\sqrt{2m_2}\right)}{A_{rep}/A_{nom}}$

In particular for low fractal dimension, high magnification and assuming $\alpha_V \approx 3$ (neglecting the weak dependence on pressure) simple criterion for stickiness is obtained

$$\frac{h_{rms}}{\epsilon} < \left(\frac{9}{4}\frac{l_a/\epsilon}{\epsilon q_0}\right)^{3/5}$$

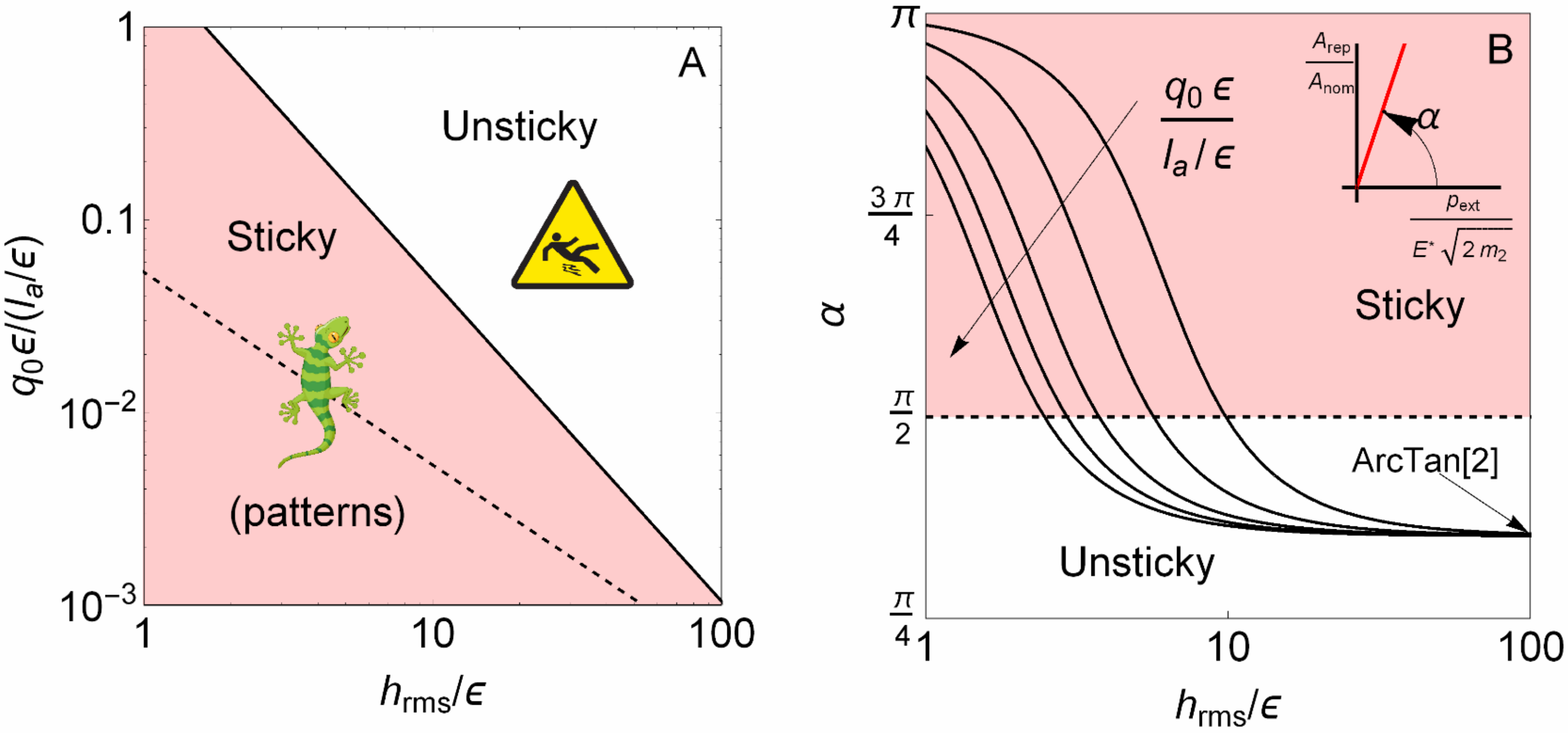
Which depends only on the shortest wavevector q_0 and on the surface RMS. Both are converging quantities with magnification. This contrast with the currently available stickiness criterion from Fuller and Tabor (1966) and Pastewka and Robbins (2014), which depend on the PSD truncation, thus are also difficult to check experimentally.

On stickiness of multiscale randomly rough surfaces



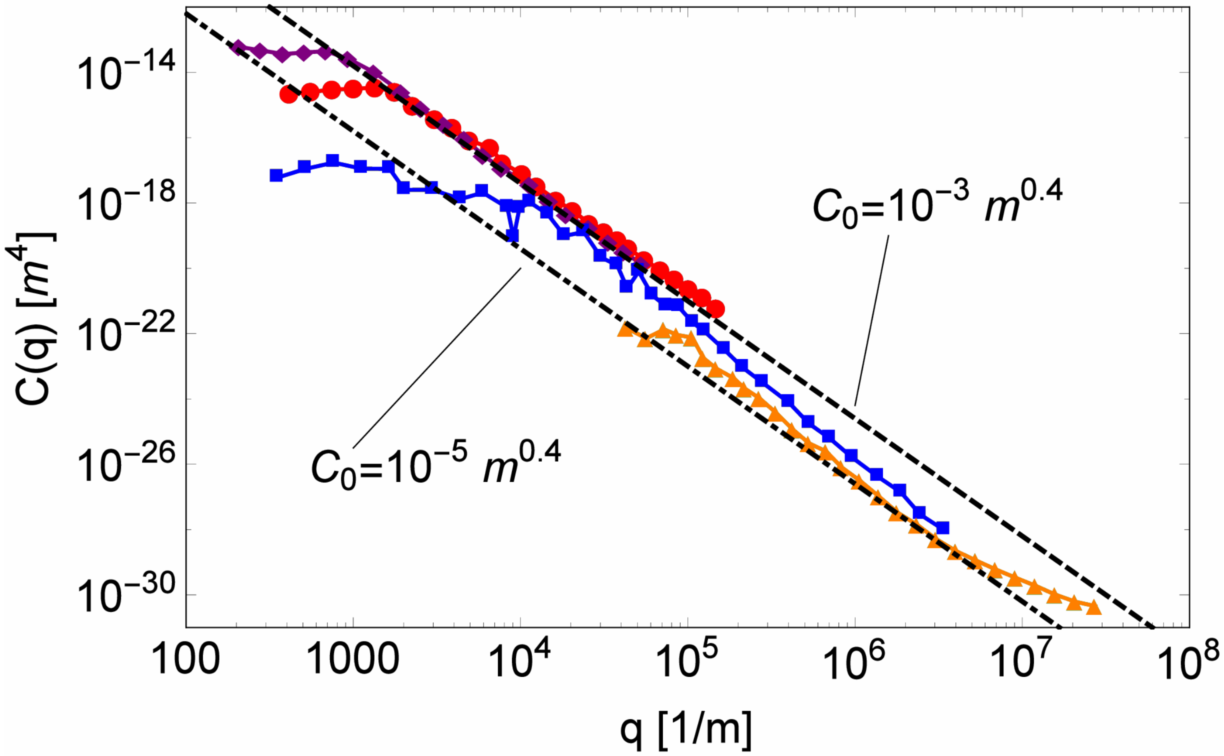
Our predictions has been checked against full numerical results obtained using the Contact-App from Lars Pastewka over a wide range of parameters.

On stickiness of multiscale randomly rough surfaces



Using our criterion for stickiness we can define a sticky and region where surfaces are expected to naturally snap into contact after a gentle approach. From JTB's results, we know that complex instabilities and patterns form at very low RMS amplitude of roughness, and hence in the sticky range, DMT type of analysis can be expected to hold only above the dashed line in panel A.

On stickiness of multiscale randomly rough surfaces



A self-affine randomly rough surface is defined by its PSD, $C(q) = C_0 q^{-2(H+1)}$. Our criterion can be written directly as a function of C_0. Assuming reasonable estimates for the rest of parameters we obtain that real surfaces should stick for $E^* < 0.3$ MPa, which is in agreement with the very well known Dahlquist criterion for adhesives.

Comparison with BAM

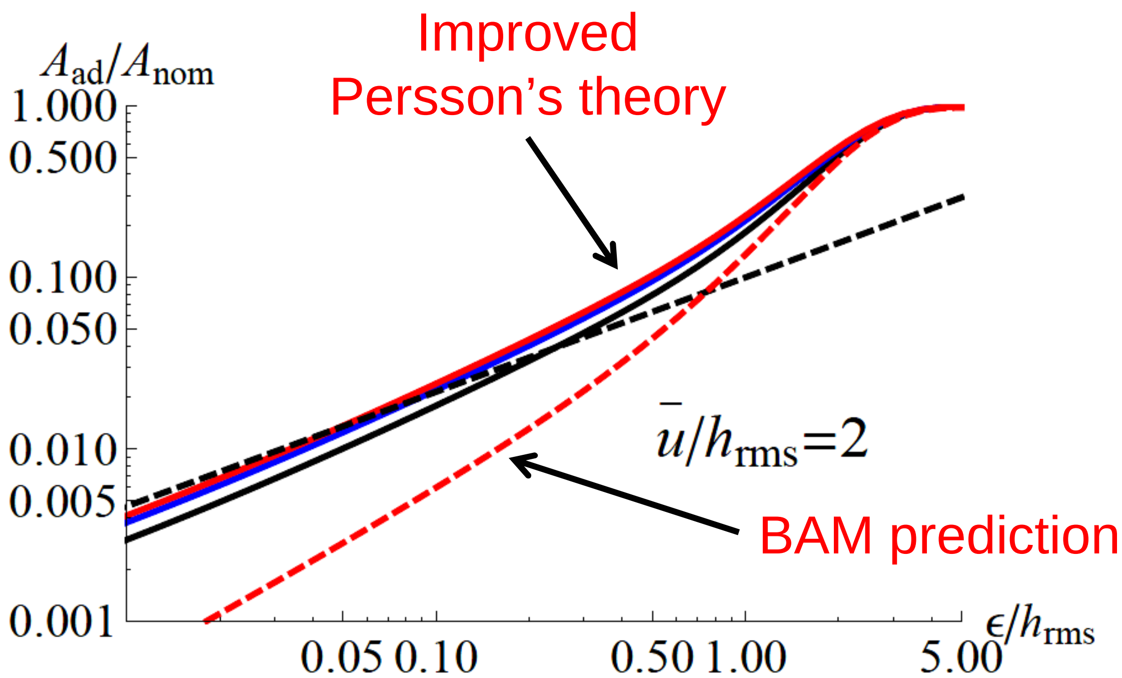


Fig.4. Comparison of the attractive area $\frac{A_{ad}}{A_{nom}}$ estimated by Persson's theory (black, blue and red line respectively for $\zeta = 10, 100, 1000$) as improved by Afferrante *et al.* (2018). Red dashed line shows BAM (Ciavarella, 2017) prediction, and dashed black line indicates a guide to the eye with $\left(\frac{\epsilon}{h_{rms}}\right)^{2/3}$. Case of Fig.2

Comparison with BAM

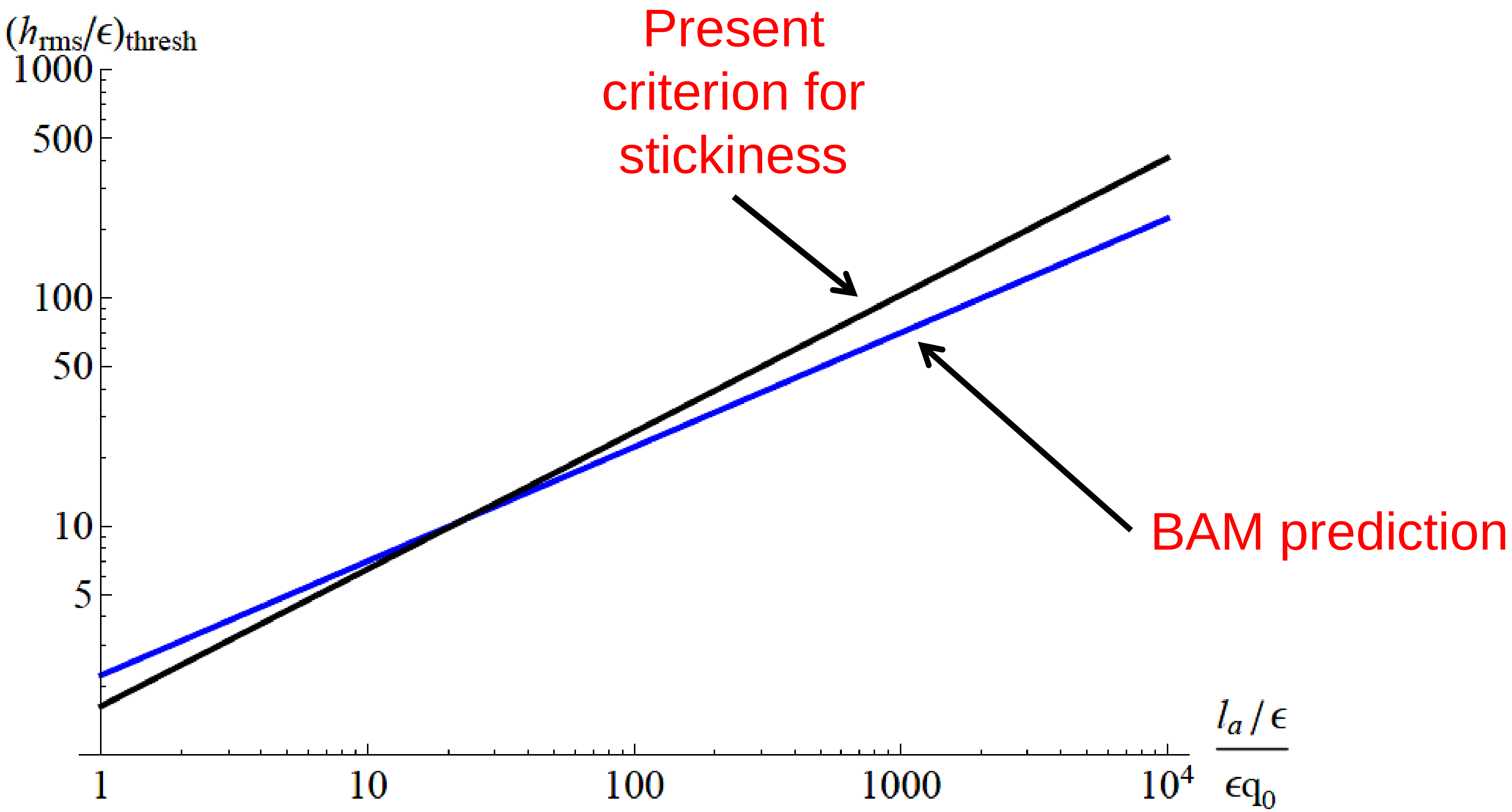
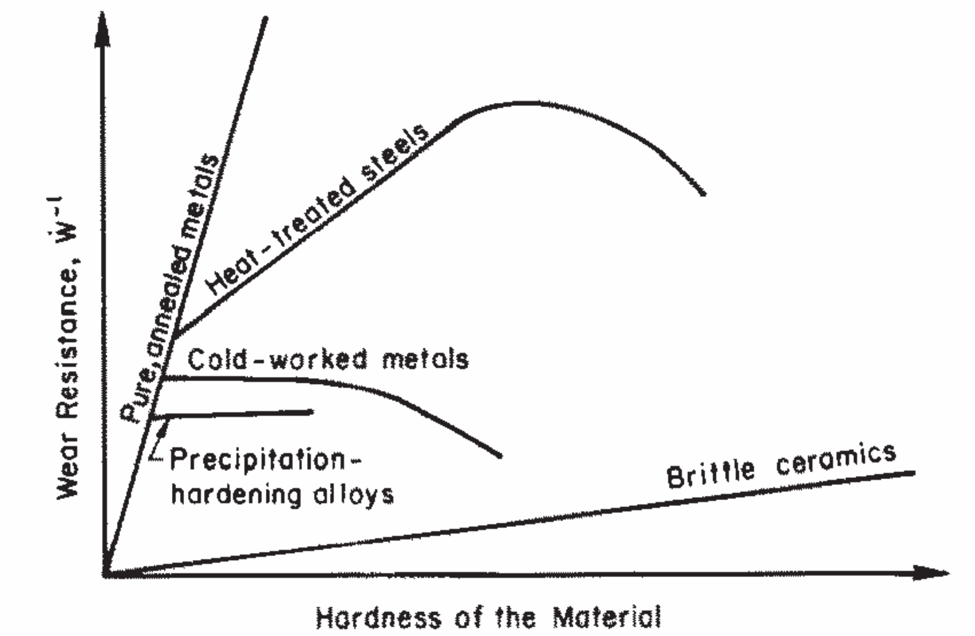


Fig.8. Estimates of the limit RMS amplitude of roughness for the BAM model (blue solid line) and the present one (black solid line).

Wear



Wear remains one the least scientifically understood tribological processes. The most common approach in wear refers to Archard [2] as wear volume *V* is proportional to the sliding length, the normal force, and inverse with the hardness *H* of the material. Hence, the wear rate is proportional to pressure *p*:

$$V' = \frac{kp}{H}$$

Critical Length Scale In Wear

- One of the interesting ideas for adhesive wear (one of the most prevalent types of wear) was suggested in 1958 by Rabinowicz [12] in *Wear* and was later forgotten (the paper has 12 citations in Google Scholar!), except rediscovered recently in numerical experiments by Molinari's group at EPFL.
- *There is a critical size (of contact radius), such that smaller fragments remain adherent while larger fragments come off in loose form.*

$$a^* = \lambda G \frac{w}{\sigma_j^2}$$

Aghababaei, R., Warner, D.H., Molinari, J.-F., 2016, *Critical length scale controls adhesive wear mechanisms*, Nature Communications, 7, 11816.

Aghababaei, R., Warner, D.H., Molinari, J.-F., 2017, *On the debris-level origins of adhesive wear*, Proceedings of the National Academy of Sciences, 114(30), pp. 7935–7940.

Predicting Archard Wear Coefficient?

- wear coefficients vary by more than 7 orders of magnitude.
- Archard *"We postulate: worn volume ~*a^3 *and effective sliding distance ~a, therefore, the contribution of this contact to the wear per cm of sliding ~*a^2*; also load supported by contact ~*a^2*. Therefore, for this contact, the contribution to the wear rate is proportional to the load supported by it. A similar argument applies to all other contacts, and the total wear rate is proportional to the load"*.
- Unconvincing: constant density of asperities with height, all asperities are wearing in his model, in contrast with the Rabinowicz–Aghababaei critical size concept.

Upscaling The Concept Of Rabinowicz Length Scale In Multiscale Contact.

- Frérot et al. [16] for example inserted the Rabinowicz-Aghababaei critical scale in the Archard model, i.e. assumed K is the probability that contact area A is larger than A^* (where A^* is the area of critical length scale a^*) for a given load W

$$K = P\left(A > A^*\right) = \int_{A^*}^{\infty} p\left(A,W\right) dA$$

- More precisely, starting from analytical predictions, using the quasi-realistic Greenwood-Williamson model with exponential distribution of asperities $\varphi=(C/\sigma_s)\exp(-z_s/\sigma_s)$ for $z_s>0$, and σ_s being a scale parameter of the order of RMS amplitude, $p(A,W)$ turns out a negative exponential independent of the applied load W

$$p\left(A,W\right) = \frac{1}{b\sigma_s} exp\left(-\frac{A}{b\sigma_s}\right)$$

Frérot, L., Aghababaei,R. Molinari, J.F., 2018, *On the understanding of the wear coefficient: from single to multiple asperities contact*, submitted, JMPS

Wear Paradox

- Therefore, (i) the wear coefficient *K* is also independent of the load,

$$K = exp\left(-\frac{A^*}{b\sigma_s}\right)$$

- (ii) there is a linear relationship between the contact area and the applied load in both elastic and plastic cases.
- However, the sensitivity to radius to resolution of the instrument (or to truncation of the fractal process) is extremely high, this would mean that in the fractal limit the *elastic model predicts always infinitesimally small wear*! Notice this is not just consequence of the asperity model

Simple Way Out Of Paradox?

- Pei et al. show that plasticity produces distribution of contact clusters closer to the very simple overlap model than to the elastic model.

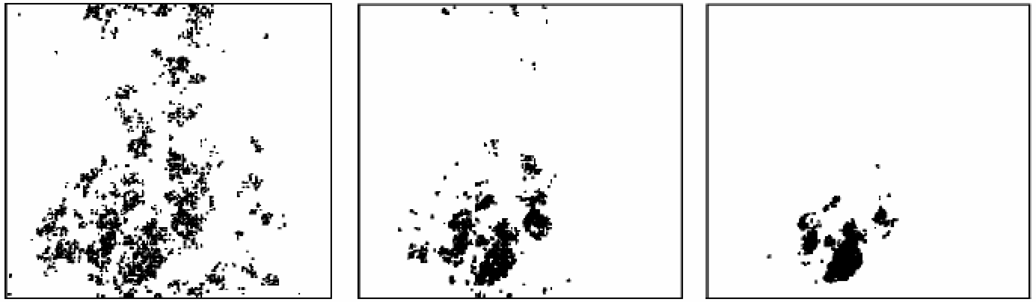


Fig. 2 An example of contact area prediction for: a) elastic; b) elasto-plastic and c) rigid overlap model (adapted from [19])

Pei, L., Hyun, S., Molinari, J. F., & Robbins, M. O., 2005, *Finite element modeling of elasto-plastic contact between rough surfaces*, Journal of the Mechanics and Physics of Solids, 53(11), 2385-2409.

Predicting Friction? A plastic model

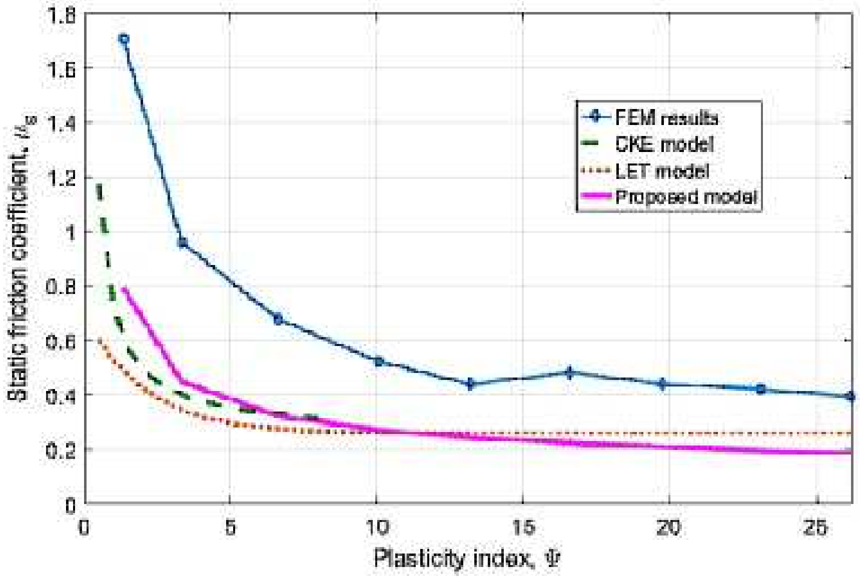


Fig. 22 Comparison of static friction coefficient for generated Gaussian surfaces with various plasticity indices under a dimensionless normal load $F_n/(A_n S_y) = 0.155$

$$\psi = \frac{2E'}{\pi C S_y}\sqrt{\frac{\sigma_s}{R}}$$

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ORIGINAL PAPER

Theoretical and Finite Element Analysis of Static Friction Betwee
Multi-Scale Rough Surfaces

Xianzheng Wang[1] · Yang Xu[1] · Robert L. Jackson[1]

Interesting that for fractal limit, friction coefficient tends to a universal value 0.25 (which is curiously enough, the value predicted by Leonardo 500 years ago!) Strangely instead, the low plasticity index and large R leads to strange infinite friction...

Intermediate conclusions - 1

- The contact of rough surfaces has been studied for a long time (from 1957), with the hope to explain friction, adhesion, wear and other tribological problems
- Some considerable progress has been made in the area of elastic contact, where we know in details the solution for nominally flat, infinite surfaces. The choice of elasticity is mainly for mathematical convenience!
- However, friction has been studied since the times of Leonardo and in 500 years, no predictive model has emerged, nor significant improvement from rough contact models. Indeed, plastic models of friction today are more well posed and recognize f=0.25 like Leonardo predicted in his notebooks!

Intermediate conclusions - 2

- For adhesion, we described progress with BAM by Ciavarella (2017) and with the criterion of Violano, Afferrante, Papangelo & Ciavarella (2018) who have found early claims to be incorrect: stickiness does not depend on truncation of the PSD spectrum
- Recently, rough contact models have also been attempted in the hope to predict the coefficient of proportionality between wear loss and friction dissipation which was observed already by Reye in 1860, and then Archard in the 1950's. (both papers were ignored in the new papers in top journals!)
- Resolution-dependence of contact area make the models very ill-defined, and many predictions quite hard.

Conclusions

- the contact “sport” of simulating elastic multiscale contact with fractal accurate models that has dominated the specialized literature (including my own contributions!), is mostly a little remote still from real tribological problems
- With fractals, it is easy to end up with paradoxical limit conclusions. We made the example of two important theories in adhesion. The way out of the paradox requires very significant effort (Persson’s solution) or simple but clever ideas (BAM).
- And after this effort, we are just about to seeing perhaps some really quantitative applications….