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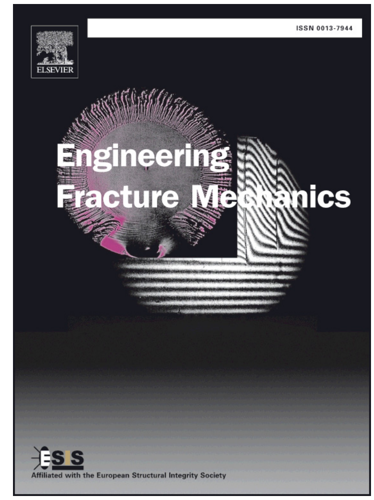

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A simple finding on variable amplitude (Gassner) fatigue SN curves obtained using Miner's rule for unnotched or notched specimen

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Abstract

In this note, starting from the SN curve under Constant Amplitude (CA) for the fatigue life of the uncracked (plain) specimen, we obtain that Gassner curves for Variable Amplitude (VA) loading using the simple Palmgren-Miner's law are simply shifted CA curves. Further, using the Critical Distance Method in a very clean and powerful form proposed by Susmel and Taylor for VA loading, we find similar result for notched specimen, the spectrum loading results in the same multiplicative term for notched, cracked and unnotched specimen. Hence, the present proposal can be considered as a simple empirical unified approach for rapid assessment of the notch effect under random loading, which simplifies the recent proposal by Susmel and Taylor. To their extensive validations, we add some specific comparison with experimental data from the Literature on our further findings.

Keywords: Fatigue, notches, Medium-Cycle fatigue, critical distance approach, random loading

Nomenclature
a = notch or crack (half) size

N= number of cycles
N_∞= number of cycles to failure for "infinite life" (fatigue limit)
a_0= El Haddad intrinsic crack size (infinite life)
$a_0\,(N)$= "finite life" El Haddad intrinsic crack size
C, m= Paris' "material constants"
K_f = fatigue strength reduction factor for infinite life
$K_f\,(N)$= "finite life" fatigue strength reduction factor
K_t = elastic stress concentration factor
ΔK_{th} = fatigue threshold for long cracks
σ_g= remote nominal (gross) stress
$\Delta\sigma_L$ = plain specimen fatigue limit (in terms of stress range)
R= stress ratio

α = geometric shape factor
k = inverse slope of the Wöhler SN curve
CA =Constant Amplitude
VA =Variable Amplitude

1 Introduction

The classical approach to fatigue (e.g. [1-4]), uses a number of heuristic but simple corrective factors, mainly for infinite life ("safe-life" design approach), but also for "finite life": one of the classical examples being the suggestion to interpolate between a static strength, and a infinite life value (fatigue limit), see e.g. Fuchs and Stephens [2]. Fracture Mechanics has introduced new material constants (and ways to test materials) namely fatigue threshold, static toughness, Paris' law constants C, m, but this results in a separate perspective with respect to standard design approaches. For infinite life, the connection between the classical approach and the new "fracture mechanics" is simple: Smith and Miller [5] suggested that notches behave like cracks if they are large and sharp ("crack-like" notches), their infinite life is ruled by ΔK_{th}, the threshold value of the stress intensity factor range for which long cracks do not propagate according to Paris' law [6][1]. For infinite life, an important transition is from short crack (fatigue-limit dominated) to long-cracks (fatigue-threshold dominated) [7], occurring at crack sizes of the order of

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_L}\right)^2 \tag{1}$$

where $\Delta\sigma_L$ fatigue limit range (at a given load $R-$ratio). This length is of the order of $100\mu m$ for many metals (for typical values of R between 0 , -1). Kitagawa and Takahashi [8] introduced the $\Delta\sigma - a$ diagram which bears now their name, and showed the transition in a clear way, and El Haddad *et al* [9] simply added a_0 as an "intrinsic" additional crack size for the threshold equation, resulting in a simple interpolating formulae for the infinite life strength $\Delta\sigma_\infty$

$$\Delta\sigma_\infty = \Delta K_{th}/\sqrt{\pi\left(\alpha a + a_0\right)} \tag{2}$$

where α is a geometrical factor introduced to correct the stress intensity factor with respect to that used in defining a_0 (usually, the central crack in the infinite plate). The El Haddad idea belongs to the class of "critical distance" heuristic methods (see also the recent book by Taylor [10]) starting from the early suggestions by Neuber and Peterson for the fatigue knock-down factor K_f. Ciavarella and Meneghetti [11] suggest a single criterion with the transition from "El Haddad" to "Lukáš and Klesnil" K_f . When dealing with finite life, one could approach the problem in various ways: we could integrate Paris' law, which requires a correction for short cracks, or try to derive a generalized "finite life" Kitagawa diagram as done in Ciavarella [12].

In particular, Ciavarella [12] defines "naturally" a power law for the "intrinsic" crack size $a_0(N)$ (dependent on life)

$$a_0(N) = a_0\left(\frac{N_\infty}{N}\right)^{2(1/r-1/k)} \tag{3}$$

[1]However, in the case of large notches of sufficiently large tip radius, the fatigue limit is really fully controlled by K_t and the notch is defined as "blunt" notch.

where a candidate for the exponent r is Paris' constant m (hence, about 2-4 for metals, and 10 or higher for ceramic materials).

This equation has similarities with the recent Susmel and Taylor [13,14,15,16] "critical distance" (Point) method where the fatigue life is evaluated considering the stress at a single "point" whose distance from the stress raiser is indicated as $L/2$ (it corresponds to $a_0(N)/2$) also given in a power law form

$$L(N) = AN^B \tag{4}$$

where *"the constants of the above equation are expected to be different for different materials and different load ratios, R".* These constants (A, B) are determined by them as a best fit directly on notched data or else from basic material properties depending on the variant of the method. In all cases, the stress useful for assessment of fatigue failure is that at a critical distance $L(N)/2$ from the notch (or crack) vertex.

1.1 Variable amplitude loading

Recently, Susmel and Taylor [15] have suggested an extension of the critical distance method for finite life of notches under Variable Amplitude (VA) loading, and have extensively validated their approach using numerous experimental results generated by testing notched cylindrical samples of a commercial medium-carbon steel containing three different stress concentration levels, and two types of spectra (a conventional Rayleigh distribution, and another emphasizing cycles of low stress amplitude). In [16], they further generalize the method to multiaxial loading, and again validate the accuracy and reliability of the technique against 124 experimental results testing notched cylindrical samples of carbon steel C40 under three type of spectra.

The critical distance method under VA loading relies on two further (quite strong, in principle) assumptions:

- that the critical distance is independent on the spectrum, and depends only on the final life of the specimen: this is, at least, the variant they propose for the generalization of the method for multiaxial loading [16], and which we shall refer to in the present paper.

- that we can apply (Palmgren-)Miner's law (linear damage rule), which suggests for a given block with a total number of cycles per block N_c that damage will be

$$D = \sum \frac{n_i}{N_i} \tag{5}$$

  where n_i is the number of cycles spent at level i on the stress amplitude, and N_i is the total number of cycles the specimen could resist at that level of stress, according to the constant amplitude (CA) SN Wöhler curve. Failure according to the Miner rule should occur at critical damage of $D_c = 1$.

Although Miner's linear rule can be quite approximate and on the unsafe side, it is by far the most well known and used damage summation law: the suggested strategy by handbooks is to simply assume a lower D_c. FKM-Guideline [17] recommends $D_c = 0.3$ for steels, steel castings, aluminum alloys, while $D_c = 1$ for ductile iron, grey cast iron malleable cast iron. Sonsino [18, 19] and Schijve [20] suggest testing is always the best choice of course when possible (modern trends to refer to "virtual testing" in fatigue are quite suspicious). However, even testing (aside from the obvious very large costs) is not obvious and involves an art on its own, which is developed in certification rules for safety-critical applications (now both for metals and composites [21]) involving the concepts of Life Factors (LF) or Load Enhancement Factor (LEF): essentially, under a given service loading, a single test can assess a given same reliability (typically assuming a Weibull distribution) only increasing the load or the life with respect to the mission; the former version is preferred of course as it is much accelerated, although accelerated testing may introduce problems if time-dependent phenomena affect fatigue process. Often small amplitude cycles are omitted for simplicity and accelerating testing (and similarly in the original Palmgren-Miner rule cycles below fatigue limit are omitted from computation of the damage), although in some design handbooks, especially in welded joints, these are known to produce fatigue damage. Indeed, low amplitude cycles can be dealt with according to the "*Palmgren–Miner–Elementary method*" with prolongation of the Wöhler curve below the knee point with the same slope, or according to Haibach [22] with a reduced slope — the former method is more conservative of all, and will be adopted in the present note, without loss of generality. In other words, *we assume the CA SN Wöhler curve to extend to infinity, for variable amplitude calculations.* This is particularly true for materials which do not show a clear fatigue limit, like light alloys, for which Haibach correction would not be required anyway.

At the other extreme, it is often believed that high amplitude cycles may be beneficial as they induce favorable residual stresses at notches – in order to avoid this, which would lead to unconservative estimates, also these are eliminated from the spectra in most testing. In our simple calculation, instead, we don't take any account of these effects, neither for cracks and notches, as we are aiming at very simple "design level" results. Moreover, in many cases, it is still debatable whether load spectra are known with sufficient accuracy, if cycle-counting methods (such as rainflow) are reliable (i.e. if load-sequence effects are not important), so this is why Miner's law is still very much used, and this is why we cannot expect our results to substitute testing by all means. They are just going to suggest better ways to plot Gassner curves than what presently done, and what we may expect when applying Miner's rule according to Critical Distance Method in complex situations, perhaps coming from Finite Element results of the stress fields. In most cases, we expect to be able to avoid the Finite Element analysis.

2 Gassner curves with unnotched material

Let us start with assuming Basquin's law holds

$$N_S\left(\sigma_S\right)^k = N_\infty\left[\sigma_L\right]^k = N\left[\sigma\left(N\right)\right]^k = C_W \tag{6}$$

where we have written the law also at the extreme points at which Basquin holds, i.e. at some value σ_S close to static strength with very low number of cycles N_S, and at at the "fatigue limit" σ_L at N_∞. Once again, in the variable amplitude case, we disregard the existence of the fatigue limit, and extend Wöhler line to infinity.

According to Miner's rule for a given block with a total number of cycles per block N_c, damage will be (using (6))

$$D = \sum\left(\frac{n_i}{N_i}\right)^\alpha = \sum\frac{n_i}{N_c}\frac{N_c}{N_i} = N_c\sum\frac{\alpha_i}{N_i} = \frac{N_c}{C_W}\sum\alpha_i\sigma_i^k \tag{7}$$

where $\alpha_i = \frac{n_i}{N_c}$ is the proportion of cycles spent at level i on the total number N_c. The life under the sum of all i blocks is N^*,

$$\frac{1}{N^*} = \frac{D}{N_c} = \frac{1}{C_W} \sum \alpha_i \sigma_i^k \tag{8}$$

Therefore, normalizing the block with the peak tension of the block $\sigma_{\max}$, so that $\sigma^* = \beta\sigma_{\max}$ and we change the factor β to obtain a full Gassner curve, it follows

$$\frac{1}{N^* (\sigma^*)} = \frac{\sigma^{*k}}{C_W} \sum \alpha_i \left(\frac{\sigma_i}{\sigma_{\max}}\right)^k = \frac{\sigma^{*k}}{C_W} G \tag{9}$$

where

$$G = \sum \alpha_i \left(\frac{\sigma_i}{\sigma_{\max}}\right)^k \tag{10}$$

is a "spectrum factor" which shifts the Basquin curve in the log-log diagram, or else we have the Basquin law (6) with a new

$$C_W^{VA} = C_W / G \tag{11}$$

Obviously, with $G = 1$ we return to the CA Wöhler curves. We can also write this as

$$N \left[\frac{\sigma^* (N)}{G^{-1/k}}\right]^k = C_W \tag{12}$$

or else we can plot the Gassner curve as superposed to the Wöhler curve, if we use instead of the usual $\sigma^* (N)$ scale, the scale

$$\frac{\sigma^* (N)}{G^{-1/k}} \tag{13}$$

which indeed will be used in all our subsequent plots, because we will see that this result will have very wide and interesting generalizations. But already in this simple form, and despite the simplicity of this result, it is not known to the authors that this result has been obtained before.

3 Gassner curve with a crack

From the introduction, we have recognized that in many cases (crack-like notches), intuitively when notches are sufficiently "sharp", the behaviour of a notch is not too dissimilar from that of a crack. Let us assume the limit case of a Griffith crack in a infinite plate for which it is possible to derive very simple results, which will inspire further numerical tests. The stress field, for the asymptotic region $r \to 0$ is given by

$$\sigma (r) = \frac{K_I}{\sqrt{2\pi r}} = \frac{\sigma_g \sqrt{\pi a}}{\sqrt{2\pi r}} \tag{14}$$

where σ_g is the remote stress. The critical distance local approach suggests we should use $\sigma (r (N))$ to evaluate fatigue life. In other words, we have a spectrum of $\sigma_{\infty,i}$ values, which give a spectrum of $\sigma (r (N))$ values, where we take (either Ciavarella [12] or Susmel-Taylor variants [13-16]) a critical distance of the form

$$r (N) = \frac{a_0 (N)}{2} = \frac{A}{2} N^B \tag{15}$$

which is calibrated according to either some tests if available, or with some material constants.

Substituting (15) into (14),

$$\sigma_i(r) = \frac{K_{I,i}}{\sqrt{2\pi r}} = \frac{\sigma_{g,i}\sqrt{\pi a}}{\sqrt{\pi A N^B}} \tag{16}$$

and for a given block, using Miner's rule

$$\frac{1}{N^*} = \frac{D}{N_c} = \frac{1}{C_W}\sum \alpha_i \left(\frac{\sigma_{g,i}\sqrt{\pi a}}{\sqrt{\pi A (N^*)^B}}\right)^k = \frac{1}{C_W}\left(\frac{\sqrt{\pi a}}{\sqrt{\pi A (N^*)^B}}\right)^k \sum \alpha_i (\sigma_{g,i})^k \tag{17}$$

Notice that this equation has unknown N^* but it is simple to solve

$$(N^*)^{\frac{Bk}{2}-1} = \frac{1}{C_W}\left(\frac{a}{A}\right)^{k/2} \sum \alpha_i (\sigma_{g,i})^k \tag{18}$$

Therefore, normalizing the block with the peak tension of the block $\sigma_{g,\max}$, so that $\sigma_g^* = \beta\sigma_{g,\max}$ we can plot the Gassner curve

$$\left(N^*\left(\sigma_g^*\right)\right)^{\frac{Bk}{2}-1} = \frac{\left(\sigma_g^*\right)^k}{C_W}\left(\frac{a}{A}\right)^{k/2}\sum \alpha_i \left(\frac{\sigma_{g,i}}{\sigma_{g,\max}}\right)^k = \frac{\left(\sigma_g^*\right)^k}{C_W}\left(\frac{a}{A}\right)^{k/2} G \tag{19}$$

where G remains as for the unnotched specimen. However, notice that the new curve can be written as

$$\left[\sigma_g^*\right]^{\frac{k}{1-\frac{Bk}{2}}} N^* = \left[\left(\frac{a}{A}\right)^{k/2}\frac{G}{C_W}\right]^{\frac{1}{\frac{Bk}{2}-1}} \tag{20}$$

and therefore would have the same slope as Basquin only if $1 - \frac{Bk}{2} = 1$ i.e. if a_0 stayed constant. Otherwise, the slope is changed for the notch and is

$$k_n = \frac{k}{1-\frac{Bk}{2}} \tag{21}$$

Obviously, a larger notch (bigger a) implies a short life. Notice that the same curve can be written as

$$\sigma_g^* (N^*)^{\frac{1-\frac{Bk}{2}}{k}} = \left[\left(\frac{a}{A}\right)^{k/2}\frac{G}{C_W}\right]^{-\frac{1}{k}} \tag{22}$$

where sometimes the notation $-\frac{1}{k} = b$ is used, especially in Coffin-Manson version of the Basquin law. Notice that this means that, in terms of stress amplitude, *the SN curve is shifted exactly of the same amount of the unnotched SN curve when obtaining VA data*. This is a second result of the present paper, although at the moment it only appears in a rather limit case of a crack. But we shall see that this result holds with an excellent degree of approximation also in much more general conditions.

4 Examples

We shall consider here the results obtained with a classical example of a hole in a infinite plate (Kirsch solution [23]), and the Griffith crack, this time using the full Westergaard's solution [24]. We underline that as the stress field for the notch has a limit value ($K_t = 3$ as well known), the stress ahead becomes in a region higher than that of a crack (see Fig.1). This will have the obvious consequence that for some intermediate regime, the Critical Distance Method predicts for the hole a more detrimental effect in fatigue than for the crack.

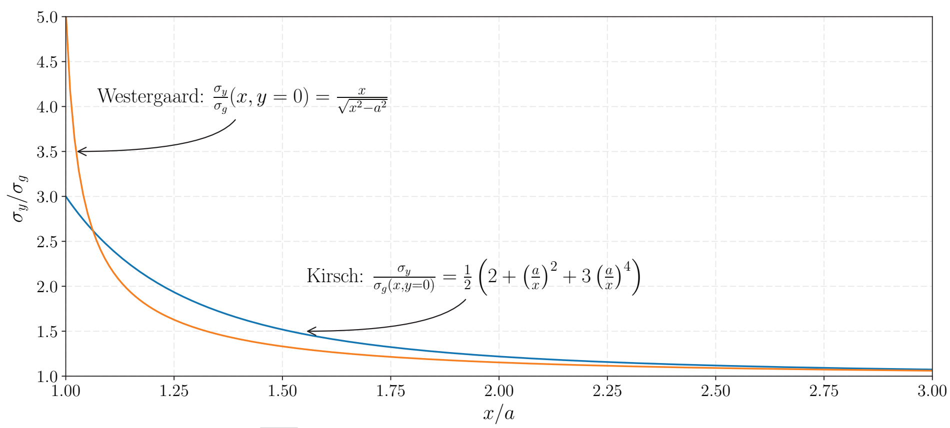

Figure 1: The stress field ahead of a central crack in the infinite plane (Westergaard solution), or of a hole (Kirsch solution)

We therefore move to consider an example material. Using $K_{Ic} = 30MPa\sqrt{m}$, $\Delta K_{th} = 2.7MPa\sqrt{m}$, ultimate strenght $\sigma_U = 800MPa$ (where at $N_S = 10^3$ we take $\sigma_S = 0.9\sigma_U = 0.9 \times 800MPa$) fatigue limit at $N_\infty = 10^6$ cycles of $\sigma_L = 400MPa$ (here by fatigue limit, we mean a conventional one, but the material is assumed to have no real fatigue limit, to simplify the presentation, but this does not affect the generality of the results). We use the equations suggested by [14] based on standard material constants, and not fitting on notched specimen: namely $a_0(N_\infty) = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{2\sigma_L}\right)^2 = \frac{1}{\pi}\left(\frac{2.7}{800}\right)^2 = 3.6\mu m$ and $a_0(N_S) = \frac{1}{\pi}\left(\frac{K_{Ic}}{2\sigma_U}\right)^2 = \frac{1}{\pi}\left(\frac{30}{1600}\right)^2 = 0.112\,mm$, from where we compute the power law between the two values, and we assume a_0 continues to decrease beyond N_∞.

Fig.2 indicates the result of our calculations. The SN curves for notched and cracked specimen with two different sizes ($a = 2a_0$ or $a = 12a_0$) are indicated (hole in a plate, red line), and correctly lead to the unnotched Wöhler curve at low number of cycles. Notice that the notched SN curves lead eventually to the Wöhler unnotched lowered by a constant value $K_t = 3$, as in the model we have assumed that $a_0(N)$ continues to decrease even below the fatigue limit, and hence eventually the stress concentration factor effect is found.

But the main, surprising result of the curves is that the data obtained with the numerical procedure (which finds the "critical distance" by iterations) indicated with discrete symbols is extremely close to a simple prediction using just the Wöhler curve in the Critical Distance Method, which does not require any numerical evaluation. Since this result has been obtained analytically for the asymptotic field of a crack, and then numerically also for the hole in the plate and with the full field of the crack (for different dimensions of the notch/crack), we believe that the conclusion is sufficiently general. Therefore, simply plotting Gassner VA curves as $\frac{\sigma^*(N)}{G^{-1/k}}$ as suggested by our equation (13) is sufficient to confuse them with the Wöhler curves under the same condition, once the spectrum factor G has been computed. The iterative solution suggested by Susmel and Taylor [15] seems not needed. Notice that the difference between a crack and a hole is not too large at low number of cycles, as it is relatively established fact, whereas starts to be much more important at longer lives, where for the crack we should eventually truncate at fatigue threshold levels.

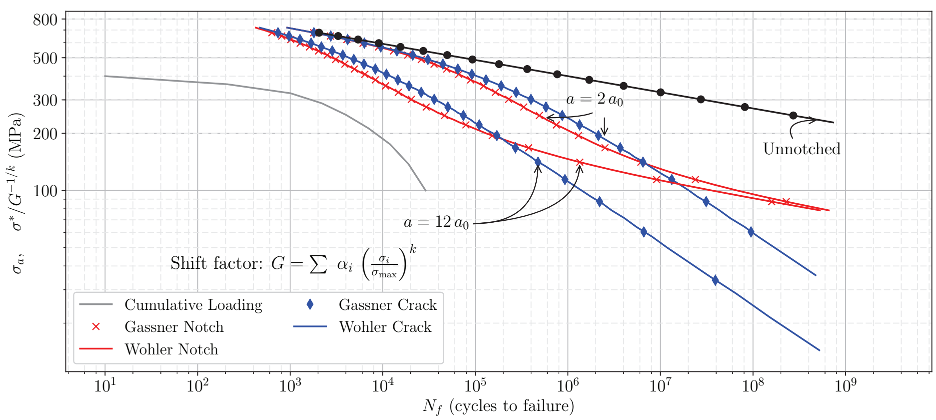

Figure 2: SN curves for notched specimen (hole in a plate, blue line), cracked specimen (red line) and unnotched specimen (black line). Discrete data with various symbols are Gassner VA curves (plotted as $\frac{\sigma^*(N)}{G^{-1/k}}$ as suggested by our equation (13)) obtained with the numerical calculation using Critical Distance Method described in the paper, for the cumulative spectrum indicate in the figure by solid gray line, and are almost indistinguishable from the Wohler lines obtained under CA loading.

5 Some quantitative validation

Susmel and Taylor [15] have already provided some evidence of the validity of their method, despite of course the main general warnings against Palmgren-Miner's law should be taken into account. Therefore, what we need to validate here is really the finding that the VA and CA curves can really be superposed as it is beautifully indicated in Fig.2 It is difficult unfortunately to find known and reliable data from the literature about Gassner curves, including the details of spectrum. One simple semi-direct way to do this is to use data from Sonsino and Dieterich [25]. To estimate the Wöhler k slope, we consider their Table 3 (Cyclic material data for unnotched specimen). There is the entire Coffin Manson law which for cast magnesium alloys AZ 91 and

AM50 and strain ratio $R = -1, 0$ — gives a slope factors of very similar value $k \simeq 5.60$ to $k \simeq 5$. This is extremely close to the slope $k_n = 5$ for the notched data ($K_t = 2.5$ in Fig.7,8,9 under both $R = -1, 0$ ratio, for all alloys. We therefore notice:-

- the slope of Gassner curve for notched data is indeed unchanged from CA or VA loading, see Fig.6,8,9 of [25] i.e. in the entire set of data, and within the measured life intervals (obviously near static failure, we expect Wöhler and Gassner curves to start off from virtually the same point). This confirms, independently, the (20) and (9) curve exponents
- From (21), $1 - \frac{Bk}{2} = \frac{k}{k_n} = \frac{5.6}{5} = 1.12$ or $B = -\frac{0.12 \times 2}{5.6} = -0.0429$. This could be used to further quantitative estimates.
- Considering $N = 10^5$ as reported in Table 4 of [25], the shift in stress amplitude SN data in the notched and unnotched data, according to our (22), should be identical. In Tab.1 we report the ratio CA/VA life for the notched and unnotched data. As it can be seen, the difference is only a few % (and has different signs, so it is within the scatter we would expect from fatigue data anyway), and this confirms again another aspect of our finding.
- Obviously, the knee point of fatigue limit is a more difficult quantity to estimate.

	$K_t = 1$			$K_t = 2.5$			% difference
	(CA)	(VA)	ratio CA/VA	(CA)	(VA)	ratio CA/VA	in ratios
AZ91,R=-1	84	163	0.52	61	132	0.46	-11.5
AZ91,R=0				38	91	0.42	
AM50,R=-1	70	157	0.45	49	101	0.49	8.9
AM50,R=0				36	82	0.44	
AM20,R=-1	61	126	0.48	43	85	0.51	5.9
AM20,R=0				32	72	0.44	

Tab.1 - Elaboration of data from Tab.4 of [25]. Fatigue strength amplitudes σ_a at N=10^5 cycles with probability P=50%. The ratio of CA/VA is nearly the same for notched and unnotched data, as we predict.

6 Discussion

6.1 Calibrating constants

The Critical Distance Method (CDM), well described in its general principle and historical origins in the recent book by Taylor [10], has apparently a simple implication for VA loading, within the assumptions made by the present note, which in turn are exactly those proposed by Susmel and Taylor, at least in one of the variants of the method. CDM has really a great potential of unifying many of the known effects of fatigue, as it appears from the fact that, using the full stress field ahead of a notch or of a crack, the entire SN curve is obtained naturally. Clearly, the calibration of the A, B constants is a critical point, in assessing the method, although it is not crucial for the scopes of the present note, and therefore we give only some comments. Ciavarella [12] and Susmel-Taylor [13-16] propose slightly different approaches, where both have received some validation with respect to experimental data. In the case of a large crack in a infinite plate, we derived in Ciavarella [12] that Susmel-Taylor best-fitting "free constants" would be

$$A = \frac{1}{2\pi} C_K^{2/m} / C_W^{2/k} \tag{23}$$
$$B = 2\left(1/r + 1/k\right) \tag{24}$$

where C_W is the Wohler-Basquin constant in (6), k is the slope of the Basquin SN curve, and there is only one free parameter in our approach, namely the constant r, which we estimate roughly to be of the order of Paris' constant. Indeed, the other constant, C_K came in Ciavarella [12] from assuming a second power law

$$N'_S \left(\Delta K_{Ic}\right)^r = N'_\infty \left[\Delta K_{th}\right]^r = C_K \tag{25}$$

where r was therefore the Paris law constant. This is very close to the method Susmel-Taylor suggest as we used in the example, by defining a power law between two values of a_0.

Notice that some complications arise in the presence of fatigue limits. The *true* knee points can vary considerably in the presence of notches, and this where inevitably some approximation will appear. Equation (25) essentially replaces Paris' law since it is equivalent to an integrated crack propagation law, resulting in a SN curve. Notice however the absence in (25) of any information on the size of the crack itself, in contrast to the established Paris' law.

Obviously if we only make use of material constants, without calibration of notched fatigue data, the approach is easier to use. It is possible however that calibrating with notched data could be more precise (as Susmel-Taylor seem to suggest).

In some cases, the Gassner curve is seen to have a slope which differs from Wöhler law — this may be a sign of invalidity of the Palmgren Miner's rule: indeed, the simple use of non-unitary value of critical damage sum doesn't change the slope of Gassner curves. Unfortunately, despite some data are available in the literature indicating a (rather modest) change of slope [26], they are not sufficiently detailed for us to comment on the frequency and relevance of this effect.

7 Conclusions

We show that the SN curves under CA or VA loading can be obtained by a simple shift factor, which depends on the spectrum histogram, within the assumptions of Palmgren-Miner's law. This holds true for both unnotched and notched specimen, and seems confirmed by some experimental data taken from the Literature. The finding is based on a Critical Distance simple method proposed and validated by Susmel-Taylor. However, in light of this result, there is no need to apply the iterative calculations Susmel-Taylor propose, as the VA curves can be obtained directly from the CA curves, for which many proposals have already been put forward, also in closed form. Even the computation of the stress field from Finite Element Method does not seem necessary in many cases, as it does not add much accuracy to a problem where the number of assumptions is already quite strong, and more important, than the details of the stress field. As a first approximation, spectrum loading effects in notched or even cracked structures can be estimated easily from reduced amount of testing.

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Highligths for review

- We obtain Gassner curves using the simple Palmgren-Miner's law and Point Critical Distance Method
- We find (either analytically or numerically) Gassner curve are simply shifted CA curves.
- Spectrum loading results in the same multiplicative term for notched, cracked and unnotched specimen.
- A simple empirical unified approach for rapid assessment of the notch effect under random loading is proposed
- Experimental data from the Literature seem to confirm these simple findings, despite the strong assumptions
- The iterative solution suggested by Susmel and Taylor seems not needed.