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BOOK REVIEW

JR Barber, Contact Mechanics [1]: a review

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It is not the first time that Jim Barber has written an excellent book. He had already written "Elasticity" in [2], an advanced treatise of elasticity solutions in plane and three-dimensional cases, and one for undergraduates "Intermediate mechanics of materials" [3]. The former has been very successful with a number of editions, and accompanying software in Mathematica and Maple. The second is a much simpler book, but very pleasant and never trivial, which however competes with many other books in the same area, including some in Italian.

Both showed the style of a lucid, clear, elegant writer, who has a preference for exact, analytical solutions, simple examples, and an overall beautiful balance in the choice of the subjects he writes about, with nothing left unexplained or inadequately explained. Good homework and manuals for the problems' solutions complement the books.

With this "Contact Mechanics" book, Jim Barber goes well *beyond* the excellent level of his previous efforts. Clearly, the subject is what he has dedicated his entire career on (perhaps 50 years now), both in research and teaching, and this is reflected by the quality of *each single one* of the over 550 pages, none of which is boring, redundant, or less than carefully written.

The book is a good complement to the excellent books by Johnson [10], of Sackfield et al. [14] and Popov [12].

The book concentrates on elastic behaviour but has also some parts about other constitutive behaviour, and contains a very authoritative update for the subject, which has expanded significantly since 1985. It reviews most "clever" approaches to contact mechanics, alternative to the brute-force methods of, say, Finite Element or Boundary Element Models, which are always extremely demanding in this field, and do not permit a clear picture.

The book starts, very sensibly, by defining clearly what is a contact problem, in the kinematics (Chapt. 1) and directly moves to the three-dimensional halfspace configuration (Chapt. 2) which, by far, is the most important case. Concepts of similarity, integral equation formulations, and Galin's theorem are gradually introduced, with special emphasis on the case of elliptical contact area, which obviously is the case originally considered by Hertz during the famous vacations when, at only 24, he devised the theory which bears his name.

Chapter 3 therefore is dedicated to a precise analysis of Hertzian contact, while Chapt. 4 attempts to consider more general cases, although perhaps this never went very far, despite the large effort of Fabrikant and others. In contrast, axisymmetric

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contact, as described in Chapt. 5, permits very general solutions, see also Popov and Hess [13].

Then, Chapt. 6 deals with two-dimensional (frictionless) contact. Solutions by Chebyshev polynomials, and Fourier series methods, are not new, but it is useful to see them all together in Barber's unified formulation. More interesting for the researcher are the "cotangent transform" and "Manners' solution" which are not well known. Also useful are treatments of anisotropic problems.

Chapter 7 then moves to tangential loading, again well introduced in many respects. In Chapt. 8 friction laws like Amonton's law are introduced, and their implications for existence and uniqueness of solution of frictional problems are carefully described, with particular reference to Klarbring's model. Also, wedging (a situation in which a state is reached after pushing into a wedge, such that a non trivial state exists without external loads), shakedown—a steady state under cyclic loading where recent studies show the non applicability of theorems like those of Melan in plasticity, for friction, not only because contact can separate and all residual stresses are then lost, but also because of the non-associative nature of friction law. Some paragraphs explain velocity-dependent friction coefficient, and its implication as one of the possible causes of stick–slip vibrations. It is remarked that, as recently found by Papangelo et al. [11], slip-weakening laws imply a Fracture Mechanics interpretation of friction.

Chapter 9 deals with a topic of a great interest to me, that of Cattaneo's problem, for which I obtained a simple result during my Ph.D. time when in Oxford. Barber calls this the "Ciavarella-Jäger theorem", i.e. the solution of the contact problem for any geometry for constant normal load and increasing tangential load, using superposition of normal pressure results. Barber not only improves my original derivation, but discusses more advanced cases (not all of which have been discussed in the Literature so far), recollecting that residual locked-in tractions are always in the "permanent" stick zone, when a cyclic problem is considered. The effect of bulk stress acting in the body as in fretting fatigue experiments where contact stresses may nucleate an embryonic crack, but which is propelled eventually only for sufficient bulk stress, is considered. Jim Barber doesn't discuss the problem of fretting fatigue, as outside the scope of a contact mechanics book, but this is the subject of another book by Hills and Nowell [9], although a little outdated. The cases of unloading and cyclic loading, which is of course of great interest, close the chapter.

Chapter 10 deals with asymptotic methods, starting from the famous Williams' expansion of the stress field in the vicinity of a corner. Considering a contact, and without or with friction, several cases of the dominant singularity can be obtained.

Chapter 11 deals with "receding" contacts, for which, upon application of the load, the contact area shrinks to a value and does not change thereafter (so the problem becomes linear). A more general category is "regressive" contacts, on which the contact area shrinks and "progressive" contacts, the more standard one. It can be proved that for conformal geometries, all these contacts tend to the receding contact area for high enough loads and the analysis can be simplified by a single non-dimensional parameter.

Chapter 12 deals with adhesion, starting from the solution by JKR, explained very clearly, in terms of the original energy derivation. Accounts are given of recent findings that, although the JKR solution is known to be a good for Tabor parameter greater than about 5, this is not the case for hysteretic energy dissipation, due to the jump-in condition being extremely simplified in the JKR model. For finite Tabor parameter, the DMT solution is briefly discussed, which heavily relies on adhesionless case and permits simple results, also for rough contacts. But a number of results, like for a rigid sinusoid, are perhaps new in the literature and Barber adds them naturally with exact treatments.

Chapter 13 deals with contact of beams, plates, membrane and shells. Here the topic is classical but specialistic even for tribologists—at least, even for me the material was quite new. The cases are often important in the real world where contact occurs in structures which cannot be approximated by halfspaces like Hertzian contacts.

Chapter 14 deals with layered bodies. And here the most elegant part is the simple treatment Johnson introduced in his book, and others including Barber generalized for more complex cases. The case of Winkler or non-linear layer on an elastic foundation, and of functionally graded materials are then superbly covered.

Chapter 15 deals with hardness testing from Barber's distinctive perspective, that is from the exact mathematical solutions to the non-linear problems.

Chapter 16 deals with contact of rough surfaces, a subject which has seen an explosion of interest recently [8], although it has perhaps failed to advance our tribology understanding in a quantitative way. Barber has a balanced account of the important theories, from Bowden and Tabor's theory of friction, to Archard's model, the asperity model of Greenwood and Williamson, the fractal models of Majumbdar and Bhushan, and the Weierstrass model of Ciavarella et al. [5], and Persson's theory. A recent "contact challenge" [4], is interesting here, which has been further discussed in Ciavarella [6], showing efforts are still rather academic.

Barber gives equal importance to the plastic old ideas, to the asperity models, and to the fractal models. Profilometry, bearing area curve, and essentials of fractals and random process theories are described. The effect of roughness in adhesion, the results for load-separation curves, and bounds on incremental stiffness are described. Contact of rough spheres is described, although it is a problem which still requires much effort with respect to the mathematically much more abstract problem of infinite rough bodies where unfortunately most effort is devoted probably for its much greater simplicity but not for any actual real practical interest. I would have cited the theory of "Bearing Area Model" of Ciavarella [7] which results in a single simple equation, also available as APP on the web site http://www.tribonet.org/adhesion-calculator-for-rough-surfaces-bearing-area-model/.

Another area of Barber's expertise is in Chapt. 17, dealing with thermoelastic contact. Although it was covered in Johnson's book, Barber here has a very complete account of the progress. Application to solidification problems, to brake and clutch systems, and to general tribological situations is important.

Chapter 18 deals with rolling and sliding contact, from Johnson's belt drive problem, to tractive rolling of elastic cylinders, also in the case they are misaligned. Kalker's strip theory and application to railways are described, as in the case of the dynamic wear instability problem causing rail corrugations. There is then a discussion of Archard wear law (perhaps Barber should have indicated that it is really known as the much earlier Reye's law in Europe), and flash temperatures and bulk temperatures in sliding contacts.

Another advanced chapter is Chapt. 19 on elastodynamic contacts, where Barber introduces Rayleigh waves, the moving line force problem, which already has some difficulties, and the Hertz dynamic problem. Adams instability and rate-state friction laws are not discussed in details, although they have an enormous importance in the modelling of earthquakes.

Chapter 20 deals with impact, both normal and oblique. An appendix gives a range of details about potential functions and integral equations.

Much of the very significant literature is re-written in a manner which is better than the original (at least, this is certainly the case for the few cases where Jim Barber writes about my own papers!). This is remarkable, and I will certainly refer to *this* book, rather than my own papers, from now on, both for teaching at Ph.D. level or directing research, or for my own research ideas. It is a must in the library of any researcher in the flourishing area of contact mechanics, and a possible choice for advanced courses in the subject (but also, for its clarity, it could even serve the purpose of less advanced courses).

Perhaps it is a pity there are no accompanying power point presentations ready to use, or available Mathematica codes, as in his "Elasticity". But we will probably expect this in later editions. And there are useful web sites now collecting interesting material in this area, like tribonet.org.

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