

The Study by Mr. Wu etc Concerning Saint-Venant's Principle Is Insignificant

Zhao Jian-zhong

(Geophysics Department, Yunnan University, Kunming, China, 650091)

Abstract: This paper reviewed the series of four works of “proof of Saint-Venant's Principle” published by Wu etc .The four articles proved neither the discrete Saint-Venant type decay of section-forces of the finite elements in the chain model of discretization of the cylinder nor the discrete Saint-Venant-type decay of section-displacements, posed in their papers. Furthermore, the discrete Saint-Venant type decay of section-forces is not provable. Nevertheless, the proposition of the discrete mixed Saint-Venant type decay of the cylinder was discussed in Wu's articles by taking the “super” forms of the two single decays mentioned above as a priori assumptions, and so the discussion is of uncertainty, or meaningless, for proof of Saint-Venant's Principle.

Key Words: Saint-Venant's Principle, mathematical problem, mathematical truthfulness, proof, chain model, propositions, a priori assumptions, uncertainty

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***Biography:** Zhao Jian-zhong (1944-), professor

Tel : 86-0871-5032983-12 (O) , 86-0871-5032825 (H)

Email : jzhzhaokm@yahoo.com.cn

Introduction

Saint-Venant's Principle in elasticity^[1,2] has its over 100 year's history. The early and important researches contributed to the principle are the articles^[3-7]. Boussinesq^[3] in 1885 and Love^[4] in 1927 announced general statements of Saint-Venant's Principle, but Mises^[5] and Sternberg^[6] argued, by citing counterexamples, that the statements are not very clear, suggesting that Saint-Venant's Principle should be proved or given a mathematical formulation. Truesdell^[8] asserted in 1959 that if Saint-Venant's Principle of equipollent loads is true, it “must be a mathematical consequence of the general equations” of linear elasticity. That means that Saint-Venant's Principle has been considered as a mathematical problem since then and its truthfulness needs to be proved. It is no doubt that mathematical proof of Saint-Venant's Principle became an academic attraction for contributors and much effort has been spent for exploring its mysterious implications, or, deciphering the puzzle. Zanaboni^[7], Toupin^[9,10] and Berdichevskii^[11] ... developed theorems, trying to find proofs or mathematical formulations for the principle or its generalized forms in various areas of continuum mechanics, and

Horgan and Knowles reviewed the developments^[12-14]. The purpose of our present paper is to review those four articles, published by Wu etc, which intended, in discrete form, to discuss about proof of “Saint-Venant’s Principle” of the cylinder or, virtually, about a proposition of discrete mixed Saint-Venant type decay of the cylinder, by chain models posed by the authors^[15-18]. However, their discussion is meaningless because of uncertainty.

1. Love Statement and Propositions of Discrete Saint-Venant type Decays of Section-forces and Section-displacements

Love Statement of Saint-Venant’s Principle is: “According to this principle, the strains that are produced in a body by the application, to a small part of its surface, of a system of forces statically equivalent to zero force and zero couple, are of negligible magnitude at distances which are large compared with the linear dimensions of the part.”^[4]

It must be pointed out that the long cylinder constructed by Wu etc is imaginarily sectioned into n longitudinally arranged segments (finite elements) and it is the distributed forces on the cross-sections that are discussed by means of the chain model^[15-18]. That implies that only the three stress components on the cross-sections rather than the six stress components in the body of the cylinder are studied. Based on the chain-discretization in Wu’s articles^[15-18], following the way of presentation and according to Love’s Statement of Saint-Venant’s Principle above, the proposition of discrete Saint-Venant type decay of section-forces should be:

Proposition 1 : For the elastic cylinder constructed by n longitudinally-linked finite elements, loaded by a system of equilibrated forces on the near-end, with free lateral and far-end, of the surface,

$$\lim_{n \rightarrow \infty} f_{i+1} = \vartheta \quad (i < n) \quad (1)$$

as $n \rightarrow \infty$, where f_{i+1} is the forces distributed on $i + 1$ th cross-section of the cylinder and ϑ is the zero function.

For the mixed boundary value problem of the cylinder with a fixed far-end, which is studied in Wu’s articles^[15-18], the proposition of the discrete Saint-Venant type decay of section-displacements is:

Proposition 2 : For the elastic cylinder constructed by n longitudinally-

linked finite elements, loaded by a system of equilibrated forces on the near-end, with free lateral and fixed far-end, of the surface,

$$\lim u_{i+1} = \vartheta \quad (i < n) \quad (2)$$

as $n \rightarrow \infty$, where u_{i+1} is the displacements of $i+1$ th cross-section of the cylinder and ϑ is the zero function.

2. Chain Inequalities and Prerequisites for Discussion

One of the chain inequalities in Wu's articles^[15-18],

$$U_{f1} > U_{f2} > U_{f3} > \dots > U_{fn} \geq U_n, \quad (3)$$

comprises U_{fn} , which means that the problem of the cylinder with free far-end is discussed. By means of f -type discretization^[18]

$$f_{i+1} = \vartheta \quad (i < n). \quad (4)$$

Then it is known by comparing Eq.(4) and Eq.(1) that the “super” (there is no sign of limit in (4)) discrete Saint-Venant type decay of section-forces is one of the prerequisites for discussion.

Similarly, another chain inequality in the articles^[15-18],

$$U_{r1} < U_{r2} < U_{r3} < \dots < U_{rn} \leq U_n, \quad (5)$$

and $u_{i+1} = \vartheta \quad (i < n) \quad (6)$

under r -type discretization^[18] indicate that the “super” discrete Saint-Venant type decay of section-displacements is the second prerequisite for discussion. (see Proposition 2).

3. Equivalence and a Priori Assumptions

It is proved^[17] explicitly that the set of two equations,

$$\lim f_i = \vartheta \quad (7)$$

and $\lim u_i = \vartheta, \quad (8)$

is equivalent to

$$\lim W_i = \lim(U_{fi} - U_{ri}) = 0 \quad (9)$$

(i in Eq.(7) and Eq.(8) should be understood as $i+1$). The chain inequalities (3) and (5) are invoked for the proof, which implies that the “super” discrete Saint-Venant type decay of section-forces and the “super” discrete Saint-Venant type decay of section-displacements are taken as a priori assumptions for discussion.

4 . The Proposition of Discrete Mixed Saint-Venant Type Decay

Studied by Wu etc

Actually, the proposition bellow is studied in Wu's articles ^[15-18], based on the equivalence relation in Sec.3:

Proposition 3 : For the elastic cylinder constructed by n longitudinally-linked finite elements, loaded by a system of equilibrated forces on the near-end, with free lateral and free or fixed far-end, of the surface,

$$\lim f_{i+1} = \vartheta \quad (i < n) \quad (10)$$

and
$$\lim u_{i+1} = \vartheta \quad (i < n) , \quad (11)$$

as $n \rightarrow \infty$, where f_{i+1} and u_{i+1} are respectively the forces and the displacements distributed on $i+1$ th cross-section of the cylinder, and ϑ is the zero function .

5. Uncertainty of the Discussion of Wu etc

Equation (1) is not proved to be “a mathematical consequence of the general equations” of linear elasticity^[8], and therefore the discrete Saint-Venant type decay of section-forces (Proposition 1) is not proved in the Wu's works^[15-18]. The discrete Saint-Venant-type decay of section-displacements (Proposition 2) is not proved either. However, the discussion of Proposition 3 is based on a priori assumptions, the “super” discrete Saint-Venant type decay of section-forces and the “super” discrete Saint-Venant type decay of section-displacements. And so the discussion of Proposition 3 lacks theoretical support from mathematical logic because Proposition 1 and Proposition 2 are not proved in Wu's articles^[15-18], let alone the validity of the two “super” conditions. That is to say that the discussion of Proposition 3 by Wu etc is of uncertainty.

6. Impossibility of Proof

Early in 1965 Toupin pointed out by using counterexamples that Saint-Venant's Principle stated by Love is not generally true ^[9,10]. Therefore, the discrete Saint-Venant type decay of section-forces (Proposition 1) is false, or can not be proved to be true.

7. Conclusion

- (A) The discrete Saint-Venant type decay of section-forces (Proposition 1) is not proved by Wu etc, let alone discussion of “general proof” of “Saint-Venant's Principle”. In fact, the discrete Saint-Venant type decay of section-forces (Proposition 1) is false.

- (B) The discussion of the discrete mixed Saint-Venant type decay (Proposition 3) in the articles of Wu etc is meaningless because of uncertainty.
- (C) So it is unfair to mention such sayings^[18] like “ general proof of Saint-Venant’s Principle” and “ a well-known puzzle has been solved...”.

APPENDICES

Even the chain inequalities in Wu’s articles do not hold true^[15-18]. They are not proved in the first article of Wu etc^[15] because it is impossible to deduce Eq. (3) and Eq.(5) from the Lemma 1 of the article. Then the author tries to prove them by using variational principles^[18]. But the difficulties of the “proofs” are:

A. The Principle of Least Work ^[18,19] (Wu calls it Principle of Minimum Complementary Energy) is invoked to prove Eq. (3) , and so U_{fi} in Eq.(3) is the complementary energy of the body. For linear elastic body, potential energy and complementary energy are equal in the equilibrium state, but not necessarily equal in the states that variations of that two kinds of energy occur. That means that minimum complementary energy does not imply or guarantee minimum potential energy. If U_{fi} expresses potential energy, the inequality of Eq. (3) does not stay.

The Principle of Minimum Potential Energy should be applied to the boundary value problem with free far-end discussed in Wu’s articles. According to the principle, the total potential energy (the sum of the potential energy of the body and the potential energy of the external forces) rather than the potential energy of the body alone takes minimum, which serves as another evidence that U_{fi} in Eq.(3) is “not” the potential energy of the body.

B. Generalized variational principles^[20,21] should be applied to the mixed boundary value problem with fixed far-end discussed in Wu’s articles^[15-18]. Generalized variational principles are those of stationary values of functionals rather than that of the minimum potential energy of the body, and so it is impossible to prove the chain inequality of Eq. (5) .

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武建 先生等有 圣文南原理的 不成立

作者： 建中（教授）

位和地址： 云南省昆明市翠湖北路2号

云南大学 源、 境与地球科学学院

地球物理系

: 650091

: 0871-5032983-12 (), 0871-5032825 (家)

: jzhzhaokm@yahoo.com.cn