A brief overview of the mechanical response of rubber-like materials and the corresponding constitutive equations

IMechanica Journal Club, 11/01/2008

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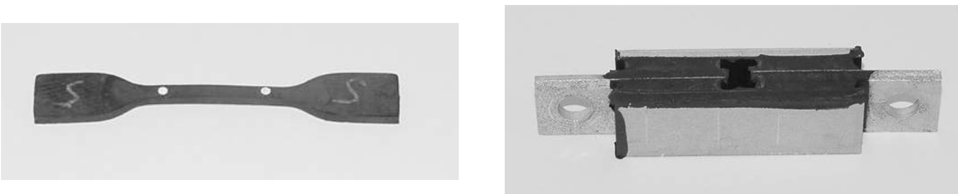
1 Description of the macroscopic mechanical response of rubbers

This section presents some of the characteristics of the mechanical behaviour of elastomers. The aim of this short presentation is to exhibit the complex mechanical response of these materials. The characteristics will be presented only through experimental results. The samples used are presented in Figure 1. The material is a carbon black-filled Natural Rubber.



(a) (b)

Figure 1: SAMPLES. (a) Uniaxial tensile test sample, (b) Four block-shear test sample.

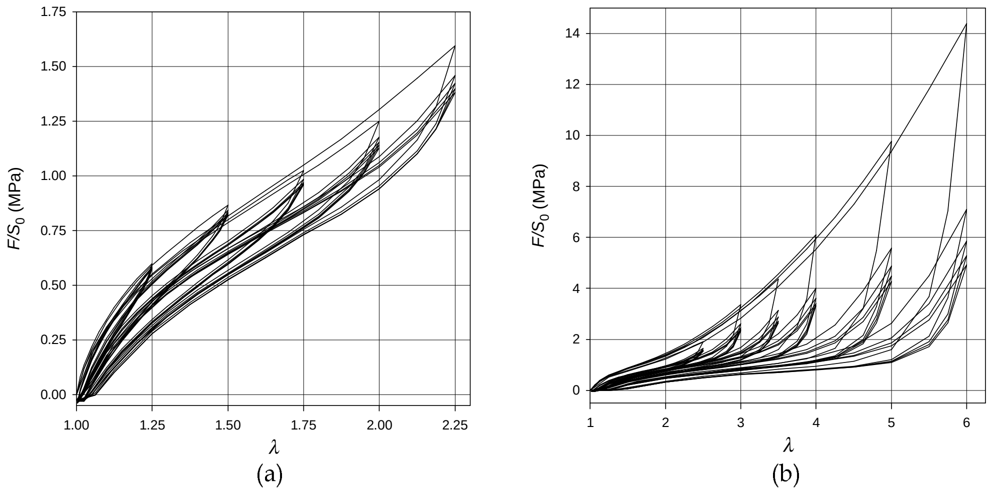


Figure 2: Global response. Cyclic uniaxial extension: (a) from 25% to 125%, (b) from 150% to 500%. In these graphs, results are expressed in terms of principal stretch ratio in the tensile direction, i.e. $\lambda = l/l_0$ (l_0 and l being respectively the undeformed and deformed length of the sample), and of the 1st Piola-Kirchhoff stress in tensile direction, i.e. F/S_0. In the bibliography, experimental results are classically plotted using these variables

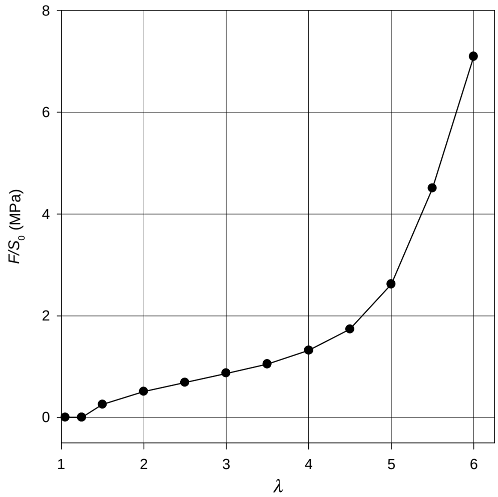


Figure 3: Large strain elasticity. 500%-uniaxial extension, loading curve of the 2nd cycle. In the bibliography, this sort of curve are classically called S-shaped curve to describe its non-linearity. Here, this S-shape is not completely clear . . .

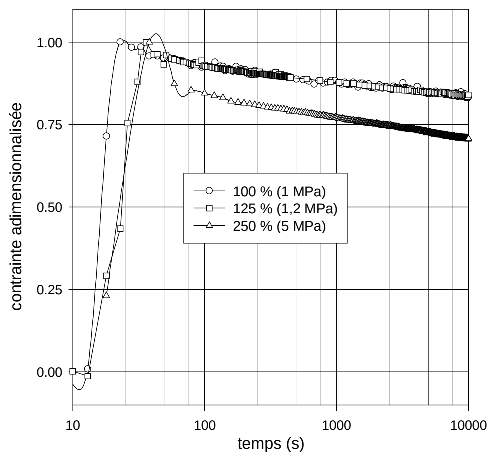


Figure 4: Non-linear viscoelasticity. Relaxation test. Two block-shear tests with three stretching levels: 100%, 125% and 250%. For each levels, the time evolution of the 1st PK stress is recorded. In order to compare the results, the stress is normalized, i.e. divided by the maximum stress of the tests. In the figure, the value of the maximum stress is given in brackets.

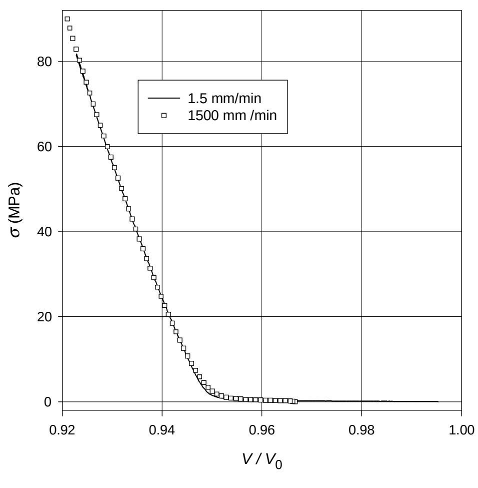


Figure 5: Incompressibility. Compressibility test. A cylindric sample (radius: 8.8 mm and height: 25.8 mm) is placed in a cavity and compressed at constant strain rate. Two strain rates are considered and the curves show the compression stress vs. the volume change of the sample. The value of the compressive stress should be compared with stress levels involved in uniaxial extension (Fig. 3)

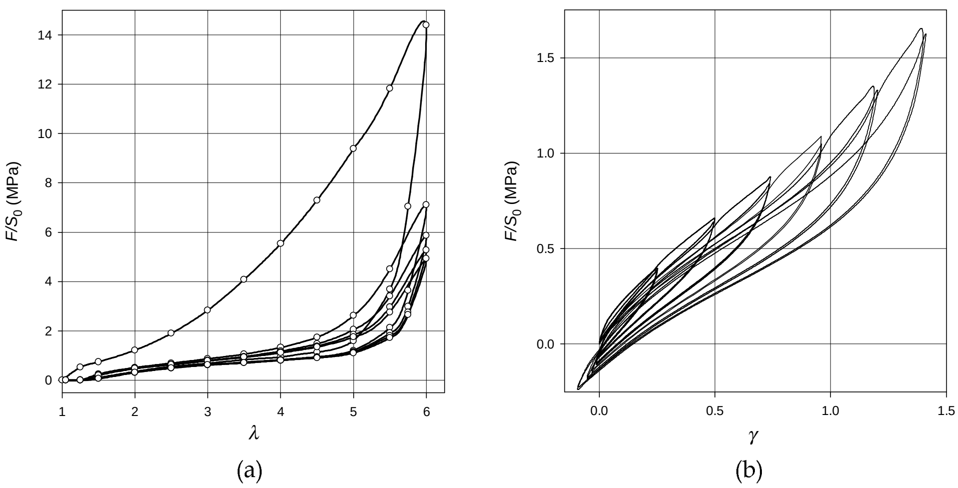


Figure 6: Cyclic response of elastomers: stress-softening (Mullins effect) and hysteresis. (a) Uniaxial tensile test: 5 cycles at 500%. (b) Four block-shear test: 5 cycles at 50%, then 5 cycles at 100%, and so on until 300%. On the graph, only the two first cycles for each stretching level are plotted.

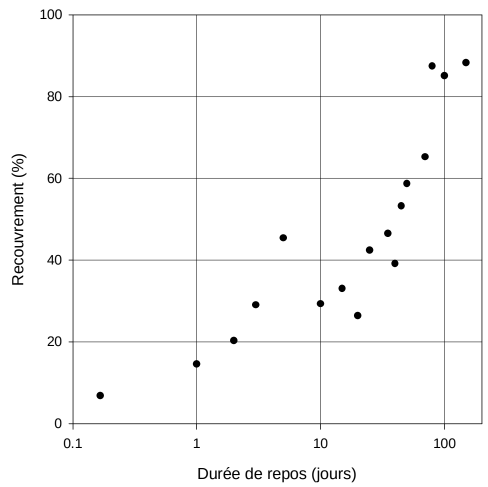


Figure 7: Recovery of the Mullins effect. At time 0, a series of samples is subjected to 5 uniaxial loading cycles up to 200%. The samples are stocked at 23°C and a similar test is conducted after a given time. The recovery is defined as the ratio of stress values at 200% during the first loading cycle obtained for the second and the first experiments.

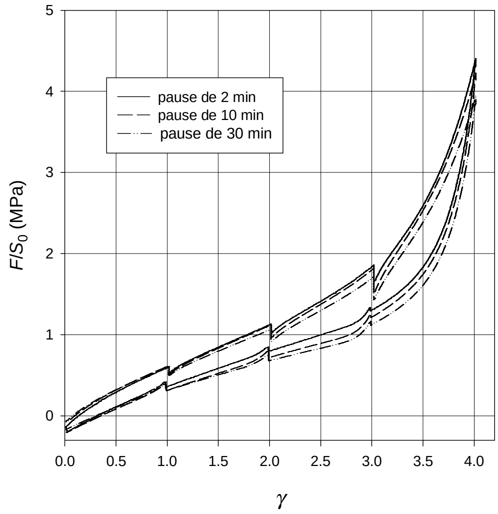


Figure 8: Complex hysteretic response. Two block-shear tests: hysteresis. The samples are first softened during 5 cycles at 500%. Afterwards, the test consists in stretching the sample up to 100% then stopping to relax the stress during a given time, stretching again up to 200% then stopping to relax the stress, and so on up to 400%. A similar deformation history is considered during unloading. Three relaxation durations are considered: 2 mn, 10 mn and 30 mn.

2 Modelling the mechanical response of elastomers: state of the art, 2003

This list of bibliographical references is a summary of the Appendix of my Habilitation which was published in 2003 (Verron, 2003). As it was written in French, I only retain here the bibliographical references. Moreover, it should be updated to take into account more recent publications.

The chronological presentation is adopted in order to underline the evolution of the approaches through years, especially to emphasize the influence of numerical tools on the studies. The three main periods are:

- foundations: 1940-1970,
- rise of numerical methods: 1970-1990,
- new challenges: 1990- .

In this review, we highlight the mechanical approaches, i.e. the constitutive equations that can be used during design or finite element simulations of industrial parts.

In most of the references, elastomers are assumed to be homogeneous, isotropic and incompressible.

Preliminaries

- Entropic nature of rubber elasticity: Meyer and Ferri (1935)
- Books devoted to large strain continuum mechanics: Ogden (1984), Bonet and Wood (1997), Basar and Weichert (2000),Holzapfel (2000).

2.1 Foundations: 1940-1970

2.1.1 Elasticity

Molecular approaches

- Generalities on chain statistics: Doi (1996), Kausch et al. (2001)
- Neo-hookean model: Treloar (1943), Flory (1944)
- Phantom model: James and Guth (1947), Mark and Erman (1988)
- Non-Gaussian approaches: Kuhn and Grün (1942), James and Guth (1943), Flory and Rehner (1943), Treloar (1946), Treloar (1954), Treloar and Riding (1979)

Phenomenological approaches

- Definition of phenomenology as applied to rubber constitutive equation: Treloar (1975)
- Mooney model: Mooney (1940), Fried (2002)
- Expansion of the strain energy density: Rivlin (1948)
- Hypothesis of Valanis and Landel: Valanis and Landel (1967)

2.1.2 Viscoelasticity

Preliminaries

- Books on the general theory of viscoelasticity (small strain): Christensen (1982), Tschoegl (1989)
- Maxwell and Kevin-Voigt models: Wineman and Rajagopal (2000)
- Boltzmann superposition principle: Ward (1983), Haddad (1988)

Finite strain viscoelasticity

- Extension of rheological models: Green and Tobolsky (1946), Tobolsky et al. (1944), Sidoroff (1974)
- Extension of the Boltzmann principle, integral models: Green and Rivlin (1957), Coleman and Noll (1961), Pipkin (1964), McGuirt and Lianis (1970), Sidoroff (1982)
- K-BKZ model: Lodge (1956), Kaye (1962), Bernstein et al. (1963), Zapas and Craft (1969), Tanner (1988)

2.1.3 Cyclic response

Mullins effect

- Experimental description: Bouasse and Carrière (1903), Mullins (1948), Harwood et al. (1967), Mullins (1969)
- Two-phase model: Mullins and Tobin (1957), Bonart (1968), Lee and Williams (1985)
- Molecular model of Bueche: Bueche (1960), Bueche (1961), Mullins (1969)
- Molecular model of Dannenberg: Dannenberg and Brennan (1966), Dannenberg (1966), Harwood et al. (1967)

2.2 Rise of numerical methods: 1970-1990

Some general results

- Volumetric-isochoric separation: Flory (1961), Ogden (1984)
- A book for finite element applications in large strain hyperelasticity: Bonet and Wood (1997)

2.2.1 Elasticity

Molecular approaches

- Slip-link model: Ball et al. (1981), Edwards and Vilgis (1986)
- Constraint junctions model: Flory and Erman (1982), Mark and Erman (1988)
- Van der Waals model: Kilian (1981), Ambacher et al. (1989)

Phenomenological approaches

- Extensions of the Mooney model to consider very large strain: Gent and Thomas (1958), Hart-Smith (1966), Chagnon et al. (2004a), Alexander (1968)
- Ogden model: Ogden (1972), Ogden (1986), Blatz et al. (1974), Seth (1964), Morman Jr. (1986), Twizell and Ogden (1983), Seibert and Schöche (2000)

2.2.2 Viscoelasticity

Integral models

- Viscoelastic counterpart of the Neo-Hookean model: Christensen (1980), Feng (1986), Feng (1992), Shrivastava and Tang (1993), Verron et al. (2001)
- Viscoelastic counterpart of the Ogden model: Chang et al. (1976a), Chang et al. (1976b), Chang et al. (1977), Morman Jr. (1988)
- Another proposal: Sullivan (1987), Sullivan and Mazich (1989)

Differential models

- Use of internal variables for viscoelasticity: Sidoroff (1974)
- Models: Lubliner (1985), Simo (1987), Govindjee and Simo (1992)

2.2.3 Cyclic response

Mullins effect

- Two papers which consider Damage Mechanics: Gurtin and Francis (1981), Simo (1987)

- Molecular models: Roland (1989c), Roland (1989a), Roland (1989b)

2.3 New challenges: 1990-

Proceedings of a dedicated conference: Dorfmann and Muhr (1999), Besdo et al. (2001), Busfield and Muhr (2003), Austrell and Kari (2005), Boukamel et al. (2007)

2.3.1 Hyperelasticity

Molecular approaches

- Chain models: Arruda and Boyce (1993), Wu and van der Giessen (1993)

- Tube models: Heinrich et al. (1988), Edwards and Vilgis (1988), Heinrich and Kaliske (1997), Kaliske and Heinrich (1999)

Phenomenological approaches

- High-order Rivlin expansion: Yeoh (1990), Yeoh (1993)

- Gent model: Gent (1996), Boyce (1996)

- Association of different models: Yeoh and Fleming (1997), Meissner (2000), Boyce and Arruda (2000)

2.3.2 Viscoelasticity

- General theories: O'Dowd and Knauss (1995), Schapery (1969)

- Extension of the Green and Tobolsky (1946) model: Johnson et al. (1992), Johnson et al. (1995), Johnson and Quigley (1993), Septanika and Ernst (1998a), Septanika and Ernst (1998b)

- Thermodynamical phenomenological models: Le Tallec et al. (1993), Holzapfel (1996), Govindjee and Reese (1997), Reese and Govindjee (1998), Holzapfel (2000), Bonet (2001)

2.3.3 Cyclic response

Mullins effect

- Damage mechanics approaches: De Souza Neto et al. (1994), Miehe (1995), Kaliske et al. (2001), Chagnon et al. (2004b)
- Models motivated by the two-phase model: Wineman and Rajagopal (1990), Rajagopal and Wineman (1992), Wineman and Huntley (1994), Huntley et al. (1996), Huntley et al. (1997), Beatty and Krishnaswamy (2000)
- Pseudo-elastic models: Ogden and Roxburgh (1999), Elías-Zúñiga and Beatty (2002)
- Molecular models: Govindjee and Simo (1991), Govindjee and Simo (1992), Klüppel and Schramm (2000), Drozdov and Dorfmann (2001) Marckmann et al. (2002)

Hysteresis

- Viscoelastic approaches: Dafalias (1991), Spathis (1997), Bergström and Boyce (1998), Bergström and Boyce (2000), Bergström and Boyce (2001)
- Elasto-(visco)-plastic approaches: Lion (1996), Kaliske and Rothert (1998), Miehe and Keck (2000), Drozdov and Dorfmann (2001), Bikard and Desoyer (2001)

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