ME 7953: Simulations in Materials

Fall 2002

Project II (Friday, 10/25/2002)

Projects II is on Material Point Method (MPM) two-dimensional simulation. Problems are due at the beginning of the class on Friday, 11/8/2002. You have two weeks to finish the work.

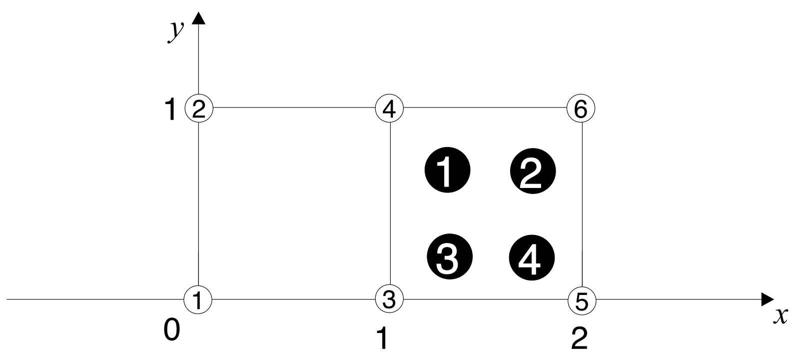
1) Shape function

Consider a two-dimensional MPM problem with background grid shown below. The *x* and *y* coordinates of grid nodes are saved in `x_g` and `y_g` vectors.

```
x_g=[0.0, 0.0, 1.0, 1.0, 2.0, 2.0];
y_g=[0.0, 1.0, 0.0, 1.0, 0.0, 1.0];
```

The body is divided into 4 particles. The coordinates of particles are saved in `x_p` and `y_p`.

```
x_p=[1.2, 1.7, 1.3, 1.8];
y_p=[0.75, 0.81, 0.26, 0.24];
```



Answer following questions:

(A) Write a MATLAB function to calculate shape function $N^{(n)}(x,y)$, that relate node n with the point (x,y).

```
function [N] = shape(n,x,y)
```

(B) Write a MATLAB function to calculate the derivatives of the shave function, $\frac{\partial}{\partial x}N^{(n)}(x,y)$ and $\frac{\partial}{\partial y}N^{(n)}(x,y)$.

```
function [dN_dx, dN_dy] = dshape(n,x,y)
```

(C) Calculate the value of $N^{(3,2)}$, which means a shape function between node 3 with coordinates (1.0, 0.0), and particle 2 with coordinates (1.7, 0.81)?

(D) For each particle *p*, calculate all the shape function between particle *p* and grid nodes. Show that $\sum_n N^{(n,p)} = 1$ for *p*=1,…,4.

2) Mass matrix

For the above grid and particles, assume all the particle masses are the same to be 1. The mass-matrix on the grid is defined as

$$\overline{m}^{(n,n')} = \sum_p N^{(n',p)} N^{(n,p)} m^{(p)}, \qquad n,\ n'=1,\ldots,6, \qquad p = 1,\ldots,4$$

where $m^{(p)}$ is the mass of particle *p*, and $N^{(n,p)} = N^{(n)}(x^{(p)}, y^{(p)})$ is the shape function of node *n* at the location of particle *p*.

Answer the following questions with the help of MATLAB.

(A) Compute the details of the 6 by 6 mass-matrix.

(B) Compute the lumped mass $\overline{M}^{(n)}$ at each node *n*. Lumped mass is defined as $\overline{M}^{(n)} = \sum_p N^{(n,p)} m^{(p)}$.

(C) Show that $\overline{M}^{(n)} = \sum_{n'} \overline{m}^{(n,n')}$ for every grid node *n*.

3) Solving dynamic equations

The body-force densities for all particles are the same,

$b_x^{(p)} = 5$

$b_y^{(p)} = 10$, *p*=1,…,4.

Answer the following questions with the help of MATLAB.

(A) Compute the external force at grid. Grid external force at node *n* is defined as

$$\overline{F}_x^{(n)} = \sum_p N^{(n,p)} m^{(p)} b_x^{(p)}$$

$$\overline{F}_y^{(n)} = \sum_p N^{(n,p)} m^{(p)} b_y^{(p)}.$$

(B) Compute the internal force at grid. Grid internal force is defined as

$$\bar{f}_x^{(n)} = -\sum_p V^{(p)} \left(\frac{\partial N^{(n,p)}}{\partial x} \sigma_{xx}^{(p)} + \frac{\partial N^{(n,p)}}{\partial y} \sigma_{xy}^{(p)} \right)$$

$$\bar{f}_y^{(n)} = -\sum_p V^{(p)} \left(\frac{\partial N^{(n,p)}}{\partial x} \sigma_{xy}^{(p)} + \frac{\partial N^{(n,p)}}{\partial y} \sigma_{yy}^{(p)} \right).$$

Assume the volume of all the particles are the same, $V^{(p)} = 2$, for all p.

(C) Using the lumped mass, calculate the acceleration at nodes where the lumped mass is not zero.

$$\bar{a}_x^{(n)} = \frac{\bar{F}_x^{(n)} + \bar{f}_x^{(n)}}{\bar{M}^{(n)}}$$

$$\bar{a}_y^{(n)} = \frac{\bar{F}_y^{(n)} + \bar{f}_y^{(n)}}{\bar{M}^{(n)}}$$

4) System updating

For the above problem, the particle velocities before this step are

```
vx_p=[1.25, 1.1, 0.9, 0.8];
vy_p=[0.25, 0.3, 0.33, 0.34];
```

Assume the current stresses developed in particles are

```
sigxx_p=[-5.1, -4.7, -4.1, -3.5];
sigyy_p=[-0.1, -0.2, -0.25, -0.3];
sigxy_p=[2.1, 2.2, 2.3, 2.5];
```

Answer the following questions with the help of MATLAB. Take time step $\Delta t = 0.001$.

(A) Map the velocities from particles to grid. Compute the grid velocities at nodes where the lumped masses are not zero. Grid velocities are defined as

$$\bar{v}_x^{(n)} = \frac{1}{\bar{M}^{(n)}} \sum_p N^{(n,p)} m^{(p)} v_x^{(p)}$$

$$\bar{v}_y^{(n)} = \frac{1}{\bar{M}^{(n)}} \sum_p N^{(n,p)} m^{(p)} v_y^{(p)}.$$

(B) Update the grid velocities using

$$\bar{v}_x^{(n)} \leftarrow \bar{v}_x^{(n)} + \bar{a}_x^{(n)}\Delta t$$
$$\bar{v}_y^{(n)} \leftarrow \bar{v}_y^{(n)} + \bar{a}_y^{(n)}\Delta t$$

at those nodes where lumped masses are not zero.

(C) Update the particle positions using

$$\bar{x}^{(p)} \leftarrow \bar{x}^{(p)} + \frac{\Delta t}{2}\left(v_x^{(p)} + \sum_n \bar{v}_x^{(n)} N^{(n,p)}\right)$$
$$\bar{y}^{(p)} \leftarrow \bar{y}^{(p)} + \frac{\Delta t}{2}\left(v_y^{(p)} + \sum_n \bar{v}_y^{(n)} N^{(n,p)}\right).$$

(D) Update the particle velocities using

$$\bar{v}_x^{(p)} \leftarrow \bar{v}_x^{(p)} + \Delta t \sum_n \bar{a}_x^{(n)} N^{(n,p)}$$
$$\bar{v}_y^{(p)} \leftarrow \bar{v}_y^{(p)} + \Delta t \sum_n \bar{a}_y^{(n)} N^{(n,p)}.$$

(E) Calculate the strain rates at particles using

$$\dot{\varepsilon}_x^{(p)} = \sum_n \bar{v}_x^{(n)} \frac{\partial N^{(n,p)}}{\partial x}$$
$$\dot{\varepsilon}_y^{(p)} = \sum_n \bar{v}_y^{(n)} \frac{\partial N^{(n,p)}}{\partial y}.$$
$$\dot{\varepsilon}_{xy}^{(p)} = \frac{1}{2}\sum_n \left(\bar{v}_x^{(n)} \frac{\partial N^{(n,p)}}{\partial y} + \bar{v}_y^{(n)} \frac{\partial N^{(n,p)}}{\partial x}\right)$$

(F) Update the particle stresses using particle strains. Take Young's modulus to be $E = 100$, and Poisson's ration to be $\nu = 0.3$.

$$\sigma_{xx}^{(p)} \leftarrow \sigma_{xx}^{(p)} + \frac{E}{1-\nu^2}\left(\dot{\varepsilon}_{xx}^{(p)} + \nu\dot{\varepsilon}_{yy}^{(p)}\right)$$
$$\sigma_{yy}^{(p)} \leftarrow \sigma_{yy}^{(p)} + \frac{E}{1-\nu^2}\left(\dot{\varepsilon}_{yy}^{(p)} + \nu\dot{\varepsilon}_{xx}^{(p)}\right)$$
$$\sigma_{xy}^{(p)} \leftarrow \sigma_{xy}^{(p)} + \frac{E}{2(1+\nu)}\dot{\varepsilon}_{xy}^{(p)}$$