February 27, 2007

ES 242r

Problem Set #2

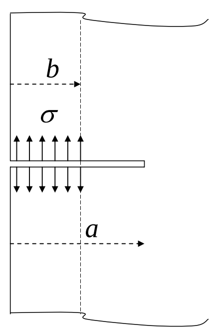
Due by submission to the TFs on March 15, 2007

7. Suggestions for fellow students and for teaching staff

Adrain Podpirka, who is taking this course from Harvard, has posted some remarks about this class (http://imechanica.org/node/931). Please write a comment in the comment section of his post, in the spirit of giving pointers to fellow students and feedback to teaching staff. For example, you can talk about any one or a combination of the following topics

- Do you have prior experience with courses taught using powerpoint slides? Have you found a good way to use the slides?
- Have you found any supplementary textbooks that complement the lectures and slides?
- For students taking the course from University of Nebraska, how well have the lectures being working? Anything that can be done to improve the lectures?
- Any other topics that have come to your mind.

8. Depth of a surface crack due to residual tensile surface stress



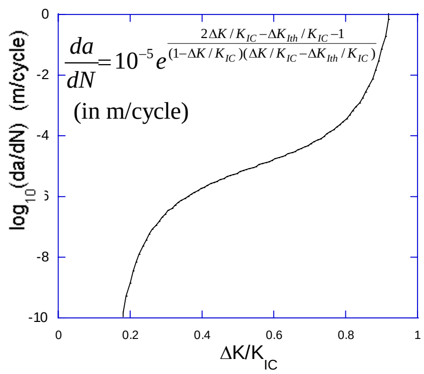
$$K = 1.12\sigma\sqrt{\pi a} \quad \text{for } \frac{a}{b} \le 1$$

$$K = \sigma\sqrt{\pi a}\,\frac{2}{\pi}\sin^{-1}\left(\frac{b}{a}\right)\left(1.3 - 0.18\frac{b}{a}\right) \quad \text{for } \frac{a}{b} > 1$$

The above approximations are given in Tada, et al. for the stress intensity factor of a surface crack of length a in a semi-infinite body having a surface layer of thickness b which is uniformly stressed to σ. Given the mode I toughness, K_{Ic}, create a nice dimensionless plot showing the depth a to which the crack penetrates. Hint: Let one variable of your plot be $K_{Ic}/(\sigma\sqrt{b})$ and be sure to indicate the range of values of this parameter such that a plane strain crack can exist satisfying $K = K_{Ic}$.

9. A fatigue crack growth problem

A steel has $K_{IC} = 100 MPa\sqrt{m}$ and a threshold cyclic stress intensity factor $\Delta K_{Ith} = 10 MPa\sqrt{m}$. It's curve of da/dN is shown below (for cycles with $K_{\min} = 0$).



where

$$\frac{da}{dN} = 10^{-5} e^{\frac{2\Delta K/K_{IC} - \Delta K_{Ith}/K_{IC} - 1}{(1-\Delta K/K_{IC})(\Delta K/K_{IC} - \Delta K_{Ith}/K_{IC})}} \quad \text{(in m/cycle)}$$

Consider the edge-cracked beam on overhead 5 of the first lecture which is of width b and is subject to a cyclic moment per unit thickness $\Delta M = M_{\max} - M_{min}$ where $M_{min} = 0$. Suppose the beam has thickness $b = 0.10m$ and suppose it has an initial crack of length $a_0 = 10^{-4} m$ (these might be considered as inherent design flaws). Assume small scale yielding.

a) What is the cyclic moment/thickness, ΔM_{th}, such that for $\Delta M < \Delta M_{th}$ there will be no crack growth?
b) Estimate the number of cycles for the crack to reach length $a = 10^{-3} m$ for $\Delta M = 1.1\Delta M_{th}$ and $\Delta M = 1.5\Delta M_{th}$.
c) Estimate the number of cycles for the crack to reach length $a = b/2 = 0.05m$ for $\Delta M = 1.1\Delta M_{th}$ and $\Delta M = 1.5\Delta M_{th}$.

10. Fully plastic crack growth

Model the material in the deeply cracked compact tension specimen below as being elastic-perfectly plastic with a tensile yield stress σ_Y. The limit yield moment/thickness in plane strain for an elastic-perfectly plastic deeply cracked section is $M_L = 0.35\sigma_Y c^2$. Assume this moment governs the fully plastic yielding of the uncracked ligament of the compact tension specimen with $M = PL$ where P is the load/thickness and $L = (a+b)/2$. Neglect the contribution of elasticity (both in the arms and in the ligament) and determine J (approximately) based on the deeply-cracked formula

$$J = \frac{2}{c}\int_0^{\Delta} M d\theta = \frac{2}{c}\int_0^{\Delta} P d\Delta \quad \text{(because } M = PL \text{ and } \theta \cong \Delta / L\text{)}$$

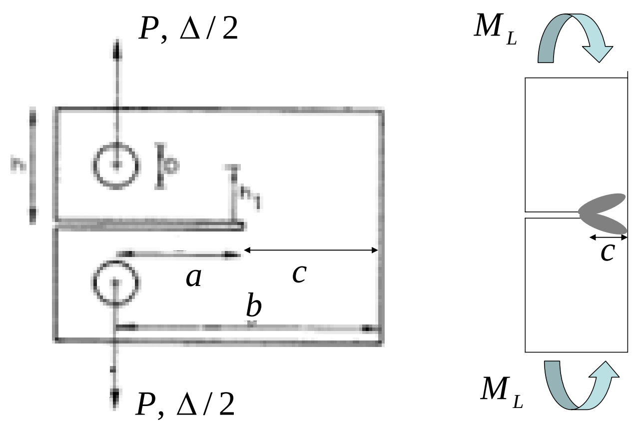
Assume the crack growth resistance is (neglecting initiation—see the data for A533B in the notes):

$$J_R(\Delta a)/J_0 = (\Delta a/\ell_0)^{1/2} \quad \text{where } J_0\ (J/m^2) \text{ and } \ell_0\ (m) \text{ are constants.}$$

a) Obtain relationships for (Δ, Δa), (P, Δa) and (P, Δ) assuming an initial crack length a_0 where $c_0 = b - a_0 << a_0$.

b) Plot the relationships for $b = 50\ell_0$, $a_0 = 40\ell_0$.

c) Assess whether the crack growth is stable or not for b).



11. Reading on the analysis of large scale yielding

Read Section 12 of the nonlinear fracture notes (pgs. 55-62) with particular attention to pages 59-62 discussing fully plastic pure power-law solutions and their use in interpolating solutions from small scale yielding (LEFM) to large scale yielding. The solution for a plane strain crack of length $2a$ in an infinite pure power-law material is

$$J = \alpha\sigma_0\varepsilon_0 a\, h(n)\left(\sigma^\infty/\sigma_0\right)^{n+1}$$

where σ^∞ is the stress at infinity and $h(n)$ has been computed (He, M.Y., Hutchinson, J.W., "The Penny-Shaped Crack and the Plane Strain Crack in an Infinite Body of Power-Law Material" Journal of Applied Mechanics 48 830-840, 1981). Use this result to derive an interpolation formula for J for the full range of σ^∞ ranging from small scale yielding to large scale yielding.

12. Residual stress in layered materials

A metal layer, of thickness t_m, is sandwiched between two identical ceramic layers, each of thickness t_c. The bonding is made at a high temperature, T_H, at which the structure is stress-free. Upon cooling to a low temperature, T_L, the metal is under tension, and the ceramic is under compression. Because of the symmetry, the structure does not bend. The lateral size is large compared to the thickness, so you can neglect the complicated stress field near the edge of the layers, and focus on the stress states in the layers far away from the edges. The layers are well bonded, and each layer is under a biaxial stress state. Show that the stresses in the two materials are

$$\sigma_c = \frac{E_c}{1-\nu_c}\frac{(\alpha_c-\alpha_m)(T_H-T_L)}{1+\frac{2t_cE_c(1-\nu_m)}{t_mE_m(1-\nu_c)}}, \quad \sigma_m = -\frac{2t_c\sigma_c}{t_m}.$$

Notes:

(i) The stress jumps across the interface.

(ii) When the metal is much thicker than the ceramics, namely, $t_m/t_c \gg 1$, the stress in the metal vanishes, and the stress in the ceramic thin layers is $\sigma_c = \frac{E_c}{1-\nu_c}(\alpha_c-\alpha_m)(T_H-T_L)$

(iii) Diamond is hard and resists wear and corrosion, but it is expensive. A recent technology is to coat metals such as steel with diamond thin films. Representative thickness of diamond coating is 1 μm. Representative thickness of the metal substrate is 1 cm. Diamond $E = 1000\,\mathrm{GPa}$, $\nu = 0.2$, $\alpha = 1\times 10^{-6}\,\mathrm{K}^{-1}$. Steel $\alpha = 11\times 10^{-6}\,\mathrm{K}^{-1}$. Deposition temperature and room temperature difference is $\Delta T = 700\,\mathrm{K}$.

13. Tunneling crack

S. Ho and Z. Suo, Tunneling cracks in constrained layers. *J. Appl. Mech.*, **60**, 890-894 (1993). (http://www.deas.harvard.edu/suo/papers/024.pdf)

For this homework problem, you will look at a special case that can be worked out analytically. A brittle layer, thickness *h*, is bonded between two thick substrates. The three materials have similar elastic constants (Young's modulus *E* and Poisson's ratio ν). The layer is under residual tensile stress σ. Cracks can tunnel in the layer.

(a) Use two methods to show that the steady state energy release rate at the tunnel front is

$$G = \frac{\pi(1-\nu^2)\sigma^2 h}{4E}.$$

Hint. The elasticity problem of a Griffith crack of size 2*a* has been solved analytically. The crack opening displacement of the Griffith crack is

$$\delta(x) = \frac{4\sigma(1-\nu^2)}{E}\sqrt{a^2-x^2}.$$

The energy release rate for plane strain crack is

$$G_{ps} = \frac{\pi(1-\nu^2)\sigma^2 a}{E}.$$

(b) The brittle layer has fracture energy $\Gamma = 10\,\mathrm{J/m^2}$, Young's modulus 100 GPa. It is 1 μm thick, and is under a tensile stress 500 MPa. Will tunnel cracks form?

(c) Discuss Fig. 4 in the above paper. Explain qualitatively the trend of the curves.