Crack Tip Fields for Power Law Hardening Materials—HRR Fields

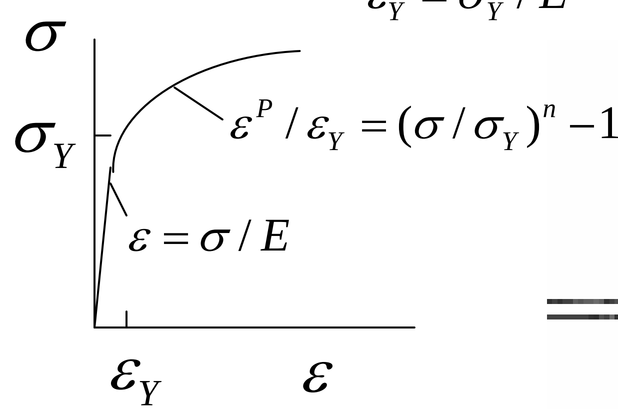
Refs. Rice and Rosengren (1968) & Hutchinson (1968); fracture notes pg. 46

$$\varepsilon_Y = \sigma_Y / E$$

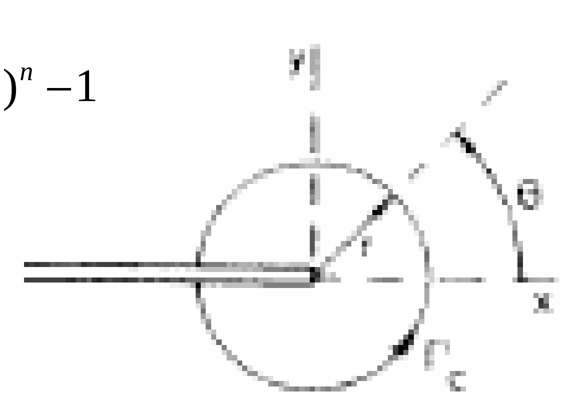
Deformation theory fit to stress-strain curve with power law plasticity

$$W \to \frac{\sigma_Y \varepsilon_Y}{1+N}\left(\frac{\varepsilon^P_{eff}}{\varepsilon_Y}\right)^{1+N},$$

$$\varepsilon^P_{eff} = \sqrt{2\varepsilon^P_{ij}\varepsilon^P_{ij}/3}, \quad N = 1/n$$



Tensile stress-strain curve



$$J = \int_{\Gamma_C} \left(Wn_x - \sigma_{ij} n_j u_{i,x}\right) ds$$

$$= \int_{-\pi}^{\pi} \left(W\cos\theta - \sigma_{ij} n_j u_{i,x}\right) r\, d\theta$$

If J non-zero this requires that $\left(W\cos\theta - \sigma_{ij} n_j u_{i,x}\right) \propto (J/r)$ since J is independent of the path and Γ_C can be shrunk down to the tip, i.e. $r \to 0$.

This implies the following as $r \to 0$:

$$W \propto (J/r) \quad \Rightarrow \varepsilon^P_{ij} \propto (J/r)^{n/(n+1)} \quad \Rightarrow \sigma_{ij} \propto (J/r)^{1/(n+1)}$$

$$I_n = \int_{-\pi}^{\pi} \left[\frac{1}{1+N}\left(\tilde{\varepsilon}^P_{eff}\right)^{1+N} \cos\theta + (....)\right] d\theta$$

The crack tip fields are of the form:

$$\left(\sigma_{ij}, \sigma_e\right) = \sigma_Y \left(\frac{J}{\sigma_Y \varepsilon_Y I_n r}\right)^{1/(n+1)} \left(\tilde{\sigma}_{ij}(\theta,n), \tilde{\sigma}_e(\theta,n)\right), \quad \varepsilon^P{}_{ij} = \varepsilon_Y \left(\frac{J}{\sigma_Y \varepsilon_Y I_n r}\right)^{n/(n+1)} \tilde{\varepsilon}^P_{ij}(\theta,n)$$

More next slide

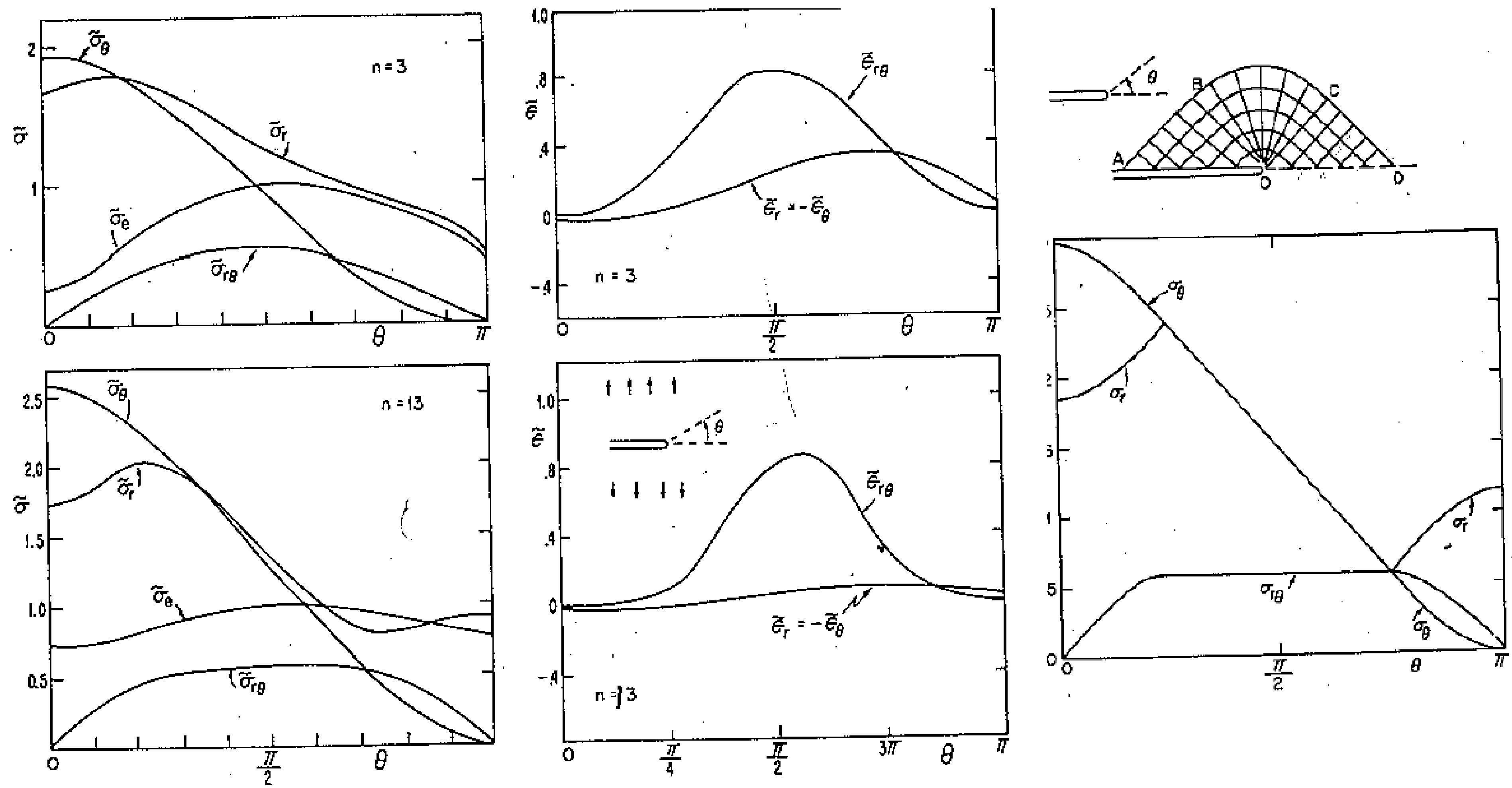
Crack Tip Fields for Power Law Hardening--continued

Plane strain, mode I fields:

The θ-variations, $\tilde{\sigma}(\theta,n)$, $\tilde{\varepsilon}(\theta,n)$ and I_n, depend on:

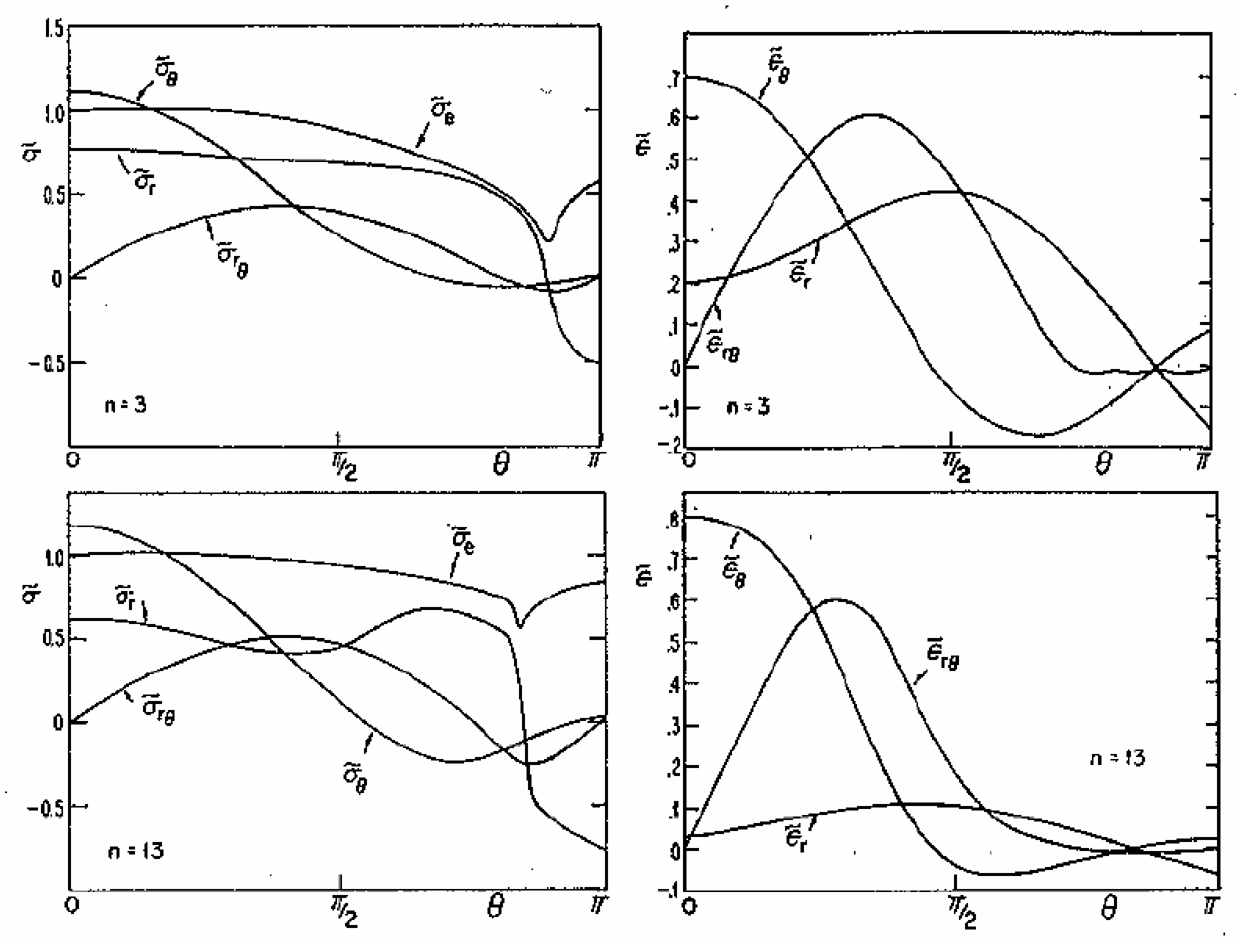
1) the mode (I, II or III); 2) whether plane strain or plane stress holds; 3) the hardening exponent *n*.

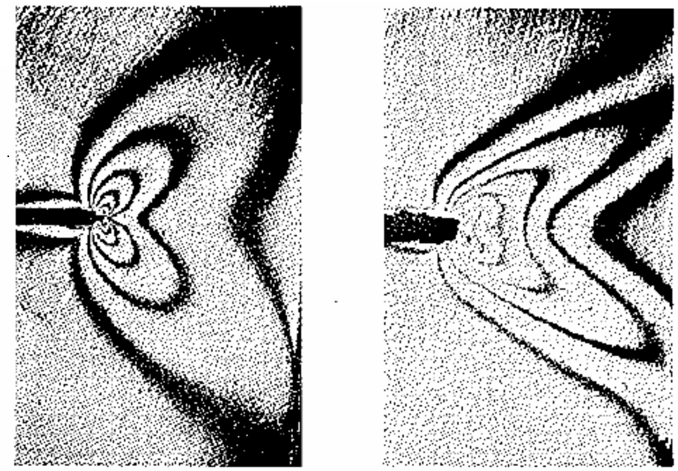
See the references for computing the θ-variations.



Crack Tip Fields for Power Law Hardening--continued

Plane stress, mode I fields:





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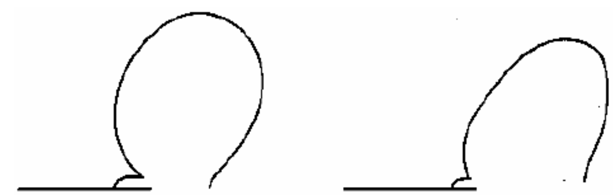
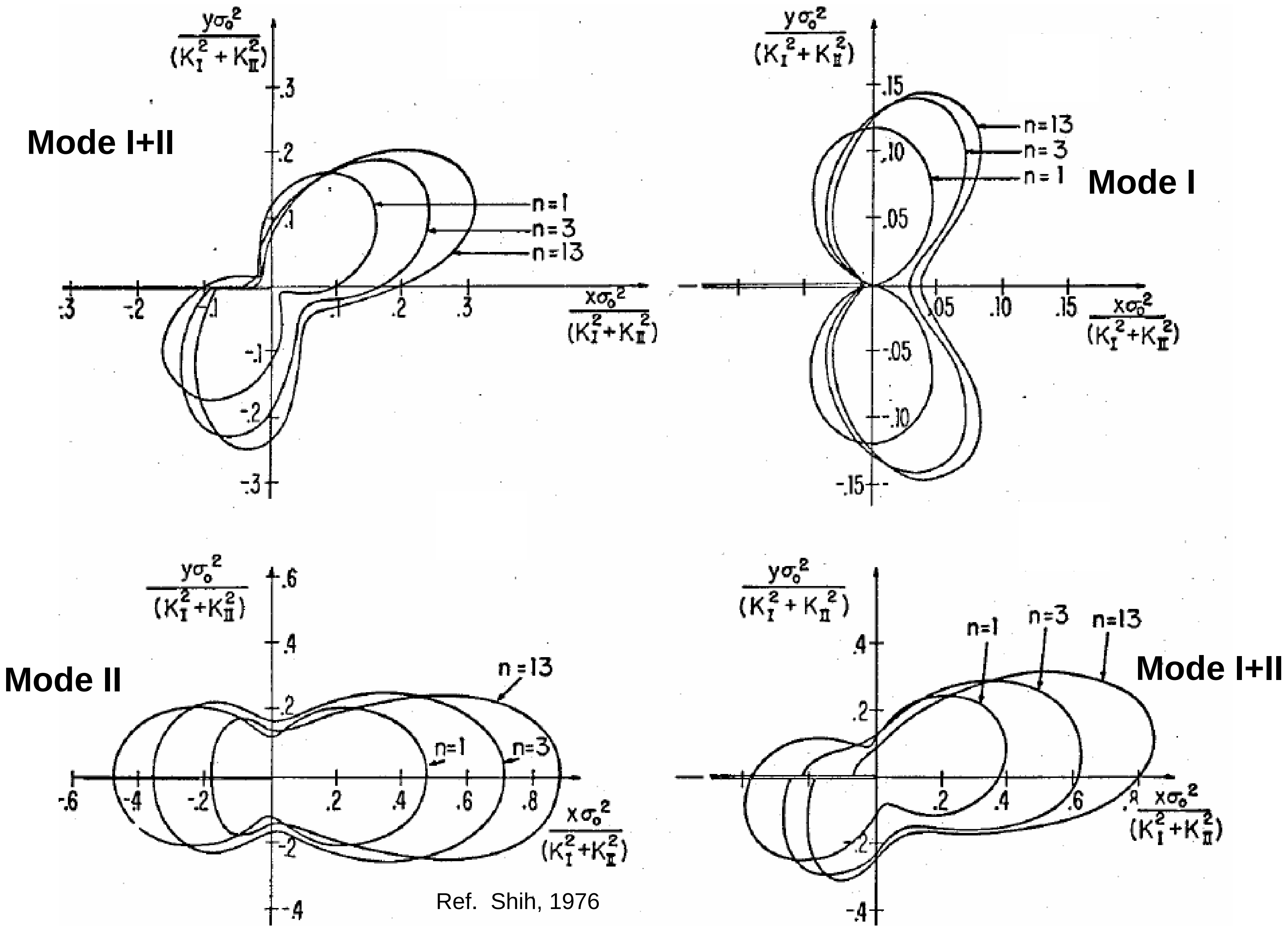


n = 3 n = 13

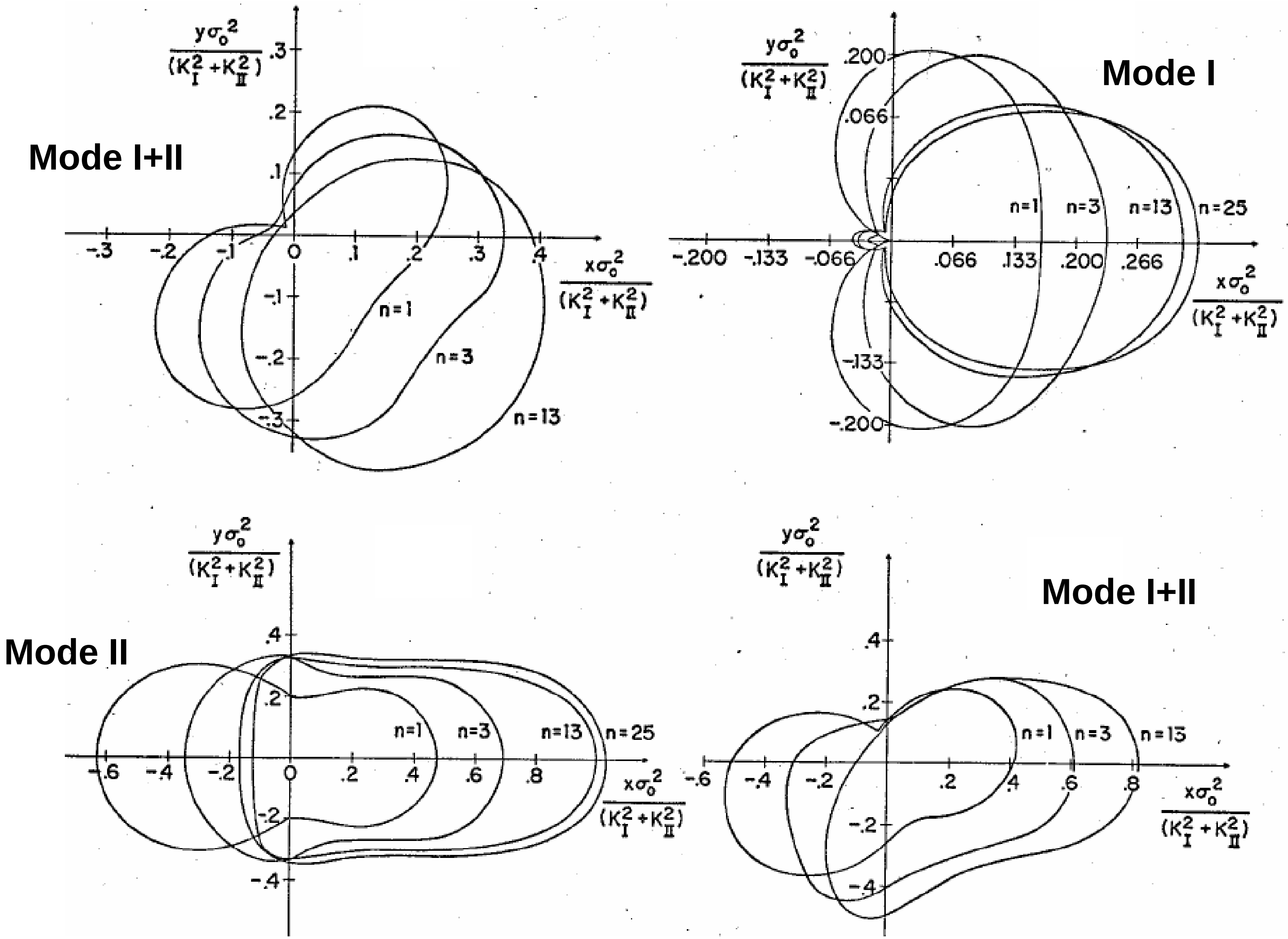
Isochromatic contours

Plastic Zones in Plane Strain Small Scale Yielding under Combined Modes I and II



Ref. Shih, 1976

Plastic Zones in Plane Stress Small Scale Yielding under Combined Modes I and II



Ref. Shih, 1976

J Characterization of Initiation of Crack Growth and JIC

The crack tip fields:

$$\left(\sigma_{ij},\sigma_e\right)=\sigma_Y\left(\frac{J}{\sigma_Y\varepsilon_Y I_n r}\right)^{1/(n+1)}\left(\tilde{\sigma}_{ij}(\theta,n),\tilde{\sigma}_e(\theta,n)\right),\quad \varepsilon^P{}_{ij}=\varepsilon_Y\left(\frac{J}{\sigma_Y\varepsilon_Y I_n r}\right)^{n/(n+1)}\tilde{\varepsilon}^P_{ij}(\theta,n)$$

Given plane strain (or plane stress) and mode I, J is a unique measure of the amplitude, or intensity, of the crack tip stress and strain distribution.

For plane strain, mode I, a criterion for crack growth initiation is

$$J=J_{IC}$$

with J_{IC} as a material property (toughness).

In small scale yielding, $J=(1-\nu^2)K^2/E$, and thus,

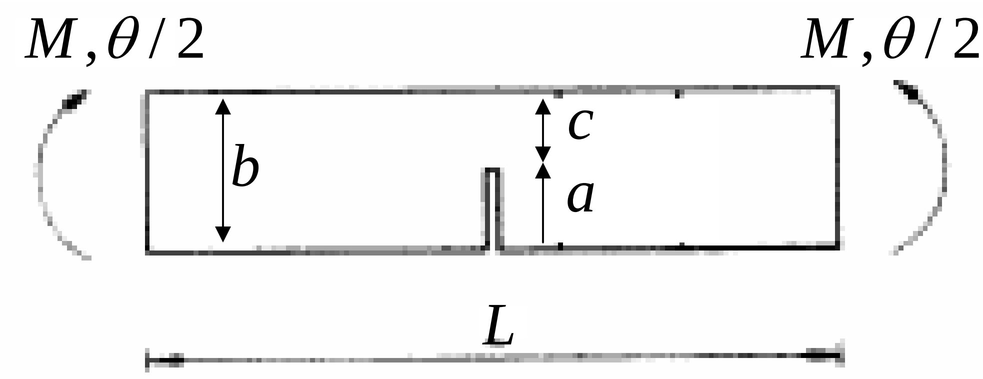
$$J_{IC}=(1-\nu^2)K_{IC}{}^2/E\equiv G_{IC}$$

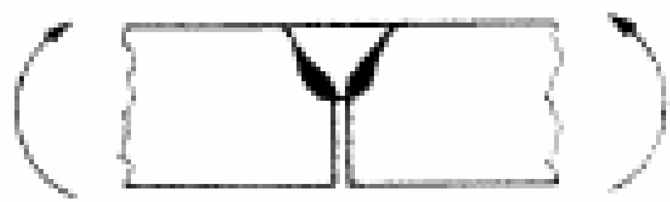
Since J is not limited to small scale yielding, it can be used in testing to determine toughness or in applications under **large scale yielding**. This requires knowledge of how J depends on load or displacement. How does one determine J?

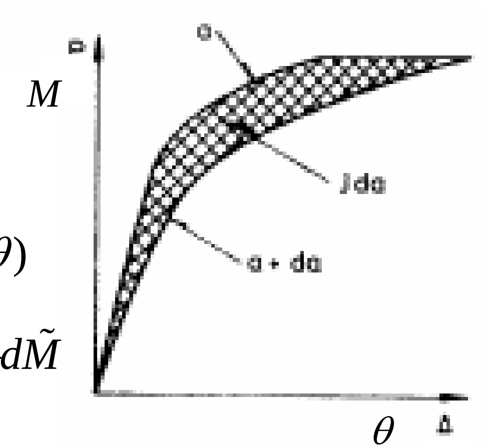
Determination of J—Analytical Methods and Experimental Methods

Various methods have been proposed to estimate J as a function of applied load or load point displacement. Some of these are surveyed in the fracture notes. It now common to use a finite element code to solve large scale yielding crack problems and to evaluate J directly using the path-independent integral. The situation is not unlike that for evaluating K or G for linear elastic problems, except that the nonlinearity due to plasticity makes the problems more difficult. We will not pursue this further in this course, however, we will derive one basic result for deeply-cracked bend specimens which allows direct experimental evaluation of J when such a specimen is used for determining JIC.

J for a deeply cracked bend specimen (pg. 55 of notes)







For a deeply cracked beam ($b >> c$), θ_{cr} depends on c but **not** b. By dimensional analysis:

$$\theta = \theta_{no\ crack} + \theta_{cr}$$

$$\theta_{cr} = f\left(\frac{M}{\sigma_Y c^2}, \frac{\sigma_Y}{E}, n\right)$$

In linear elasticity, $\theta_{cr} = 16M/(\bar{E}c^2)$

Recall, (with $P \to M$ & $\Delta \to \theta$)

$$J = -\left(\frac{\partial PE}{\partial a}\right)_M = \int_0^M \frac{\partial \theta(\tilde{M}, a)}{\partial a} d\tilde{M}$$

continued

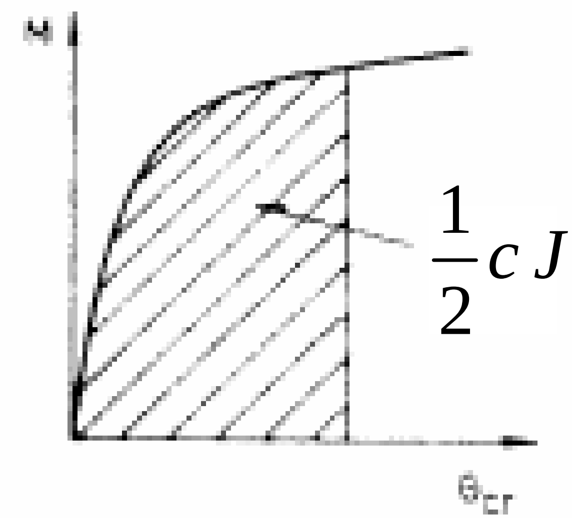
Determination of J—Experimental Method for Bend Specimens (continued)

$$J = -\left(\frac{\partial PE}{\partial a}\right)_M = \int_0^M \frac{\partial \theta(\tilde{M}, a)}{\partial a} d\tilde{M} = -\int_0^M \frac{\partial \theta_{cr}(\tilde{M}, c)}{\partial c} d\tilde{M} \quad \text{because } \frac{\partial(\ )}{\partial a} = -\frac{\partial(\ )}{\partial c}$$

Now note: $\left(\frac{\partial \theta_{cr}}{\partial c}\right)_M = -2\frac{\partial f}{\partial \mu}\frac{M}{\sigma_Y c^3}$, with $\mu = \frac{M}{\sigma_Y c^2}$, and $\left(\frac{\partial \theta_{cr}}{\partial M}\right)_c = \frac{\partial f}{\partial \mu}\frac{1}{\sigma_Y c^2}$

Thus, $\left(\frac{\partial \theta_{cr}}{\partial c}\right)_M = -\left(\frac{\partial \theta_{cr}}{\partial M}\right)_c \frac{2M}{c}$, and

$$J = \frac{2}{c}\int_0^M M d\theta_{cr}$$



Conclusion: For the deeply cracked bend specimen J can be determined directly as the area under the moment-rotation (cr) curve.