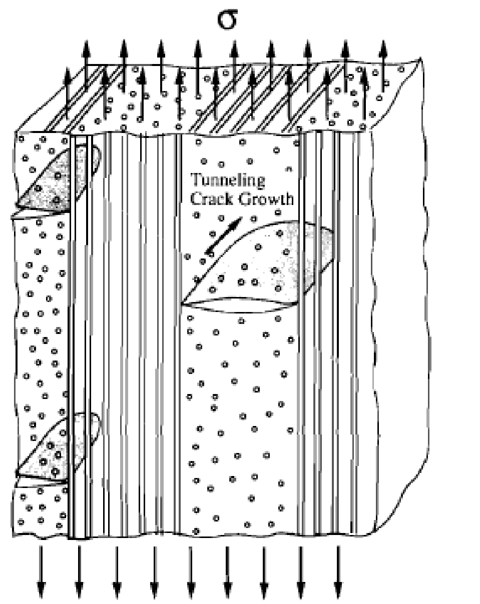
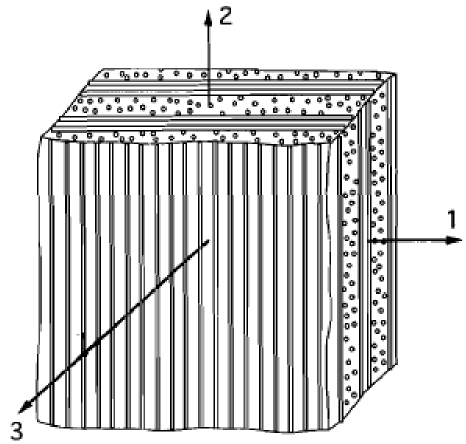
Matrix Cracking in Ceramic and Polymer Matrix Laminated Composites



.. A schematic of the 3D tunneling cracks in the 90° layers.



In-plane stress-strain relation of laminate

$$\epsilon_{11} = \frac{1}{E_0}\sigma_{11} - \frac{\nu_0}{E_0}\sigma_{22}$$

$$\epsilon_{22} = -\frac{\nu_0}{E_0}\sigma_{11} + \frac{1}{E_0}\sigma_{22}$$

$$\epsilon_{12} = \frac{1}{2\mu_0}\sigma_{12}$$

where

$$E_0 = \frac{\frac{1}{4}\left(1+\frac{E_L}{E_T}\right)^2 - \nu_L^2}{\frac{1}{2}\left(1+\frac{E_L}{E_T}\right)\left(\frac{1}{E_T}-\frac{\nu_L^2}{E_L}\right)}$$

$$\nu_0 = \frac{2\nu_L}{1+\frac{E_L}{E_T}}$$

$$\mu_0 = \mu_L$$

Stress-strain relation **for each ply** with 1-axis parallel to fibers.

$$\epsilon_{11} = \frac{1}{E_L}\sigma_{11} - \frac{\nu_L}{E_L}(\sigma_{22}+\sigma_{33})$$

$$\epsilon_{22} = -\frac{\nu_L}{E_L}\sigma_{11} + \frac{1}{E_T}\sigma_{22} - \frac{\nu_T}{E_T}\sigma_{33}$$

$$\epsilon_{33} = -\frac{\nu_L}{E_L}\sigma_{11} - \frac{\nu_T}{E_T}\sigma_{22} + \frac{1}{E_T}\sigma_{33}$$

$$\epsilon_{23} = \frac{1}{2\mu_L}\sigma_{23} \quad \epsilon_{13} = \frac{1}{2\mu_T}\sigma_{13} \quad \epsilon_{12} = \frac{1}{2\mu_T}\sigma_{12}$$

With $\nu_m = \nu_f \equiv \nu$

$$E_L = cE_f + (1-c)E_m$$

$$\mu_L = \frac{\mu_f(1-c) + \mu_m(1-c)}{\mu_f(1-c) + \mu_m(1+c)}\mu_m$$

$$\nu_L = \nu_T = \nu$$

$$E_T = \frac{1+2\eta c}{1-\eta c}E_m \qquad \eta = \frac{\frac{E_f}{E_m} - 1}{\frac{E_f}{E_m} + 2}$$

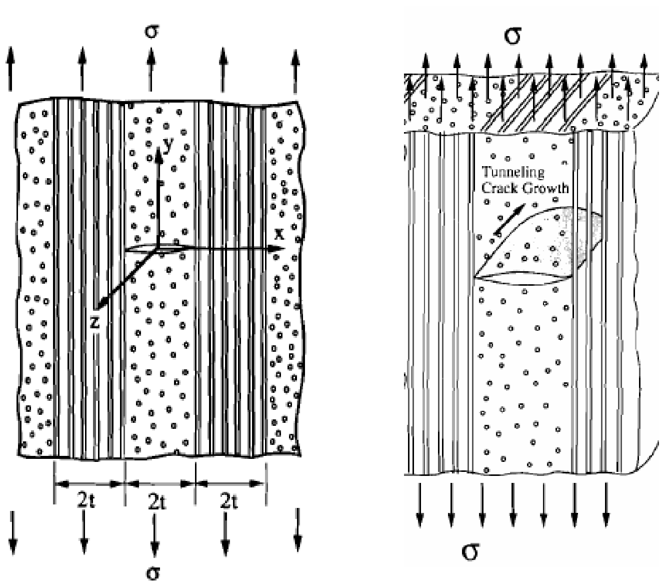
The above assumes bonded fibers.

Plane strain tensile modulus

$$\bar{E}_0 = \frac{E_0}{1-\nu_0^2}$$

Reference: Xia, Carr, Hutchinson, 1993, Acta Mater 41, 2365-2376.

Matrix Cracking, continued



Isolated tunnel crack—plane strain assumed for crack analysis.

overall (average applied stress)= σ

tensile stress in the 90° layer (prior to cracking): $\sigma_0 = \dfrac{2E_T}{E_L + E_T}\sigma$

Steady-state energy release rate for tunneling crack:

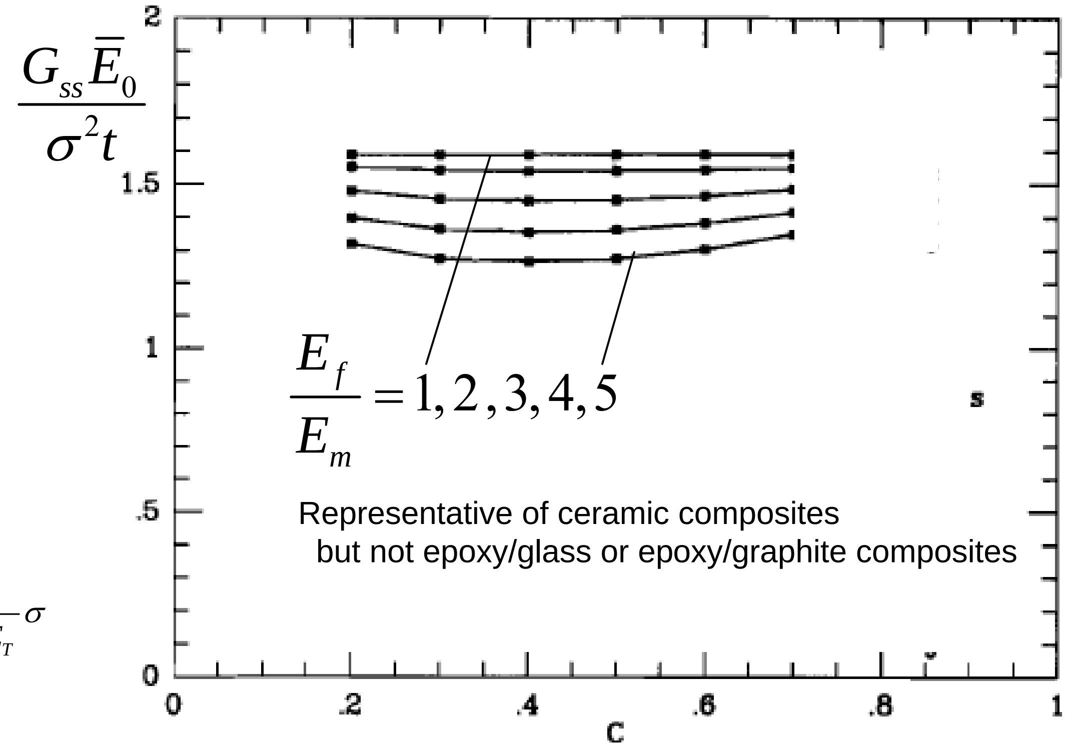
$$G_{ss} = \frac{1}{2}\left\{\frac{1}{2t}\int_{-t}^{t} \sigma_0(x)\,\delta(x)\,\mathrm{d}x\right\}$$

where $\delta(x)$ is the open displacement computed from the plane strain solution. If there is initial residual stress σ_R then it must be added to σ_0.

For specified Poisson ratios ($\nu_f = \nu_m = 0.2$):

$$\frac{G_{ss}\bar{E}_0}{\sigma^2 t} = f\left(\frac{E_f}{E_m}, c\right)$$

Finite element calculations (Xia, et al.) give



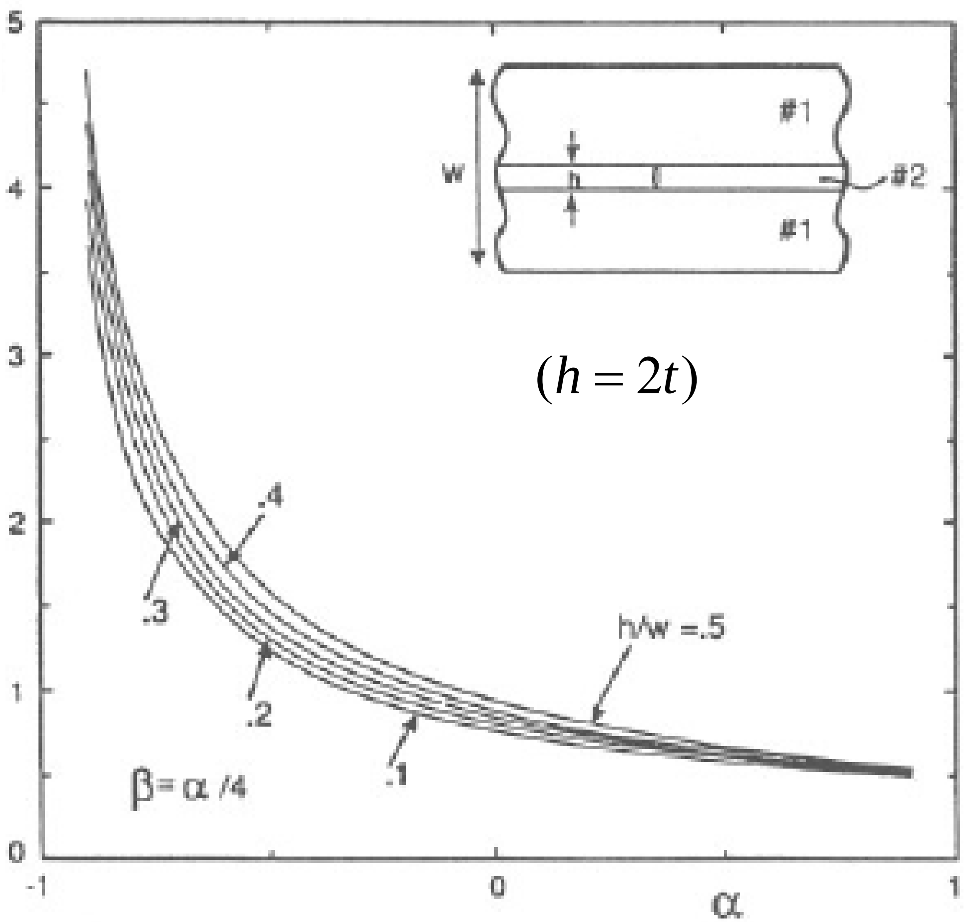
For uniform, isotropic material:

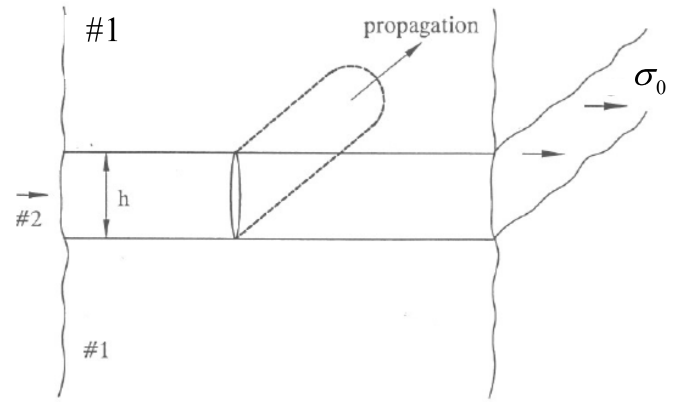
$$\frac{G_{ss}\bar{E}}{\sigma^2 t} = \frac{\pi}{2}$$

Matrix Cracking, continued—Limit of very stiff fibers

The previous method can be used for the case of very stiff fibers compared to matrix—see Xia, et al. We can also get Insight from the results of Ho and Suo (???) for one isotropic layer sandwiched between two isotropic layers of different modulus.

$$\frac{G_{ss}\bar{E}_2}{\sigma_0{}^2 h}$$





$$\alpha = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}, \ \sigma_0 = \text{stress in layer 2}$$

For $\dfrac{E_1}{E_2} >> 1, \ \ \alpha \to 1$ and

$$\frac{G_{ss}\bar{E}_2}{\sigma_0{}^2 h} \cong 0.5$$

For $\dfrac{E_2}{E_1} >> 1, \ \ \alpha \to -1$ and $\dfrac{G_{ss}\bar{E}_2}{\sigma_0{}^2 h}$ becomes large!

Matrix Cracking, continued—some representative numbers

For an **epoxy matrix** of thickness, $h = 0.5mm,\ E = 5GPa,\ \Gamma_{IC} = 50Jm^2$

$$G_{ss} = 0.5\frac{\sigma_0{}^2 h}{\bar{E}_2} = 0.5\bar{E}_2\varepsilon_0{}^2 h$$

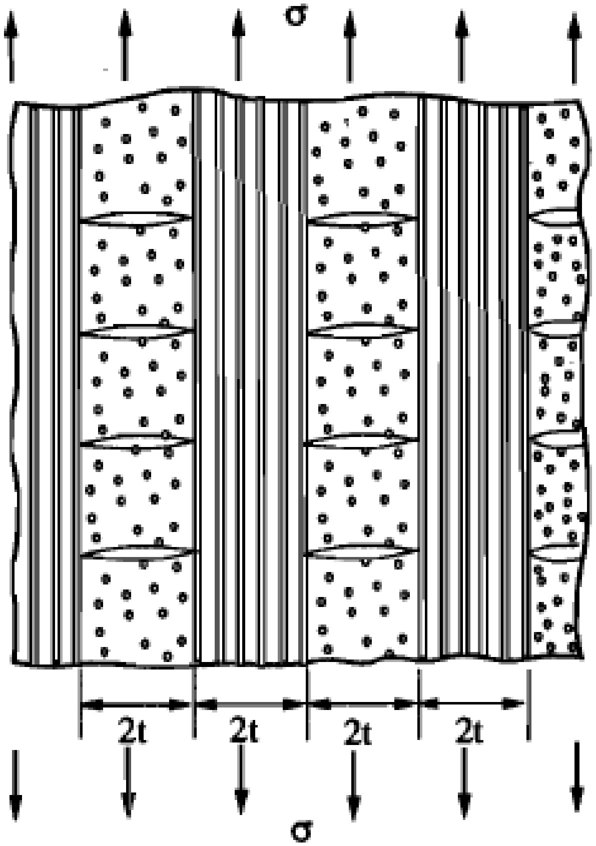
$$G_{ss} = \Gamma_{\mathrm{IC}} \Rightarrow \quad \varepsilon_0 \cong 0.02 \quad \& \quad \sigma_0 \cong 100MPa$$

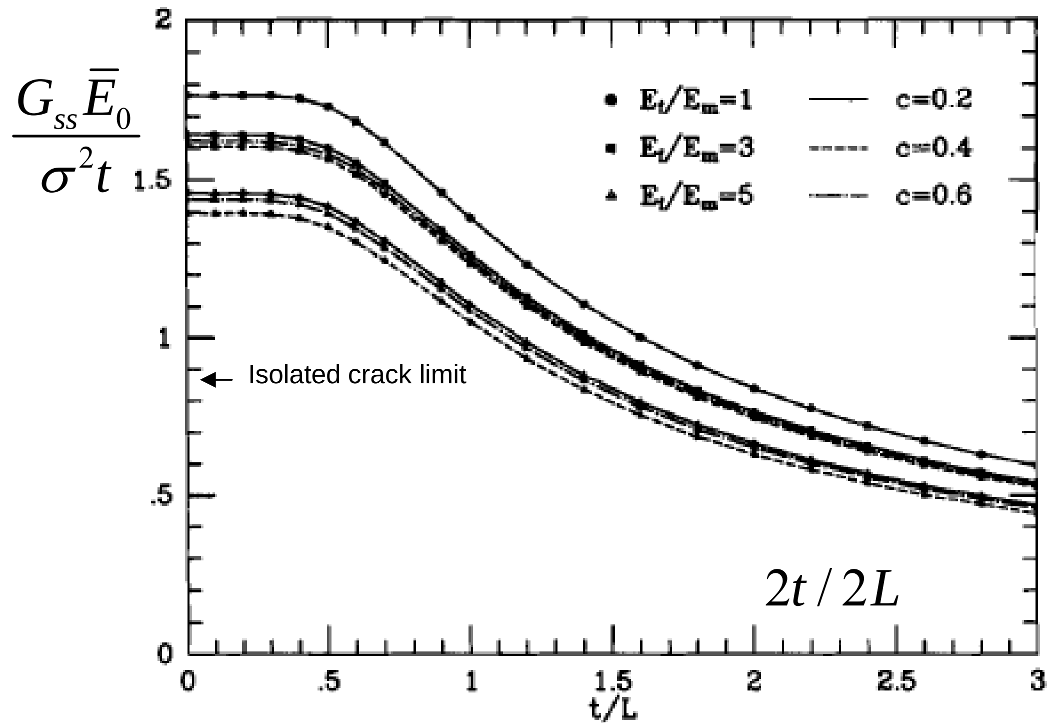
For an ceramic **matrix** of thickness, $h = 0.5mm,\ E = 200GPa,\ \ \Gamma_{IC} = 5Jm^2$

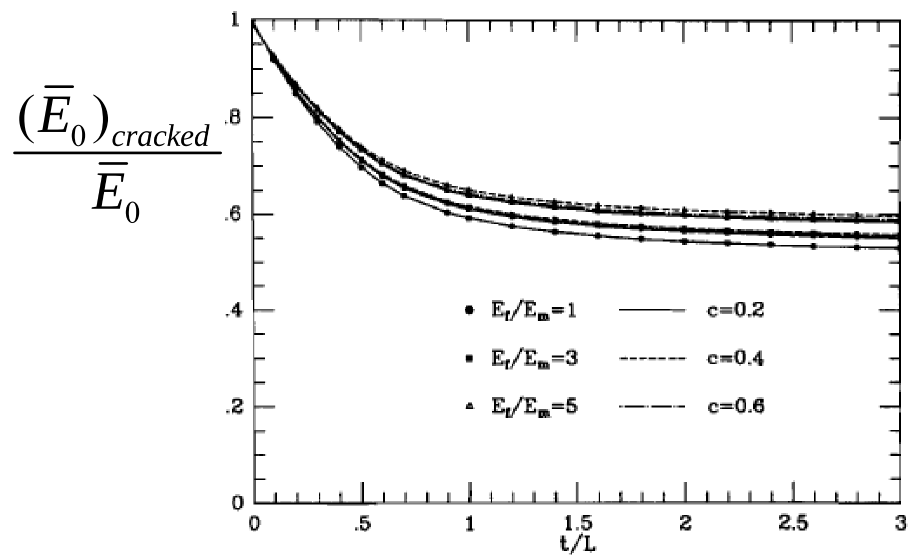
$$G_{ss} =\cong 1\frac{\sigma_0{}^2 h}{\bar{E}_2} = \bar{E}_2\varepsilon_0{}^2 h$$

$$G_{ss} = \Gamma_{\mathrm{IC}} \Rightarrow \quad \varepsilon_0 \cong 0.2\times10^{-3} \ \& \ \sigma_0 \cong 40MPa$$

Matrix Cracking, continued—periodic cracks



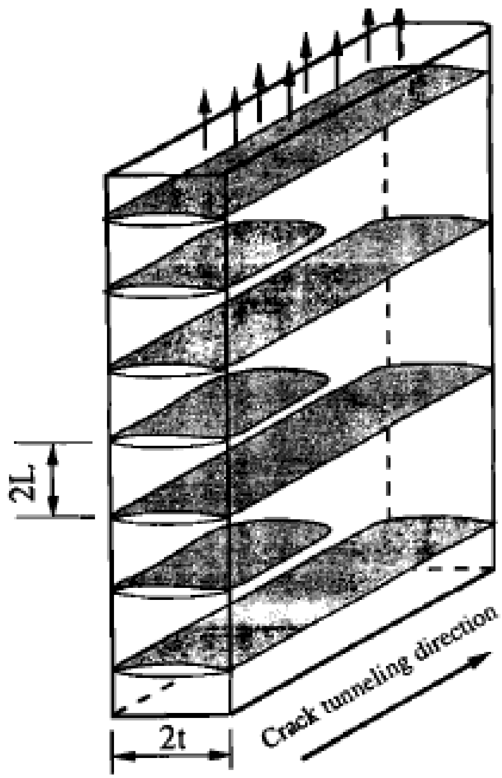




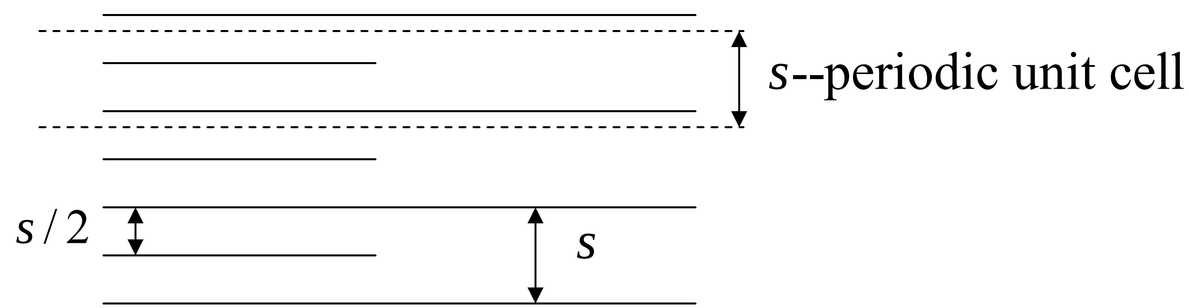
The same technique can be used to compute G_{ss} assuming **all the cracks tunnel simultaneously**.

One makes use of a unit cell with periodic boundary conditions.

Matrix Cracking, continued—Sequential periodic cracks



A new set of cracks tunneling between a set of previous cracks.



$s \equiv 4L =$ spacing between existing cracks

$s/2 \equiv 2L =$ spacing between new tunneling cracks

Recall that for a single set of cracks

with spacing s: $G_{ss}(s) = \left(SE_{no\ cracks} - SE_{cracks}(s)\right)s$

The G_{ss} for a second set of cracks bisecting an existing set with spacing s

$$\left(G_{ss}\right)_{sequential} = \left(SE_{cracks}(s) - SE_{cracks}(s/2)\right)s$$

$$= \left[2\left(SE_{no\ cracks} - SE_{cracks}(s/2)\right)s/2 - \left(SE_{no\ cracks} - SE_{cracks}(s)\right)s\right]$$

$$= 2G_{ss}(s/2) - G_{ss}(s)$$

With $G_{ss}\bar{E}_0/(\sigma^2 t) = f(t/L)$

for cracks with spacing $2L$ (see previous slide),

$$\left(G_{ss}\right)_{sequential}\bar{E}_0/(\sigma^2 t) = 2f(2t/L) - f(t/L)$$

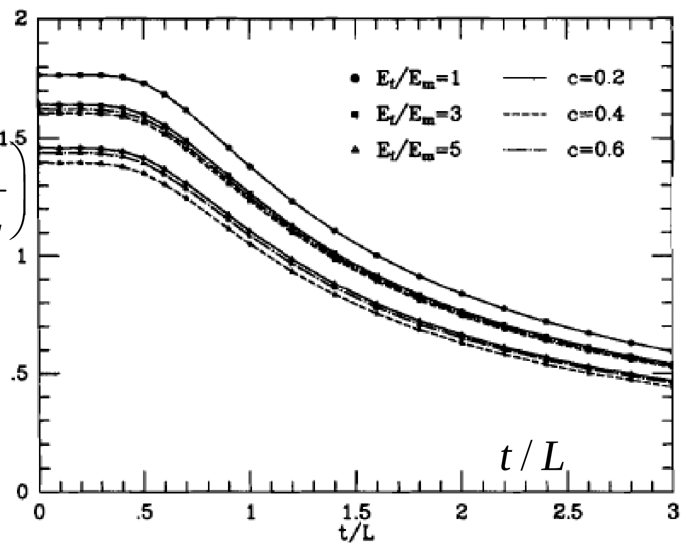
for cracks bisecting existing cracks with spacing $4L$.

Matrix Cracking, continued—crack spacing versus stress

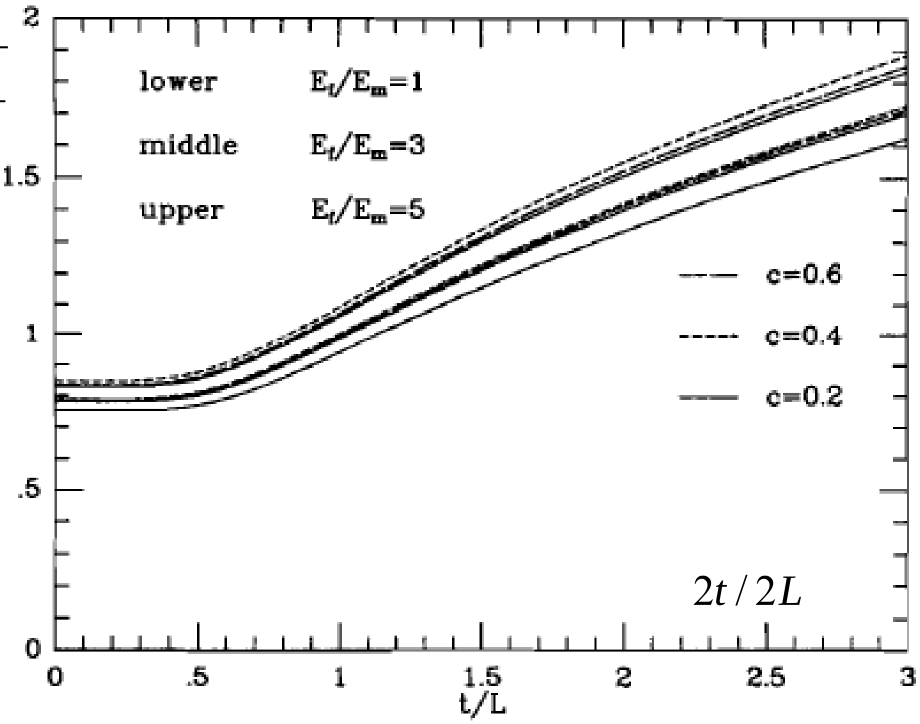
With $G_{ss}\bar{E}_0/(\sigma^2 t) = f(t/L)$ for cracks with spacing $2L$ (see previous slide),

$\left(G_{ss}\right)_{sequential}\bar{E}_0/(\sigma^2 t) = 2f(2t/L) - f(t/L)$ for cracks bisecting existing cracks with spacing $4L$.

$$\frac{G_{ss}\bar{E}_0}{\sigma^2 t} = f\left(\frac{t}{L}\right)$$



$$\sigma\sqrt{\frac{t}{\bar{E}_0\Gamma_{IC}}}$$



Relation between stress and crack spacing

$$(G_{ss})_{sequential} = \Gamma_{IC}$$

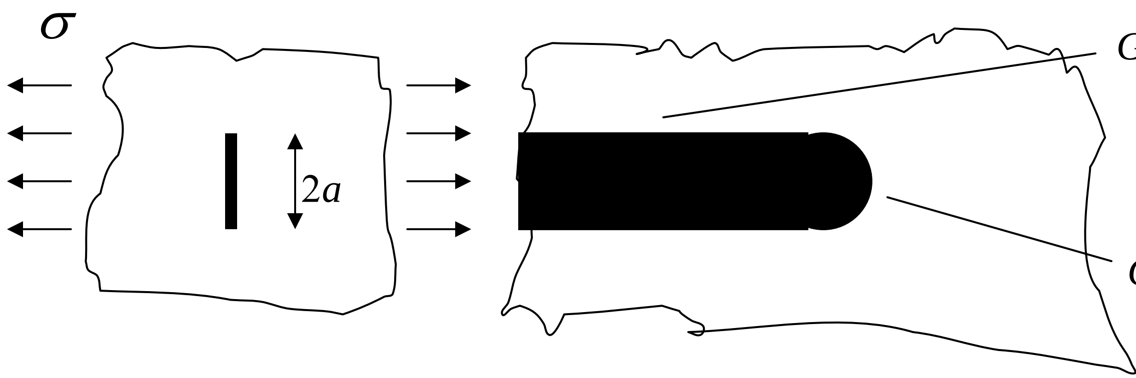
$$\Rightarrow \frac{\Gamma_{IC}\bar{E}_0}{\sigma^2 t} = 2f(2t/L) - f(t/L)$$

Note that we assume a new set of cracks is nucleated and bisects the previous set of cracks—thus it is the sequential G that is relevant.

Note, t/L increases roughly linearly with stress once the crack spacing exceeds about twice the layer thickness. The spacing does not correlate with stress for large spacing—this behavior is dominated by initial flaw statistics which is not considered here.

Competition between crack penetration and deflection at an interface

A little background: Consider the feasibility of tunnel cracking in a homogeneous isotropic material.



$$G_{sides} = \pi \frac{\sigma^2 a}{\bar{E}}$$

$$G_{front} = \frac{\pi}{2} \frac{\sigma^2 a}{\bar{E}}$$

Cracks will not tunnel in a homogeneous, isotropic material because it is more favorable for them to spread along their sides.

If the crack is in a layer and the sides are along an interface, there are three possibilities:
(i) the crack is arrested along the sides
(ii) the crack penetrates the interface
(iii) the crack deflects into the interface

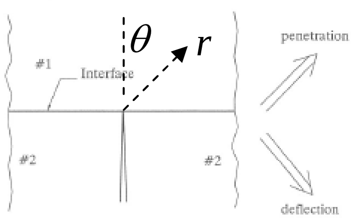


Fig. A

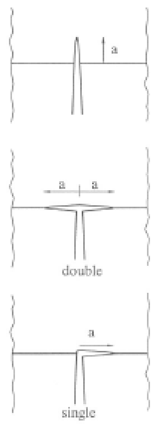


Fig. B1

Fig. B2

Fig. B3

λ is given by:

$$\cos \pi\lambda = (1-\lambda)^2 \frac{2(\beta-\alpha)}{1+\beta} + \frac{\alpha+\beta^2}{1-\beta^2}$$

(Zak and Williams, J. Appl. Mech. 30, 142-143, 1963)

See Next Slide for λ

We first consider the stress field near the crack tip in Fig. A for the tip at the interface. The stress field is symmetric about the crack assuming the loading is symmetric. The stresses have the form:

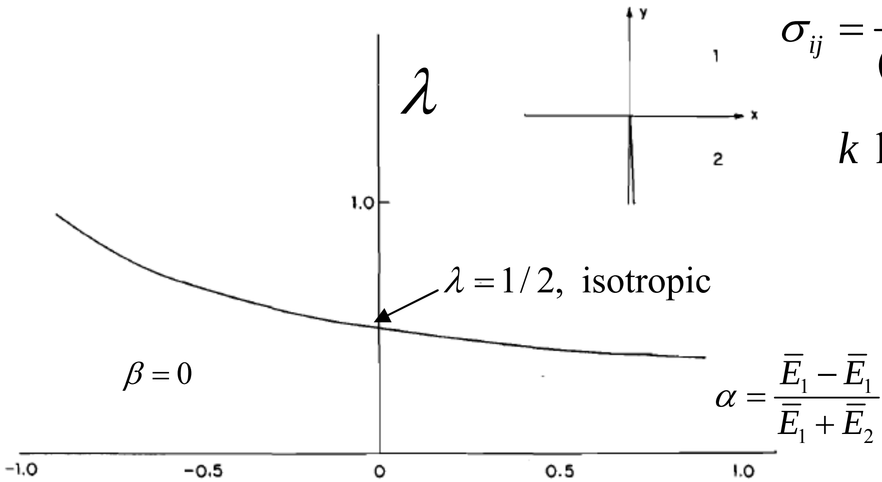
$$\sigma_{ij} = \frac{k}{(2\pi r)^\lambda} f_{ij}(\theta)$$

where the dimensionless functions $f_{ij}(\theta)$ depend on the Dundurs' mismatch parameters, α and β, as does λ--see to the left.

Note that k has units of $stress \bullet length^\lambda = Pa \bullet m^\lambda$!!

Ref: He and Hutch, IJSS 25, 1053-1067,1989
& He, Evans, Hutch, IJSS 31, 3443-34551994

Competition between crack penetration and deflection at an interface, continued



$$\sigma_{ij} = \frac{k}{(2\pi r)^{\lambda}} f_{ij}(\theta)$$

k has units $Pa\bullet m^{\lambda}$

Consider a short crack of length a in B1.

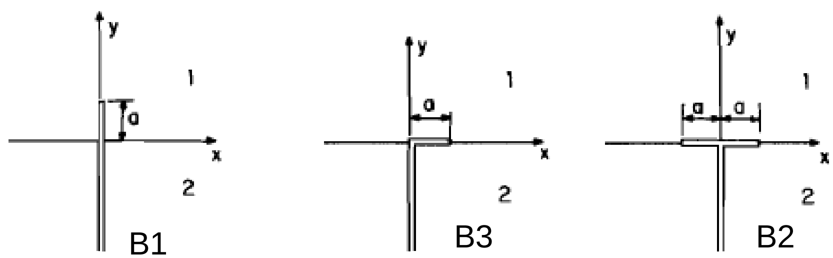
The stress intensity factor K of this crack depends linearly on k. Dimensional arguments require:

$$K_1 = c_1 k a^{-\lambda+1/2}$$

$$G_{penetration} = (c_1 k)^2 a^{1-2\lambda} / \bar{E}_1$$

where c_1 is a dimensionless function of α and β.

Note! $G \to \infty$ as $a \to 0$ if $\lambda > 1/2$ (α<0).



Now consider a short crack of length a in B2 & B3.

The stress intensity factors K1 & K2 of this crack depends linearly on k. For each case, dimensional arguments require:

$$K_1 = d_1 k a^{-\lambda+1/2}, K_2 = d_2 k a^{-\lambda+1/2},$$

$$G_{deflection} = \left[(d_1 k)^2 + (d_2 k)^2\right] a^{1-2\lambda} / \bar{E}^*$$

$$\text{for } \beta\text{=0 and } \frac{1}{\bar{E}^*} = \frac{1}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)$$

where d_1 and d_2 are dimensionless functions of α ($\beta = 0$).

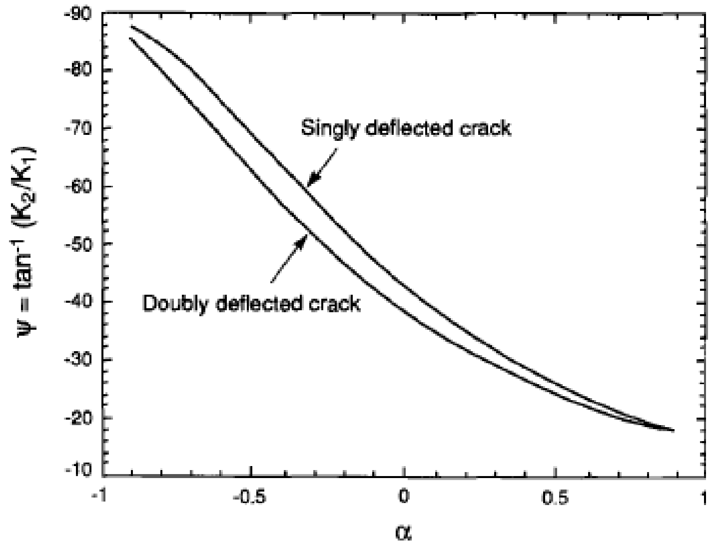
$$\Leftarrow$$

$$\frac{G_{deflection}}{G_{penetration}} = \frac{\left[{d_1}^2 + {d_2}^2\right]}{{c_1}^2} \frac{\bar{E}_1}{\bar{E}^*}$$

$$\tan\Psi_{deflection} \equiv \frac{K_2}{K_1} = \frac{d_2}{d_1}$$

Note that the above ratios are independent of load!

Competition between crack penetration and deflection at an interface, continued



Crack Advance Criteria:

Penetration $\Rightarrow$ $G_{penetration} = (\Gamma_{IC})_{\text{material1}}$

Deflection $\Rightarrow$ $G_{deflection} = (\Gamma_C(\psi))_{\text{interface}}$

Assuming roughly the same flaw size, a, for both the interface and the penetrating crack,

$$\frac{(\Gamma_C(\psi))_{\text{interface}}}{(\Gamma_{IC})_{\text{material1}}} < \frac{G_{deflection}}{G_{penetration}} \Rightarrow \textbf{Deflection wins}$$

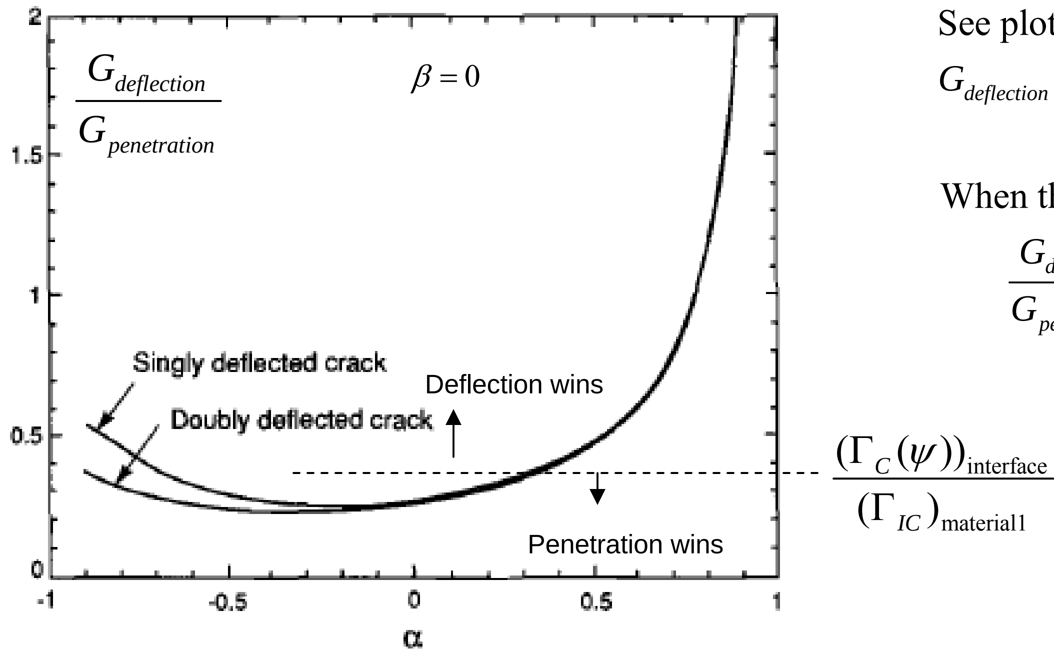
$$\frac{(\Gamma_C(\psi))_{\text{interface}}}{(\Gamma_{IC})_{\text{material1}}} > \frac{G_{deflection}}{G_{penetration}} \Rightarrow \textbf{Penetration wins}$$

See plot. Of course the load must be sufficient such that

$G_{deflection} \geq (\Gamma_C(\psi))_{\text{interface}}$ *or* $G_{penetration} \geq (\Gamma_{IC})_{\text{material1}}$

When the **elastic mismatch is small**,

$$\frac{G_{deflection}}{G_{penetration}} \cong \frac{1}{4}$$



Competition between crack advance in interface and kinking out of interface

He & Hutchinson, J. Appl. Mech. 1989, 270-278.

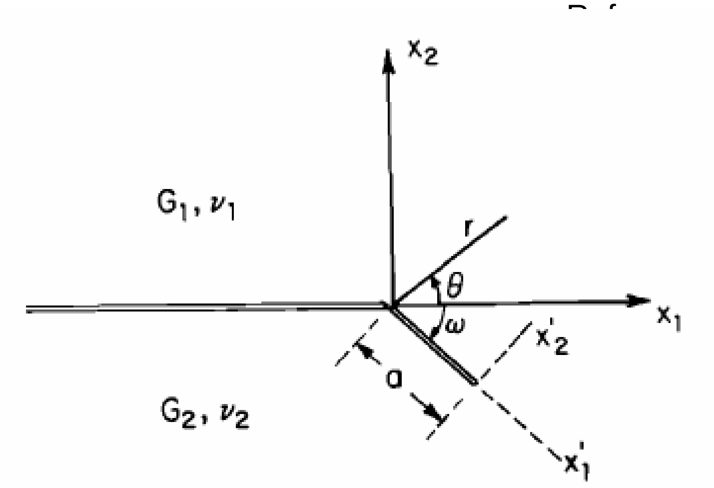


Fig. 1 Geometry of kinked crack

Crack in the interface:

Stress intensity factors & energy release for **crack on interface**:

$$K_1 \;\;\&\; K_2, \;\; \tan\psi_0 = \frac{K_2}{K_1}, \;\; G_0 = \frac{1}{\bar{E}^*}\left(K_1^{\,2} + K_2^{\,2}\right), \;\; \frac{1}{\bar{E}^*} = \frac{1}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)$$

Crack kinking out of the interface:

Stress intensity factors & energy release for **kinked crack** $(\beta = 0)$:

$$K_I \;\;\&\; K_{II}, \;\; \tan\psi = \frac{K_{II}}{K_I}, \;\; G_{kink} = \frac{1}{\bar{E}_2}\left(K_I^{\,2} + K_{II}^{\,2}\right)$$

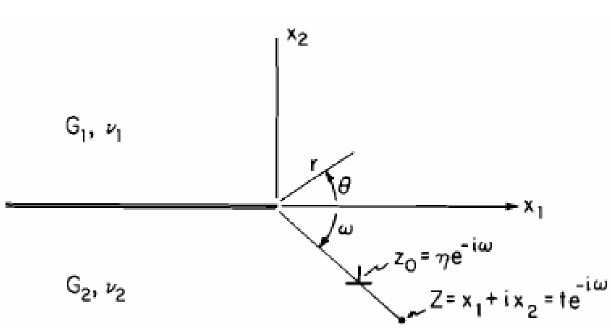
Assume $\psi_0 > 0$ so kinked crack propagates into material #2.

K_I & K_{II} are linear functions of K_1 & K_2. Dimensional analysis implies:

$$K_I = a_{11}K_1 + a_{12}K_2, \quad K_{II} = a_{21}K_1 + a_{22}K_2$$

where the $a_{ij}(\omega,\alpha)$ depend on ω and the first Dundurs' parameter α, but independent of a.

Brief sketch of solution procedures to determine KI & KII based on integral equation methods.



Let $b_r(\eta)$ & $b_\theta(\eta)$ be components of an edge dislocation at z_0. The problem noted in the figure where the dislocation interacts with a semi-infinite crack can be solved in closed form. The tractions acting on the plane at angle ω at a point $z = te^{-i\omega}$ are given by (see He & Hutch, 1989)

$$\sigma_{\theta\theta}(t) = \bar{E}\left(\left(\frac{1}{4\pi}\frac{1}{t-\eta} + H_{\theta\theta}(t,\eta,\omega)\right)b_\theta(\eta) + H_{\theta r}(t,\eta,\omega)b_r(\eta)\right)$$

$$\sigma_{r\theta}(t) = \bar{E}\left(\left(\frac{1}{4\pi}\frac{1}{t-\eta} + H_{r\theta}(t,\eta,\omega)\right)b_r(\eta) + H_{r\theta}(t,\eta,\omega)b_\theta(\eta)\right)$$

Competition between crack advance in interface and kinking out of interface: continued

The stress on the plane at z due to the applied intensity factors is (classic crack tip fields)

$$\sigma_{\theta\theta}(t)=\frac{K_1}{\sqrt{2\pi t}}f_{\theta\theta}^{(1)}(-\omega)+\frac{K_2}{\sqrt{2\pi t}}f_{\theta\theta}^{(2)}(-\omega),\quad \sigma_{r\theta}(t)=\frac{K_1}{\sqrt{2\pi t}}f_{r\theta}^{(1)}(-\omega)+\frac{K_2}{\sqrt{2\pi t}}f_{r\theta}^{(2)}(-\omega),$$

The integral equations for the distributions $b_r(\eta)$ & $b_\theta(\eta)$ are

$$\bar{E}\int_0^a\left(\left(\frac{1}{4\pi}\frac{1}{t-\eta}+H_{\theta\theta}(t,\eta,\omega)\right)b_\theta(\eta)+H_{\theta r}(t,\eta,\omega)b_r(\eta)\right)d\eta=-\frac{K_1}{\sqrt{2\pi t}}f_{\theta\theta}^{(1)}(-\omega)-\frac{K_2}{\sqrt{2\pi t}}f_{\theta\theta}^{(2)}(-\omega)$$

$$\bar{E}\int_0^a\left(\left(\frac{1}{4\pi}\frac{1}{t-\eta}+H_{r\theta}(t,\eta,\omega)\right)b_r(\eta)+H_{r\theta}(t,\eta,\omega)b_\theta(\eta)\right)d\eta=-\frac{K_1}{\sqrt{2\pi t}}f_{r\theta}^{(1)}(-\omega)-\frac{K_2}{\sqrt{2\pi t}}f_{r\theta}^{(2)}(-\omega)$$

These are called Cauchy-type integral equations. There are powerful numerial methods for solving these equations (Erdogan and Gupta, 1972, Q. Appl. Math. 29, 525-534). The desired stress intensity factors, K_I and K_{II}, and thus the coefficients, a_{ij}, are simply related to the distribution of the dislocations as $t \rightarrow a$.

Alternatively, finite element methods could be used to obtain the intensity factors and coefficients. However, given the interest in all orientations ω, integral equation methods are probably more efficient and somewhat more accurate.

Competition between crack advance in interface and kinking out of interface: continued

KINKING IN A HOMOGENEOUS MATERIAL UNDER MIXED MODE LOADING

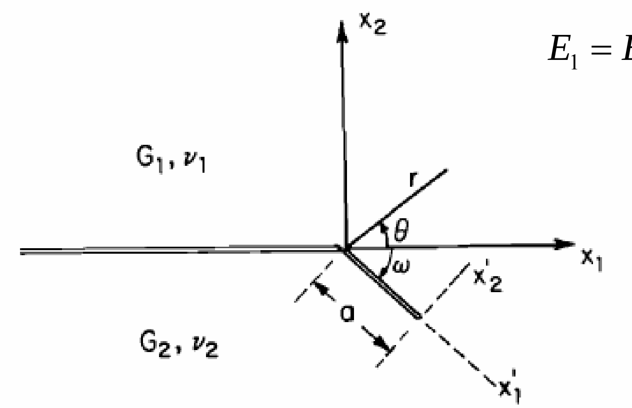


Fig. 1 Geometry of kinked crack

$E_1 = E_2 = E, \quad \nu_1 = \nu_2 = \nu; \quad K_1$ & K_2 are prescribed.

Contending criteria for advance of the kinked crack in a material with isotropic and homogenerous elastic and fracture properties.

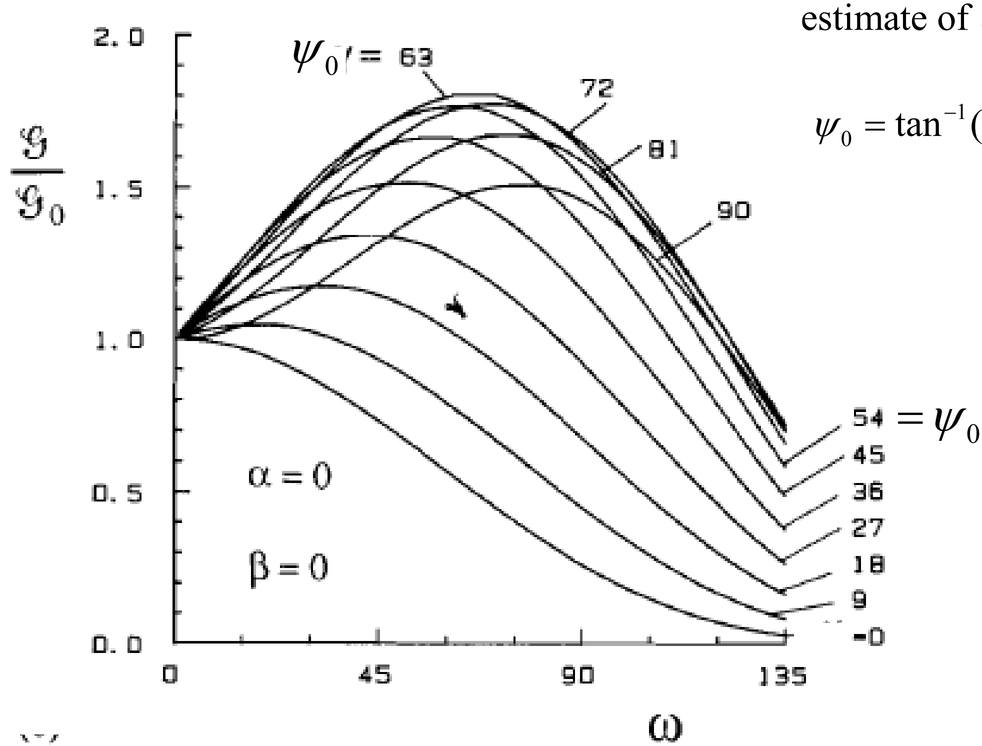
A) ω is determined by $K_{II} = 0$; advance requires $K_I = K_{IC}$

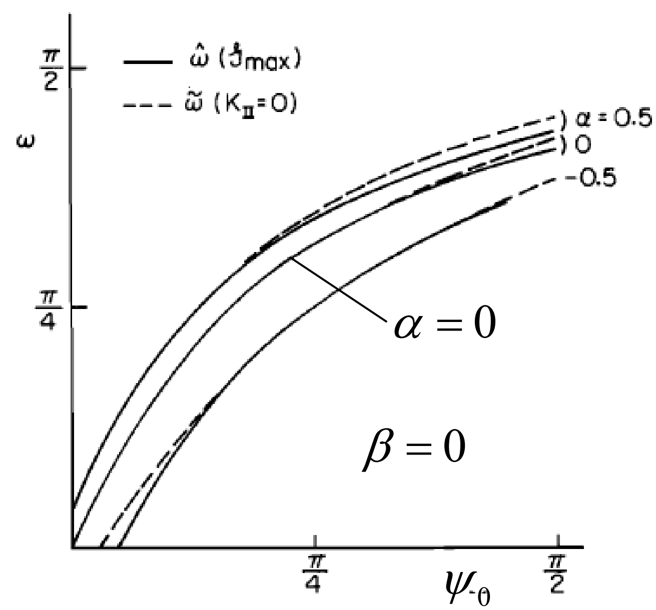
B) ω is determined by maximizing G; advance requires $G = \Gamma_{IC}$

C) ω is determined by maximizing $\sigma_{\theta\theta}$ associated with K_1 & K_2

Criterion C was set as a homework problem. It give a reasonable estimate of ω (compared to A or B) as long as $\psi_0 < 45^o$

$\psi_0 = \tan^{-1}(K_2 / K_1)$





There is very little difference between A & B

Competition between crack advance in interface and kinking out of interface: continued

Bi-material case

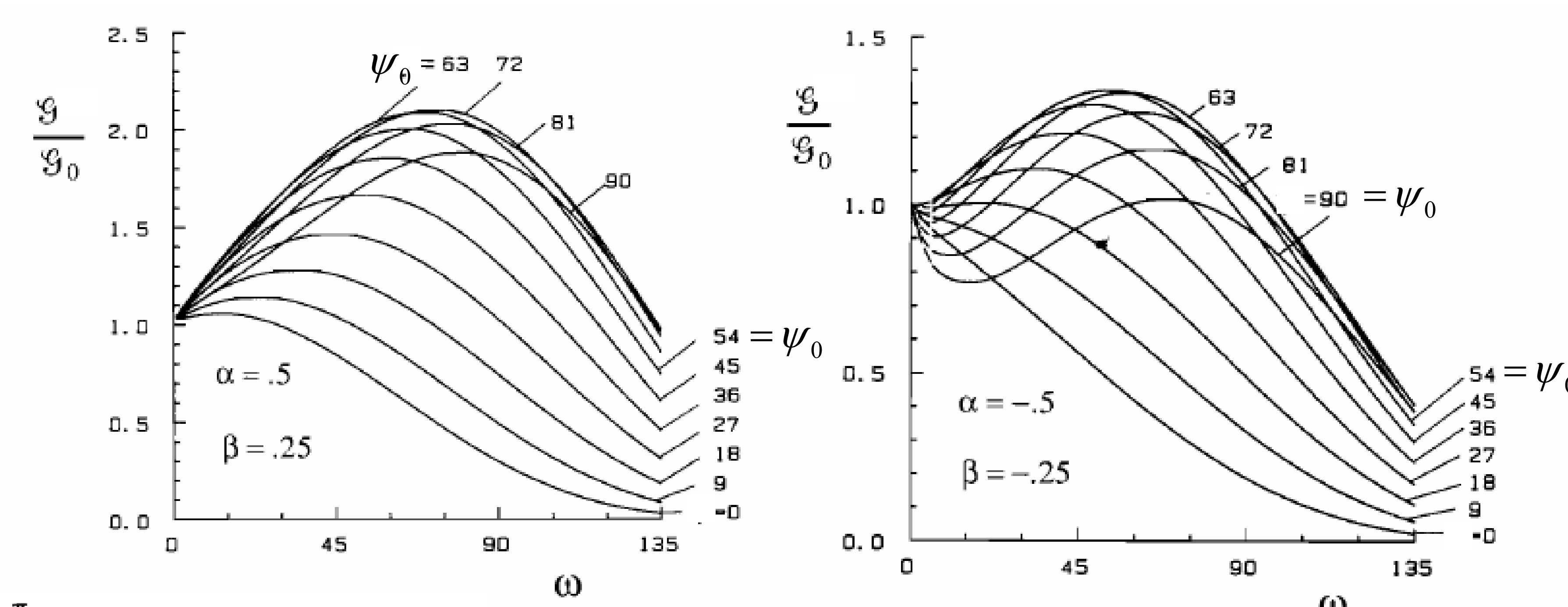
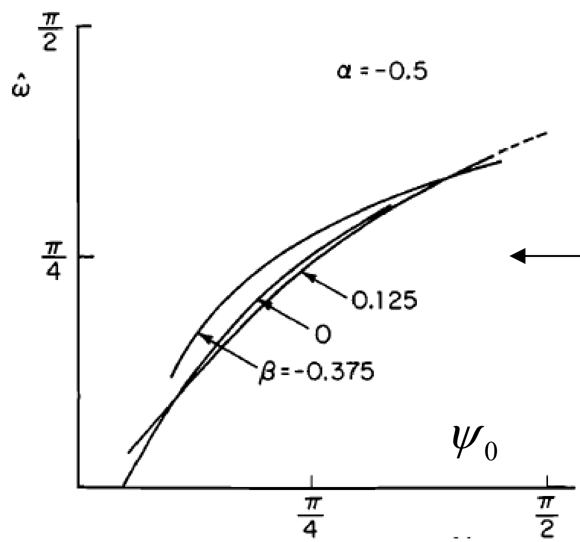


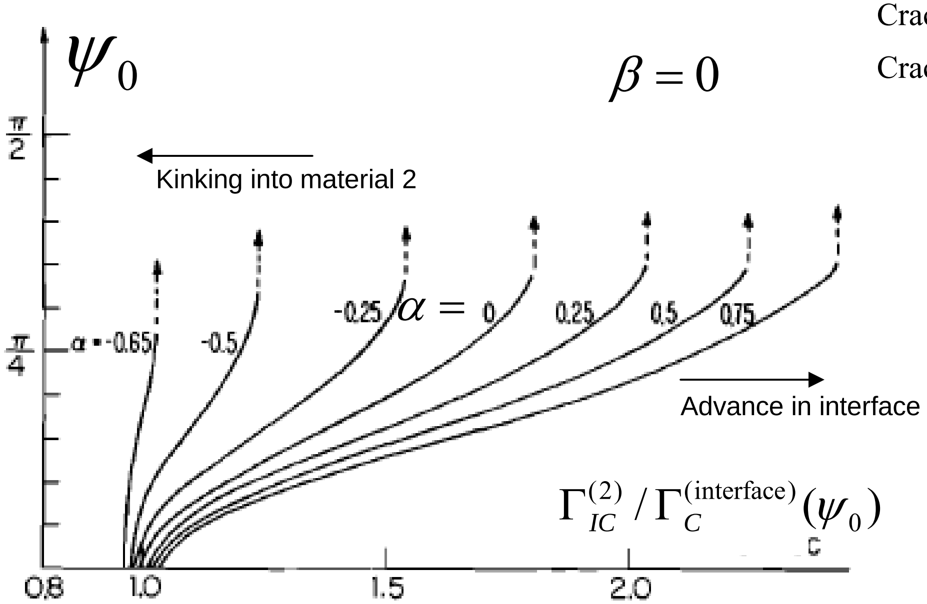
Fig. 8 $\mathcal{G}^*/\mathcal{G}_0$ versus ω for two cases in which $\beta \neq 0$



Kink angle associated with K2=0

Competition between crack advance in interface and kinking out of interface: continued

Bi-material case



Curves corresponding to equally likely kinking and advance in the interface for a wide range of elastic mismatch.

Employ maximum G criterion B--(essentially indentical to A).

Crack advance criterion for material 2 below interface: $G_{\text{kink}} = \Gamma_{IC}^{(2)}$

Crack advance criterion for interface crack: $G_0 = \Gamma_C^{(\text{interface})}(\psi_0)$

Consider curves where **kinking and crack advance in interface are equally likely** :

$$\frac{G_{\text{kink}}}{G_0} = \frac{\Gamma_{IC}^{(2)}}{\Gamma_C^{(\text{interface})}(\psi_0)}$$

These curves are plotted in the figure to the left. That is, the solid curves are ψ_0 *vs.* G_{kink} / G_0.

For a prescribed mode mix, ψ_0 :

Kinking into material 2 will occur if $\Gamma_{IC}^{(2)} / \Gamma_C^{(\text{interface})}$ is to the **left** of the curve (for a given α), while

Crack advance along with interface will occur if $\Gamma_{IC}^{(2)} / \Gamma_C^{(\text{interface})}$ is to the **right** of the curve.

For example, for α=0 and $\psi_0 = 45^o$:

Kinking will occur if $\Gamma_{IC}^{(2)} / \Gamma_C^{(\text{interface})}(45^o) < 1.6$

Interface advance will occur if $\Gamma_{IC}^{(2)} / \Gamma_C^{(\text{interface})}(45^o) > 1.6$

The more compliant is material 2, the greater must be its toughness to avoid kinking.