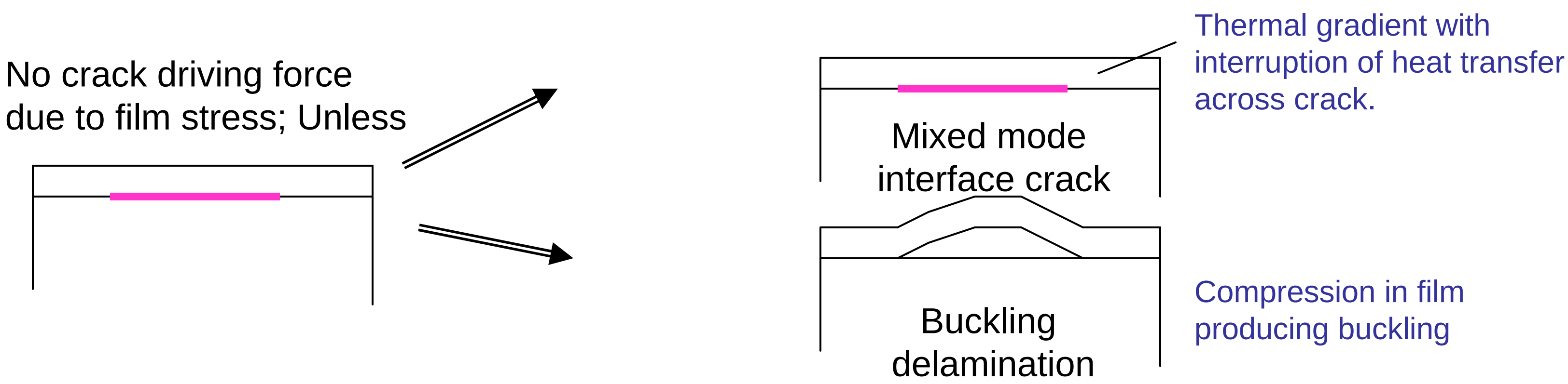
Buckling Delamination with Application to Films and Laminates

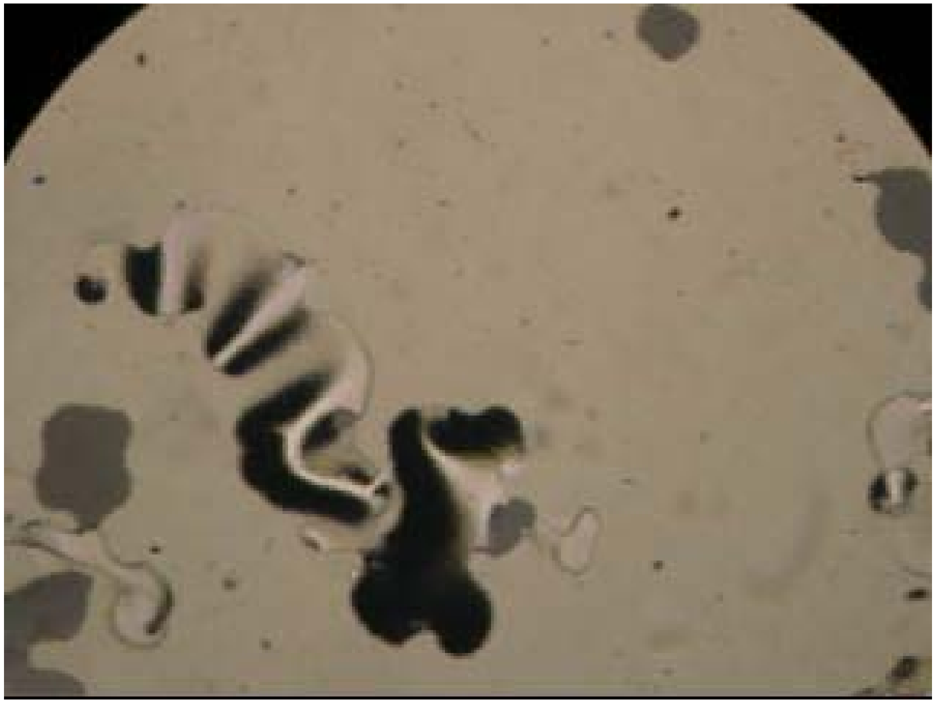


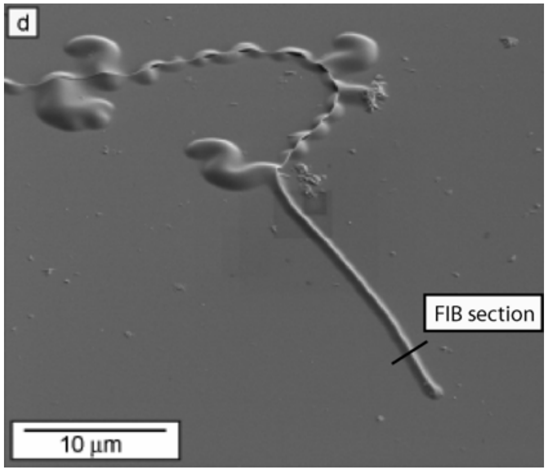
Show Volinsky movie

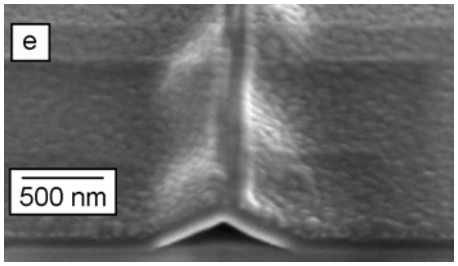
Mechanics of thin films and multilayers

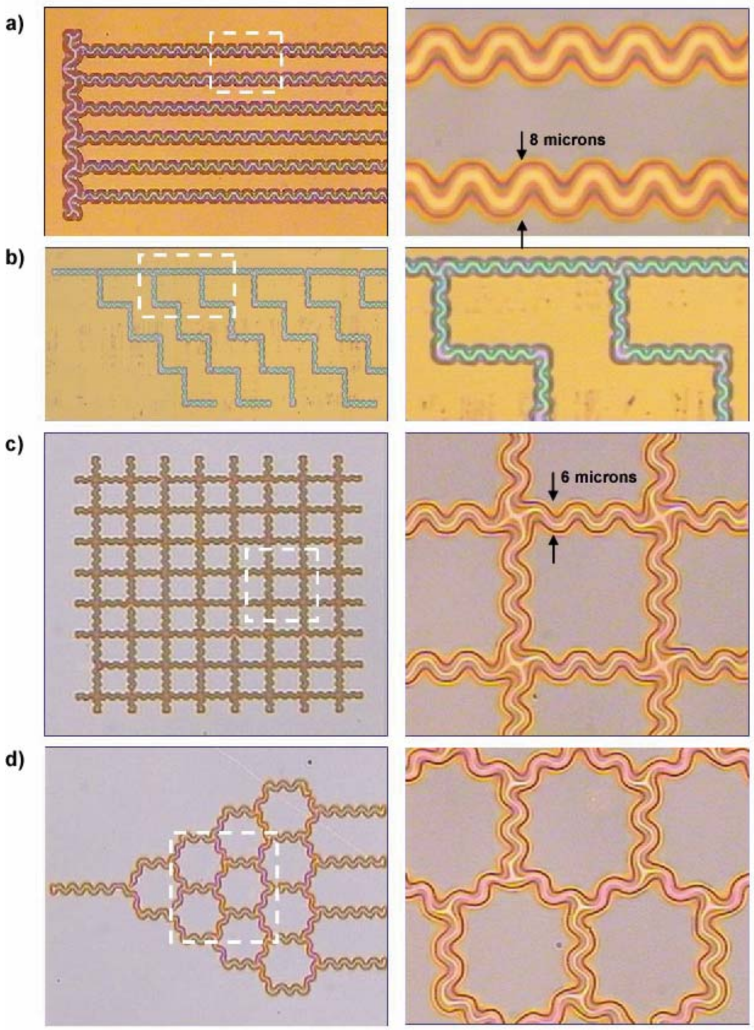
Application areas electronics, coatings of all kinds.

Example: Buckle Delaminations





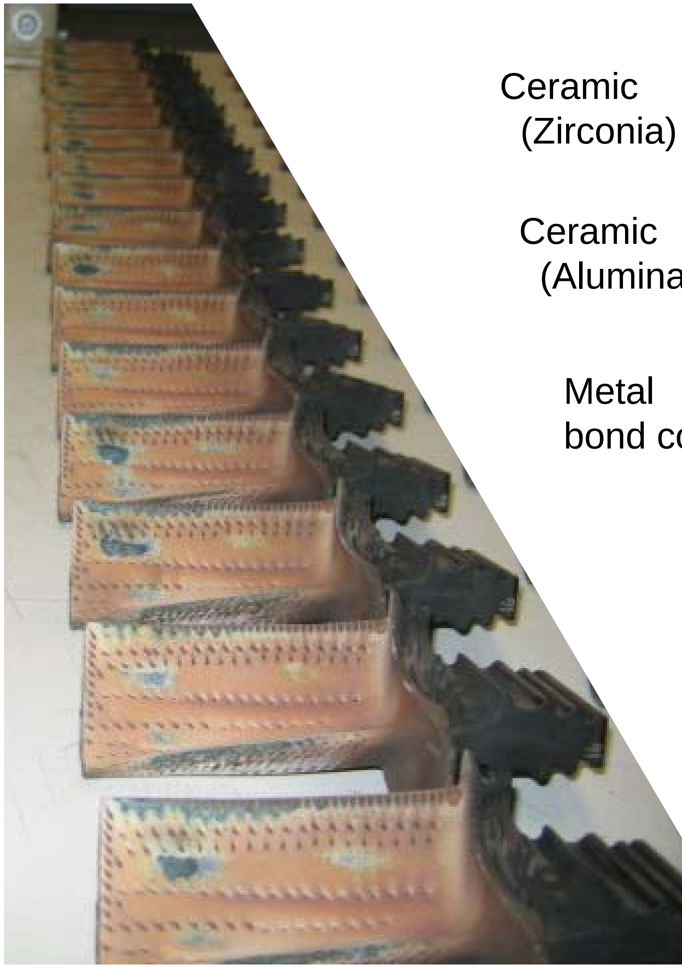




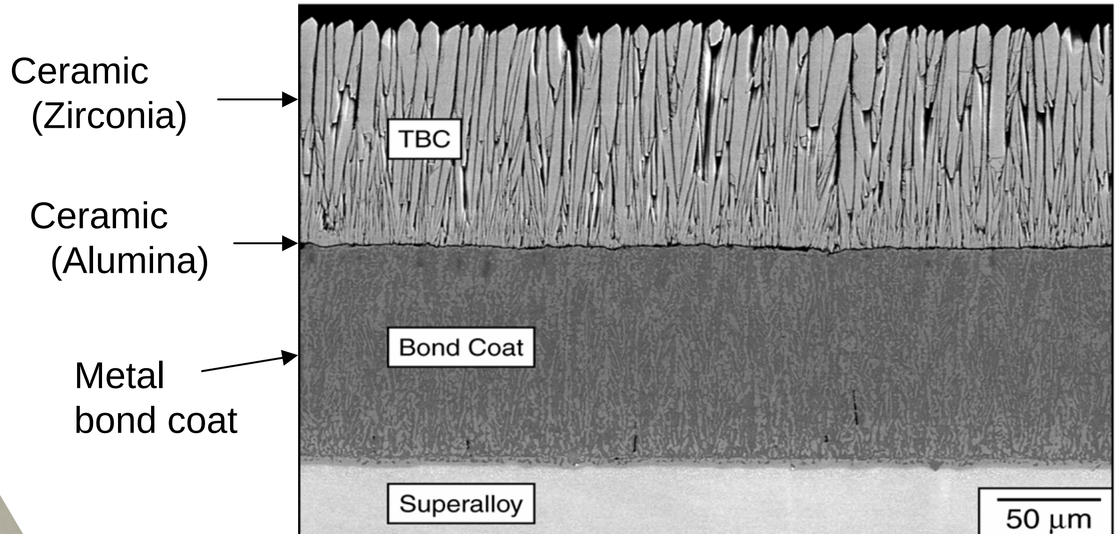
Good Delaminations on Patterned substrates

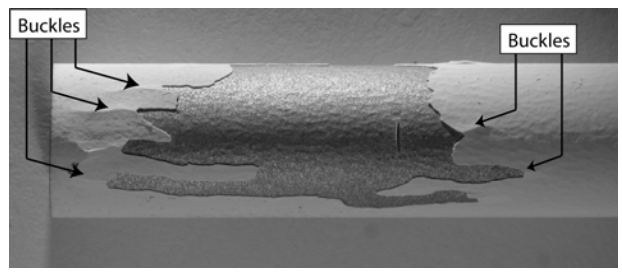
Thermal Barrier Coatings (TBCs)

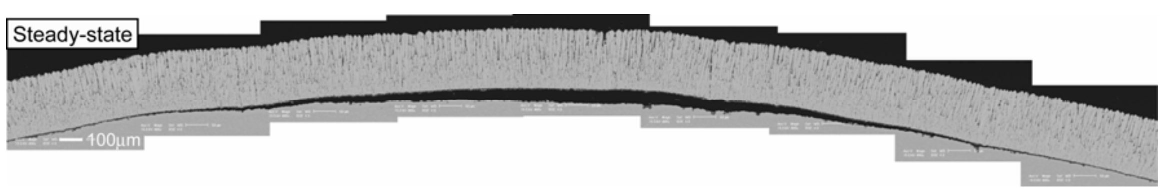
Application to jet and power generating turbines



Blades taken from an engine showing areas of spalled-off TBC

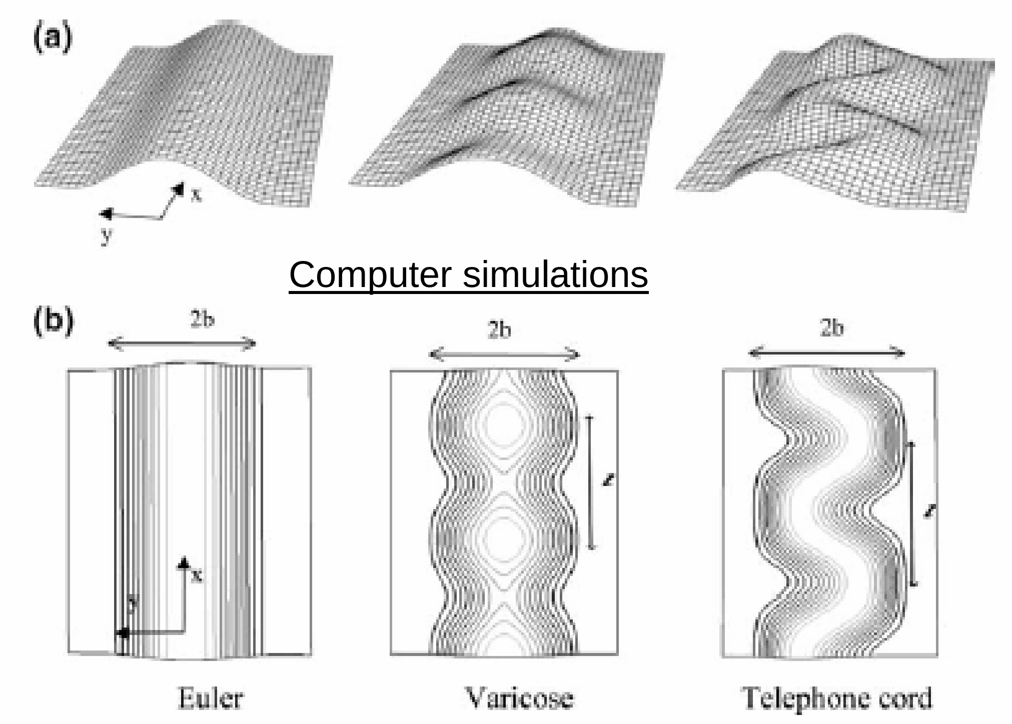






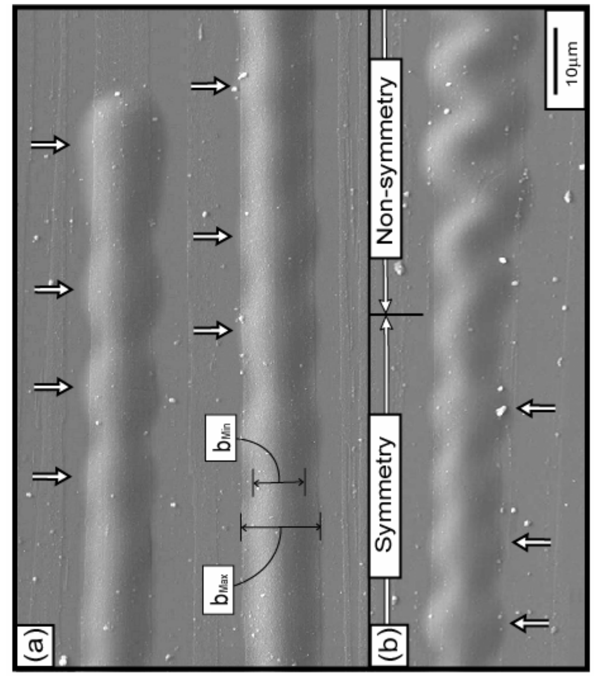
Buckle Delaminations: Interface cracking driven by buckling

Three Morphologies: Straight-sided, Varicose and Telephone Cord

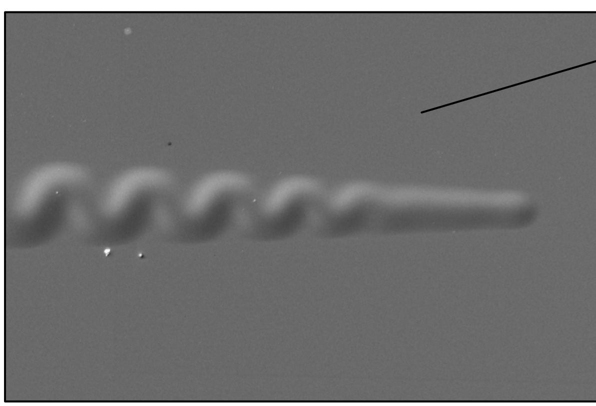


Computer simulations

Straight-sided

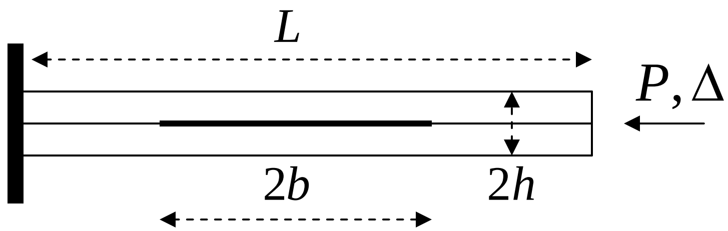


Experimental observations
200nm DLC film on silicon

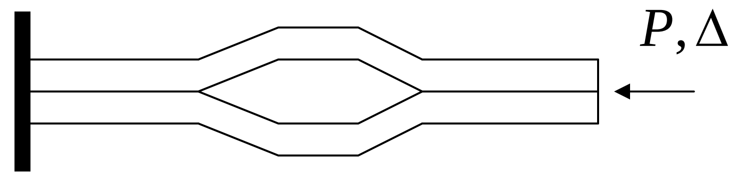


Propagation of a buckle delamination along a pre-patterned tapered region of low adhesion between film and substrate. In the wider regions the telephone cord morphology is observed. It transitions to the straight-sided morphology in the more narrow region and finally arrests when the energy release rate drops below the level needed to separate the interface.

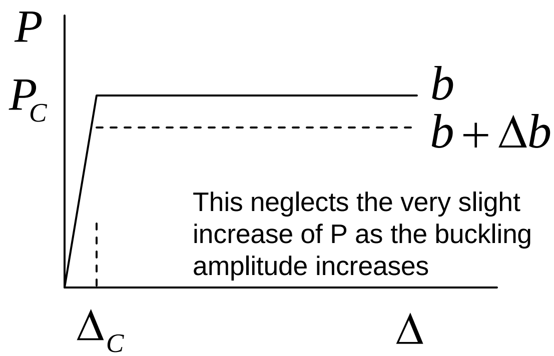
A Model Problem—Mode I Buckling Delamination of Symmetric Bi-layer



unbuckled state



buckled state



$$\text{strain energy} = SE = \int_0^{\Delta} P d\Delta = \frac{1}{2} P_C \Delta_C + P_C \left(\Delta - \Delta_C \right)$$

$$P_C = 2(hw) \frac{\pi^2}{12} \frac{Eh^2}{b^2}, \quad \Delta_C = \frac{P_C}{2hwE} L = \frac{\pi^2}{12} \frac{h^2}{b^2} L$$

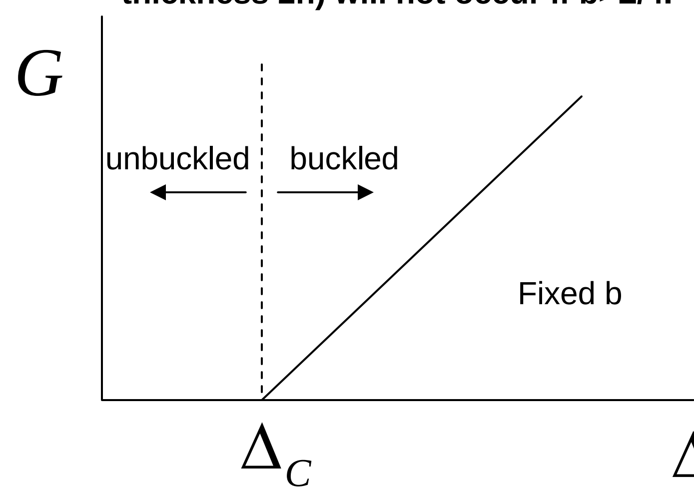
Buckling stress of clamped beam length 2b.
w is the width perpendicular to the plane.
Overall buckling of the entire laminate (with thickness 2h) will not occur if b>L/4.

$$SE = -\frac{P_C^{\,2} L}{4whE} + P_C \Delta$$

Energy release rate under prescribed Δ

$$G = -\frac{1}{2w} \frac{\partial SE}{\partial b}\bigg|_{\Delta} = \frac{\pi^2 E h^3}{6b^3} \left(\Delta - \Delta_C \right)$$

two crack tips



Mode I Buckling Delamination of Symmetric Bi-layer: continued

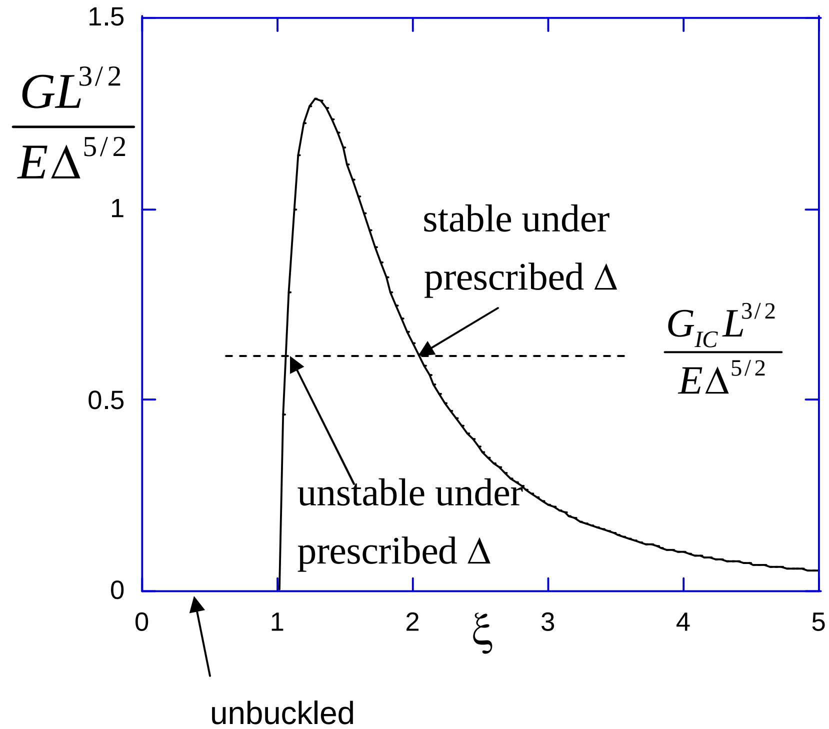
The energy release rate G can be re-written in the following non-dimensional form:

$$\frac{GL^{3/2}}{E\Delta^{5/2}} = 4\sqrt{3}\xi^{-3}\left(1-\xi^{-2}\right), \quad \text{where } \xi = \frac{b}{h}\sqrt{\frac{12\Delta}{\pi^2 L}}$$

The maximum of $\frac{GL^{3/2}}{E\Delta^{5/2}}$ is $\frac{8}{5}\sqrt{3}\left(\frac{3}{5}\right)^{3/2} = 1.288$

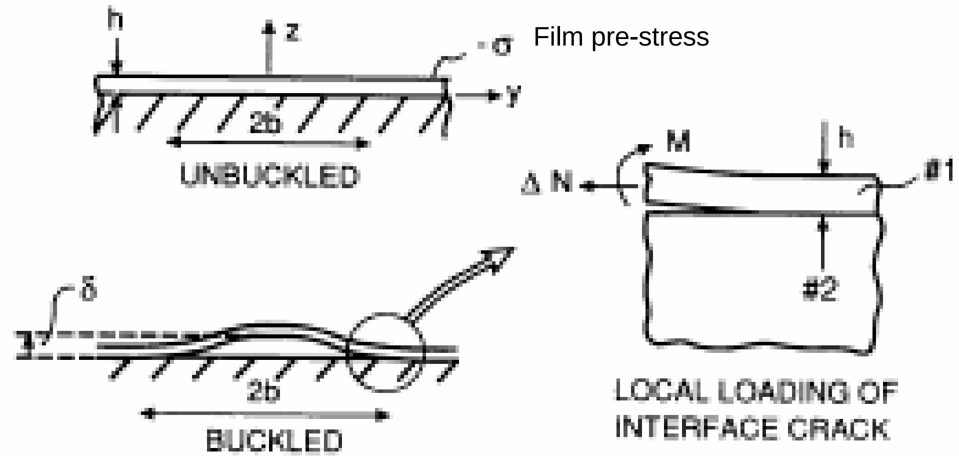
occuring for $\xi = \sqrt{5/3}$

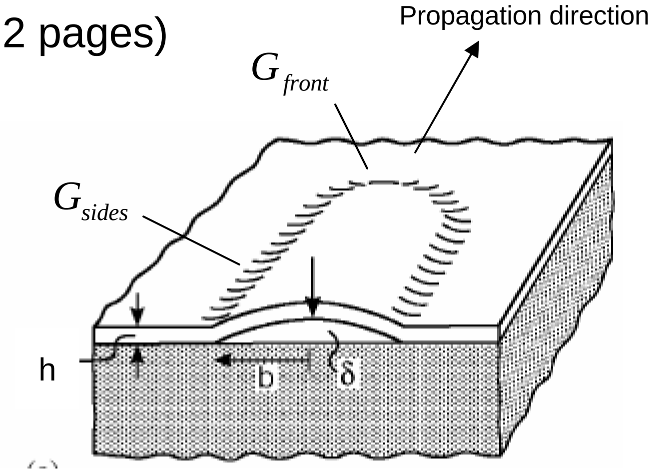
If $\frac{G_{IC}L^{3/2}}{E\Delta^{5/2}} < 1.288$, the crack will advance if b is such that $G = G_{IC}$. If b is to the left of the peak the crack is unstable under prescribed Δ and it will jump to the value of b associated with G_{IC} to the right of the peak. If Δ is then increased, the crack grows stabily with b associated with G_{IC} to the right of the peak.



Abbreviated Analysis of the Straight-Sided Buckle Delamination

A 1D analysis based on vonKarman plate theory (See next 2 pages)





Buckle deflection:

$$w(y) = \frac{1}{2}\delta\left(1+\cos(\pi y/b)\right)$$

Average stress in buckled film:

$$\sigma_C = \frac{\pi^2}{12}\bar{E}_1\left(\frac{h}{b}\right)^2$$

In-plane compatibility condition

$$\frac{1}{\bar{E}_1}\left(\sigma-\sigma_C\right) = \frac{1}{2}\int_{-b}^{b} w'^2 dy = \frac{\pi^2}{8b}\delta^2$$

Buckle amplitude:

$$\frac{\delta}{h} = \sqrt{\frac{4}{3}\left(\frac{\sigma}{\sigma_C}-1\right)}$$

At edge of buckle:

$$\Delta N = (\sigma-\sigma_C)h, \;\; M = \frac{\bar{E}_1 h^3}{12}\frac{\pi^2\delta}{2b^2}$$

Energy release rate and mode mix along sides from basic solution:

$$G_{sides} = \frac{h}{\bar{E}_1}(\sigma-\sigma_C)(\sigma+3\sigma_C) \qquad \tan\psi = \frac{4+\sqrt{3}(\delta/h)\tan\omega}{-4\tan\omega+\sqrt{3}(\delta/h)}$$

Energy-release rate can also be obtained from direct energy change calculation

Mode mix depends on the amplitude of The buckle

Energy release rate along propagating front

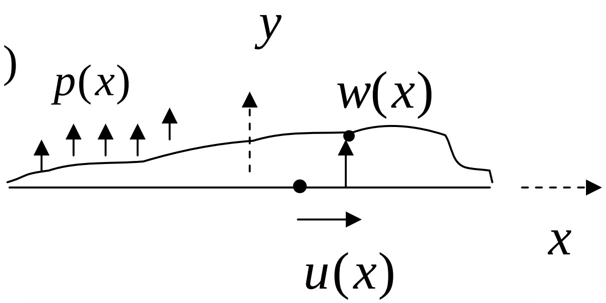
$$G_{front} = \frac{1}{2b}\int_{-b}^{b} G_{sides}dy = \frac{h}{\bar{E}_1}\left(\sigma-\sigma_C\right)^2$$

Discussed in class

Plots are given 3 slides ahead

Digression—Von Karmen nonlinear plate theory applied to clamped wide plates

Plate is infinite in z direction. Deformation is plane strain with $\bar{E} = E/(1-\nu^2)$

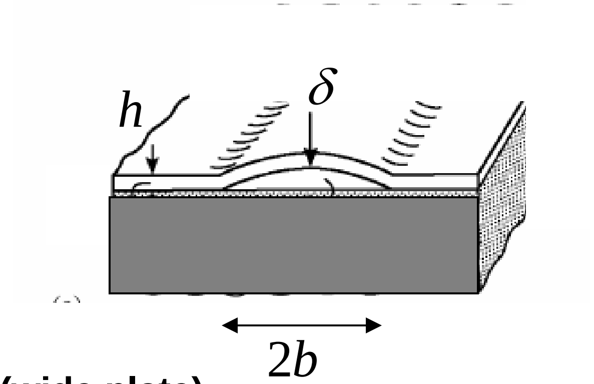


Strain-displacement relations:

mid-surface strain: $\varepsilon = u' + \frac{1}{2} w'^2$; mid-surface curvature: $\kappa = w''$

Stress-strain relations: (for plate of thickness h)

$$N \equiv \int_{-h/2}^{h/2} \sigma dy = \bar{E} h \varepsilon, \quad M \equiv -\int_{-h/2}^{h/2} \sigma y dy = D\kappa, \quad D = \bar{E} h^3 / 12$$



Equilibrium equations: (obtained from principle of virtual work)

$$\text{Moment equil.: } M'' - N w'' = p; \quad \text{Horizontal equil.: } N' = 0$$

By equilibrium, N is independent of x

Finite deflection solution for buckling of clamped-clamped beam (wide plate)

Notation: average compressive stress in unbuckled beam: $\bar{\sigma} = -N/h$

average compressive stress in buckled beam: $\bar{\sigma}_C = -N_C / h$

deflection at center of buckle: $\delta = w(0)$

beam length $= 2b \quad (-b \le x \le b)$

With the left end fixed, impose a displacement $u = -\Delta$ on the right end and then hold that end fixed.

The compressive stress in the unbuckled beam is $\bar{\sigma} = \bar{E}\Delta/(2b)$.

$$\text{Moment equil.} \Rightarrow \; D w'''' + \bar{\sigma}_C h\, w'' = 0; \quad (p = 0); \quad \text{clamped BC's} \Rightarrow \; w = w' = 0, \; x = \pm b$$

This is an eigenvalue problem with $\bar{\sigma}_C$ as the eigenvalue. Note that this stress will be independent of the amplitude of w.

Continued on next slide

Von Karmen nonlinear plate theory applied to clamped wide plates--continued

General solution $\Rightarrow\; w = c_1 + c_2 x + c_3 \sin\left(\sqrt{\frac{\bar{\sigma}_C h}{D}}\,x\right) + c_4 \cos\left(\sqrt{\frac{\bar{\sigma}_C h}{D}}\,x\right)$

For the lowest eigenvalue, the BCs $\Rightarrow c_2 = c_3 = 0,\;\; \sin\left(\sqrt{\frac{\bar{\sigma}_C h}{D}}\,b\right) = 0,\; c_1 = c_4$.

Thus, the stress in the buckled beam and the deflection shape are: $\bar{\sigma}_C = \frac{\pi^2}{12}\bar{E}\left(\frac{h}{b}\right)^2$ & $w(x) = \frac{\delta}{2}\left(1 + \cos\left(\frac{\pi x}{b}\right)\right)$

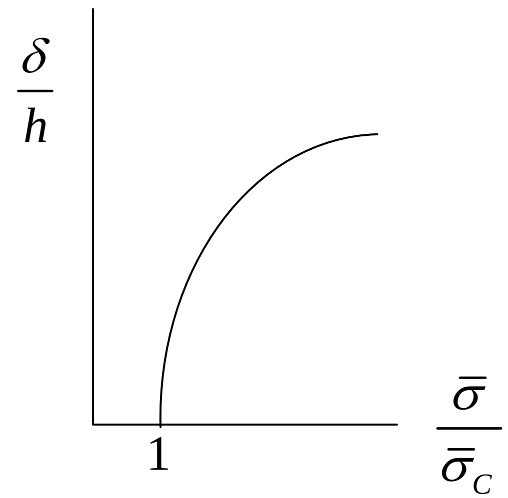
Relation between stress in buckled beam, stress in unbuckled beam and deflection

With u and w measured from the unstressed state and $\tilde{u}$ measured from the unbuckled stressed state,

$N_C = \bar{E}h\varepsilon \Rightarrow -\bar{\sigma}_C h = -\bar{\sigma}h + \bar{E}h\left(\tilde{u}' + \frac{1}{2}w'^2\right)$

Now integrate the above equation from -b to b using $\tilde{u}(-b) = u(b) = 0$:

$$\Rightarrow \bar{\sigma} - \bar{\sigma}_C = \frac{\bar{E}}{4b}\int_{-b}^{b} w'^2 dx = \frac{\pi^2}{16}\bar{E}\left(\frac{\delta}{b}\right)^2 \qquad or \qquad \delta = h\sqrt{\frac{4}{3}\left(\frac{\bar{\sigma}}{\bar{\sigma}_C} - 1\right)}$$



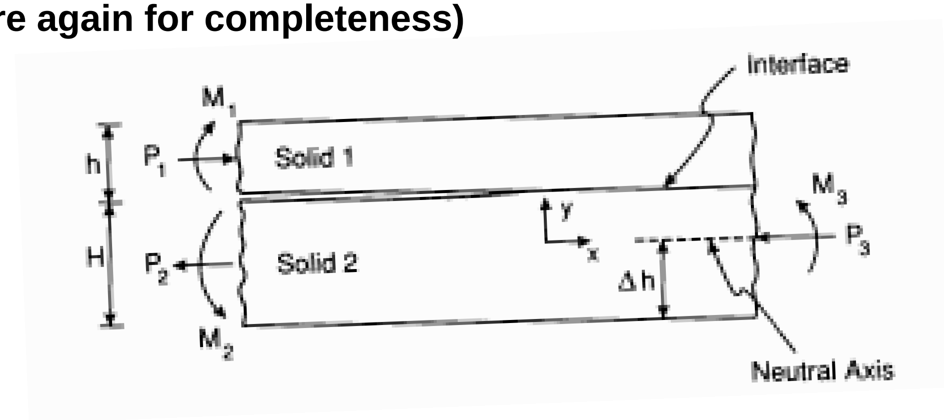
Finally, we will need the moment at $x = b$:

$$M(b) = Dw''(b) \Rightarrow \quad M(b) = \frac{\bar{E}}{24}\left(\frac{\pi}{b}\right)^2 \delta$$

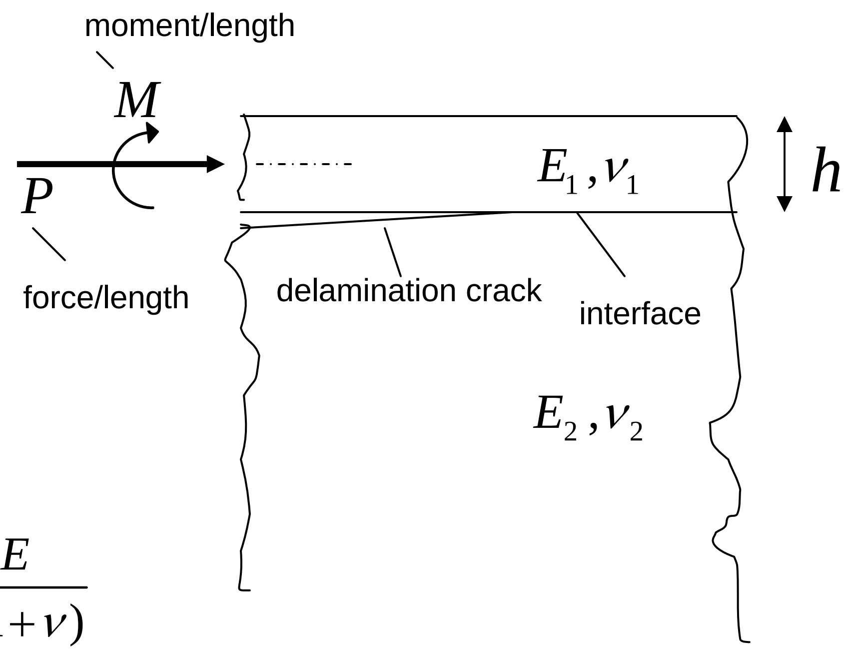
This completes the finite deflection for the clamped-clamped wide plate.

BASIC ELASTICITY SOLUTION FOR INFINITE ELASTIC BILAYER WITH SEMI-INFINITE CRACK
(Covered in earlier lectures and included here again for completeness)

Equilibrated loads. General solution for energy release rate and stress intensity factors available in Suo and Hutchinson (1990)



Infinitely thick substrate--
Primary case of interest for thin films and coatings on thick substrates



Dundurs' mismatch parameters for plane strain:

$$\alpha_D = \frac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}, \quad \bar{E} = \frac{E}{(1-\nu^2)}$$

$$\beta_D = \frac{1}{2}\frac{\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)}{\mu_1(1-\nu_2) + \mu_2(1-\nu_1)}, \quad \mu = \frac{E}{2(1+\nu)}$$

For homogeneous case: $\alpha_D = \beta_D = 0$ If both materials incompressible: $\beta_D = 0$

α_D is the more important of the two parameters for most bilayer crack problems

Take $\beta_D = 0$ if you can. It makes life easier!

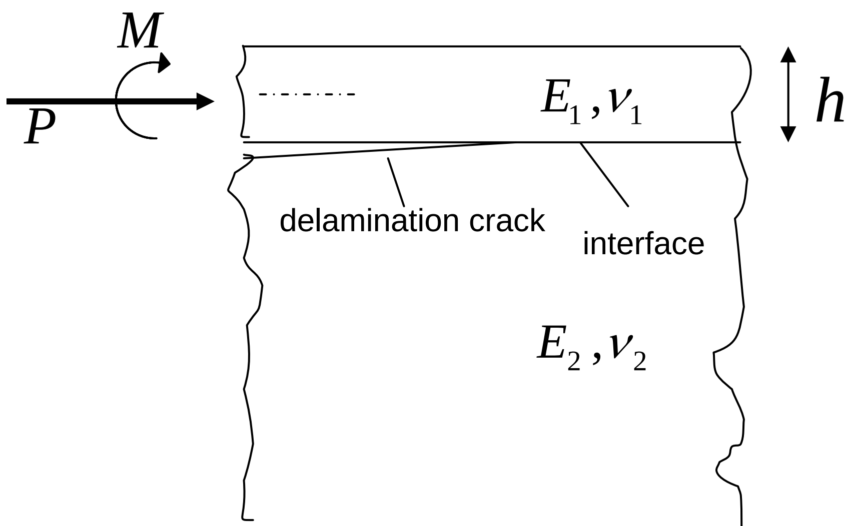
Basic solution continued:

Energy release rate

$$G = \frac{1}{2\bar{E}_1}\left(\frac{P^2}{d} + 12\frac{M^2}{d^3}\right)$$

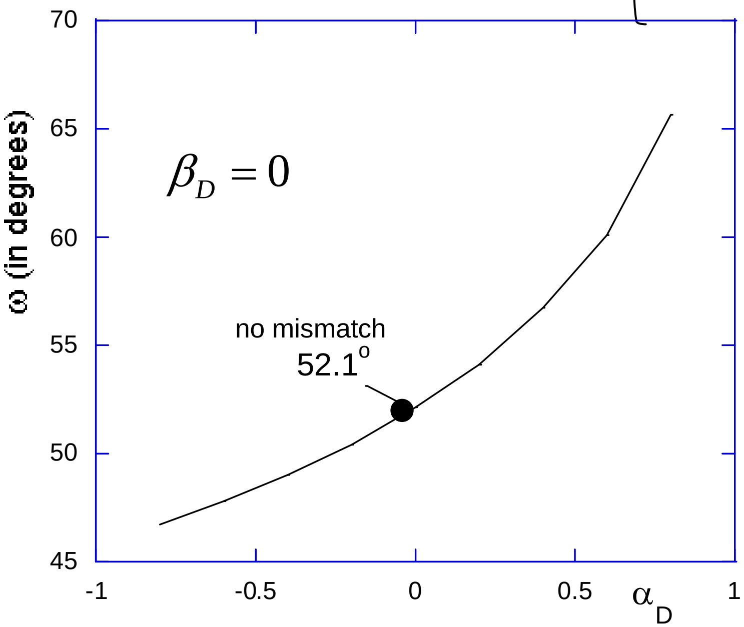
$$\bar{E} = E/(1-\nu^2)$$

Stress intensity factors: $(\beta_D = 0)$
(see Hutchinson & Suo (1992) if second Dundurs' parameter cannot be taken to be zero)



$$K_I = \frac{1}{\sqrt{2}}\left[Pd^{-1/2}\cos\omega + 2\sqrt{3}Md^{-3/2}\sin\omega\right]$$

$$K_{II} = \frac{1}{\sqrt{2}}\left[Pd^{-1/2}\sin\omega - 2\sqrt{3}Md^{-3/2}\cos\omega\right]$$

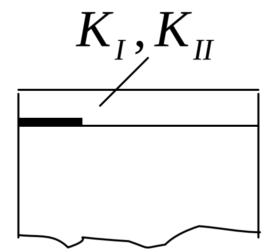


where $\omega(\alpha_D)$ is shown as a plot and is tabulated in Suo & Hutch.

Note: For any interface crack between two isotropic materials,

$$G = \frac{1-\beta_D^{\,2}}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right)\left(K_I^{\,2} + K_{II}^{\,2}\right)$$

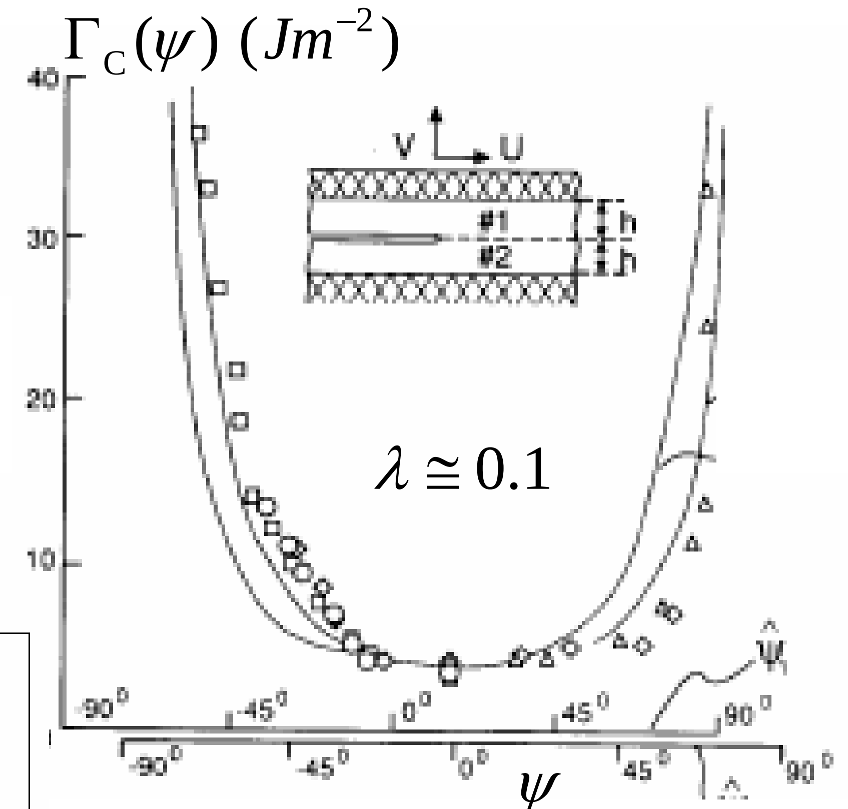
Interface toughness—the role of mode mix



Experimental finding: The energy release rate required to propagate a crack along an interface generally depends on the mode mix, often with larger toughness the larger the mode II component

Interface Toughness: $\Gamma_C(\psi)$

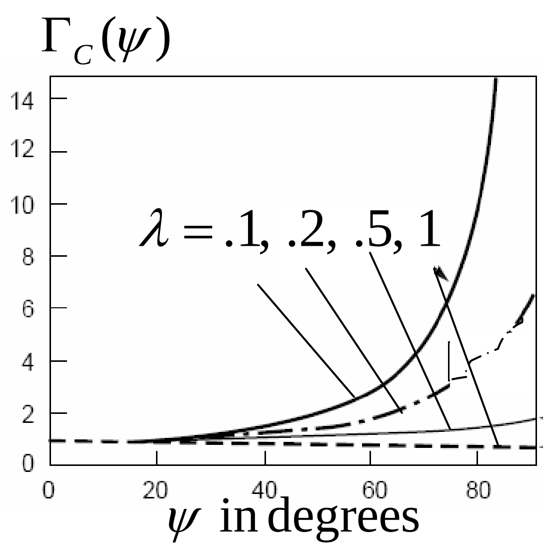
Propagation condition: $G = \Gamma_C(\psi)$

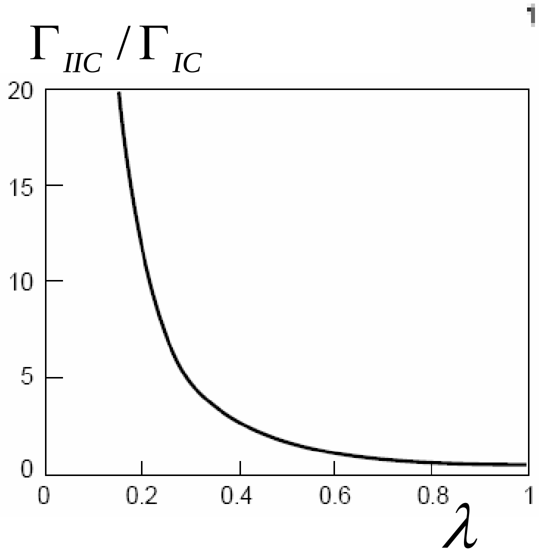


Liechti & Chai (1992) data for an epoxy/glass interface.

A phenomenological interface toughness law

$$\Gamma_C(\psi) = \Gamma_{IC}\left(1 + \tan^2((1-\lambda)\psi)\right)$$





$\lambda = 1 \Rightarrow$ no mode dependence

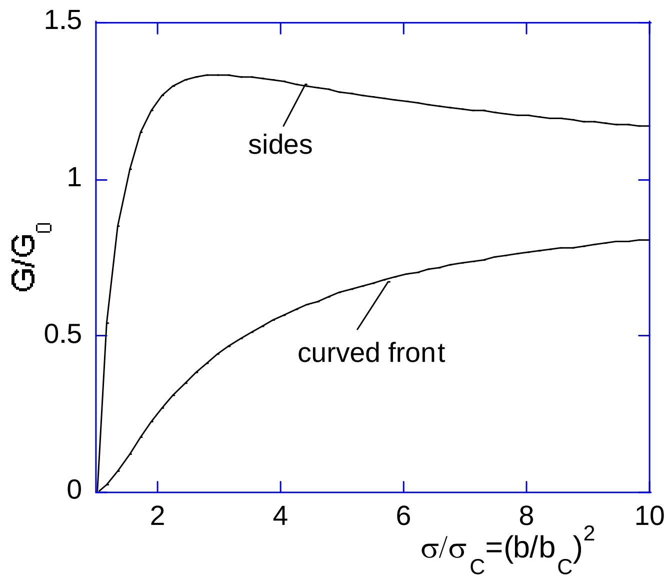
$\lambda << 1 \Rightarrow$ significant mode dependence

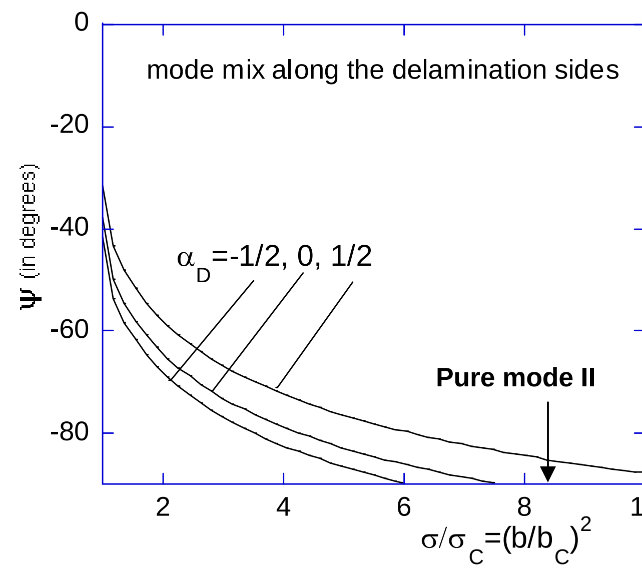
Energy release rate and mode mix on sides of Straight-sided buckle delamination

$$\sigma_C = \frac{\pi^2}{12}\bar{E}_1\left(\frac{h}{b}\right)^2$$ Stress at onset of buckling

$$\frac{b_C}{h} = \sqrt{\frac{\pi^2}{12}\frac{\bar{E}_1}{\sigma}}$$ Half-width at onset of buckling

$$G_0 = \frac{1}{2}\frac{\sigma^2 h}{\bar{E}_1}$$ Energy/area available for release in planes strain





Half-width of straight-sided delamination

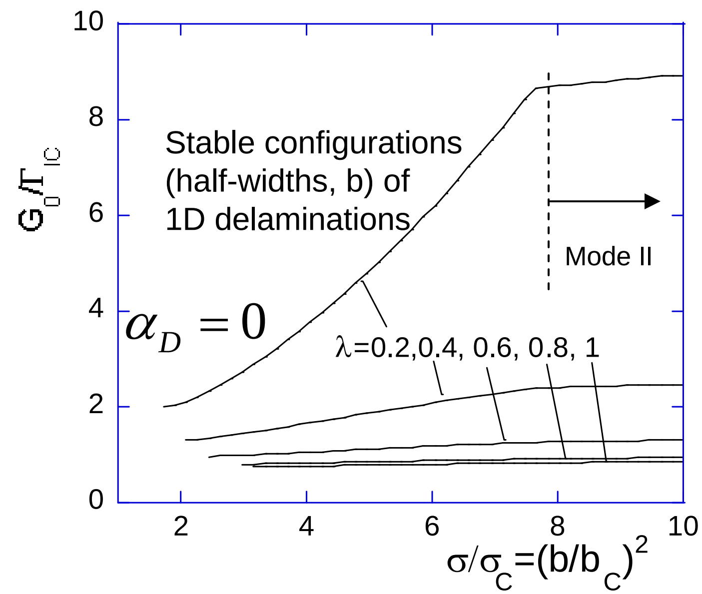
Impose: $G = \Gamma_{IC} f(\psi), \quad f(\psi) = 1 + \tan^2((1-\lambda)\psi)$

See earlier slide for interface toughness function

$$\Rightarrow \frac{G_0}{\Gamma_{IC}} = \frac{f(\psi)}{\left(1-\frac{\sigma_C}{\sigma}\right)\left(1+3\frac{\sigma_C}{\sigma}\right)}$$

Stability of crack front requires: $\frac{d}{db}\left(\frac{G}{f(\psi)}\right) < 0.$

i.e. if tip "accidentally" advances, it is no longer critical.

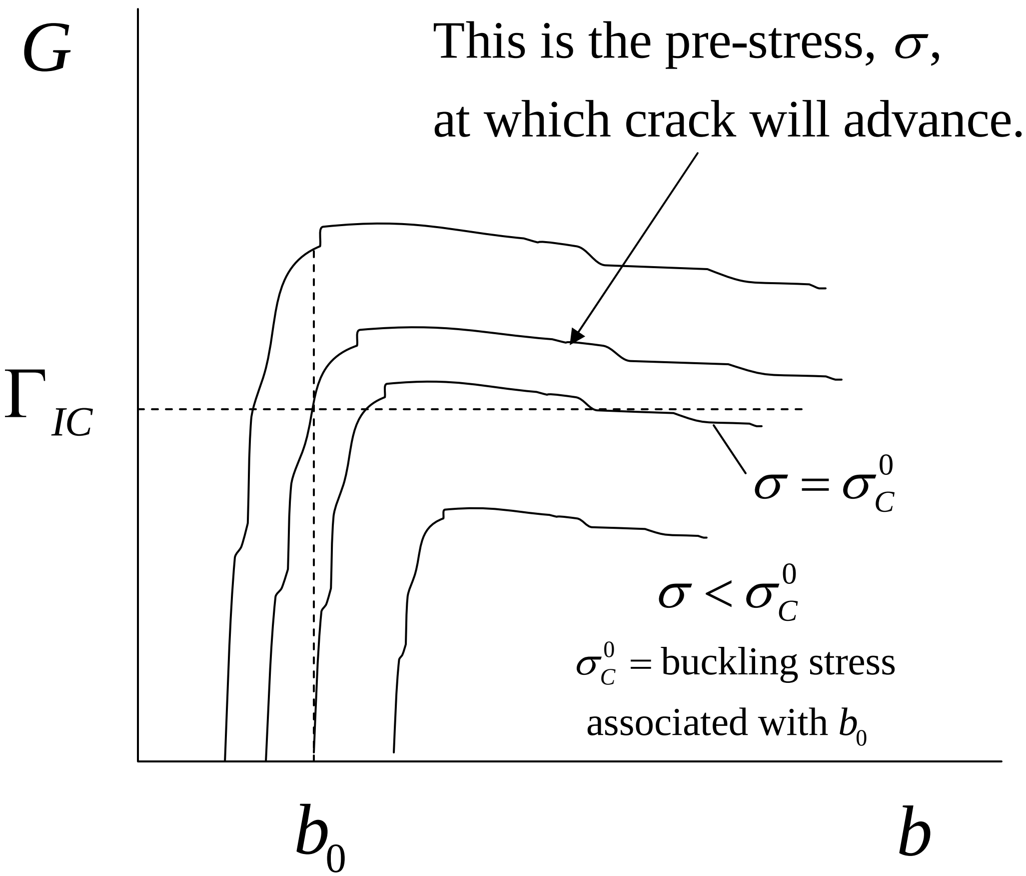


Caution! This plot is difficult to interpret because each axis depends on σ

Illustration of Spread of Delamination if no mixed mode dependence $(\lambda = 1 \;\&\; G = \Gamma_{IC})$

Scenario: Given Γ_{IC} & initial delamination flaw with length $2b_0$.

Monotonically increase the pre-stress (the stress in the unbuckled film), σ.

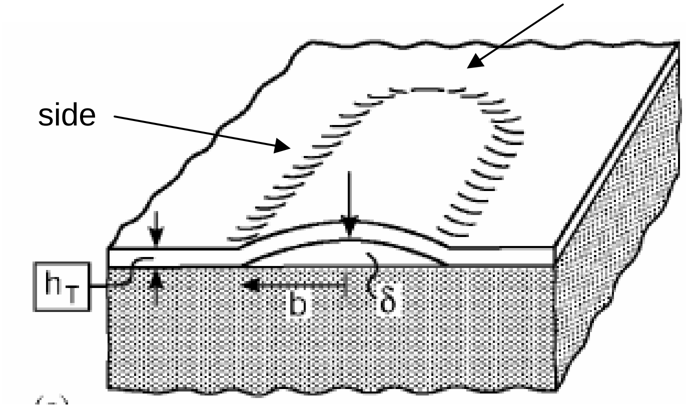


Note that once the interface crack advances, $G > \Gamma_{IC}$ and it will spread dynamically without limit. For mode-independent interface toughness the condition to ensure no "wholesale" delamination is $G_0 < \Gamma_{IC}$, or $\dfrac{\sigma^2 h}{2\bar{E}} < \Gamma_{IC}$.
Stresses well above this level can be tolerated if the interface toughness has a significant mixed mode dependence.

Since the delamination becomes mode II as it spreads, the above simple criterion against **complete delamination** can generalized when there is mode-dependence of the toughness by the requirement, $G_0 < \Gamma_{IIC}$. But such a criterion would not exclude localized delaminations such as telephone cord delaminations.

Inverse determination of interface toughness, stress (or modulus) by measuring buckling deflection and delamination width

Straight-sided delamination without ridge crack on flat substrate



$$S \sim stretching\ \ stiffness$$

$$D \sim bending\ \ stiffness$$

The basic results can be written as:

$$G_{SS} = \frac{1}{2} S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^4$$

$$G_{side} = \frac{1}{2} S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^4 + 2D \left(\frac{\pi}{b} \right)^2 \left(\frac{\pi}{4} \frac{\delta}{b} \right)^2$$

$$N_0 = D \left(\frac{\pi}{b} \right)^2 + S \left(\frac{\pi}{4} \frac{\delta}{b} \right)^2$$

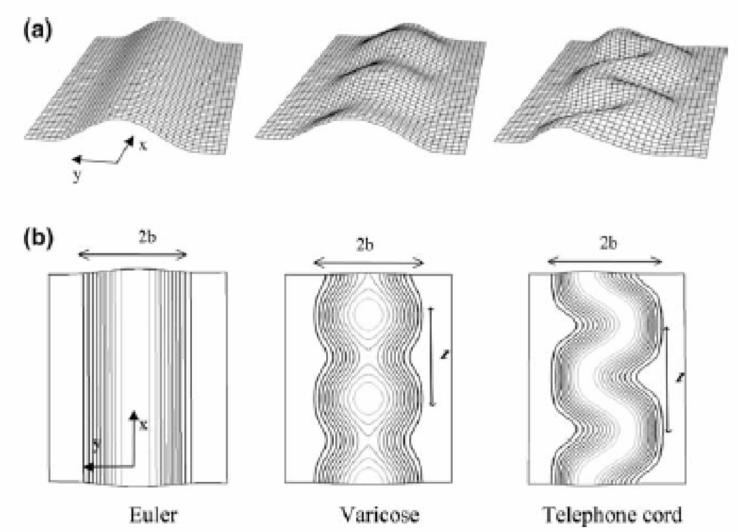
Applies to any multilayer film with arbitrary stress distribution

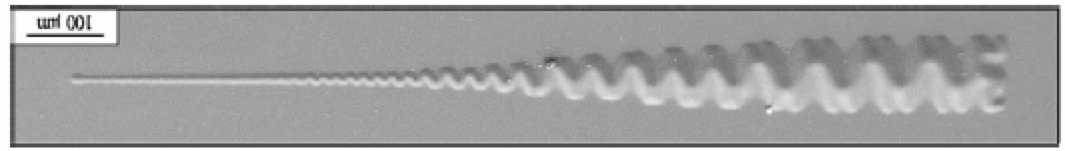
If bending and stretching stiffness of the film are known, then the energy release rates and the resultant pre-stress can be determined by measurement of the deflection and the delamination width.

If resultant pre-stress is known, then the equations can be used to determine film modulus and release rates in terms of deflection and delamination width– see Faulhaber, et al (2006) for an example.

Energy Released as a Function of Morphology

Three morphologies:



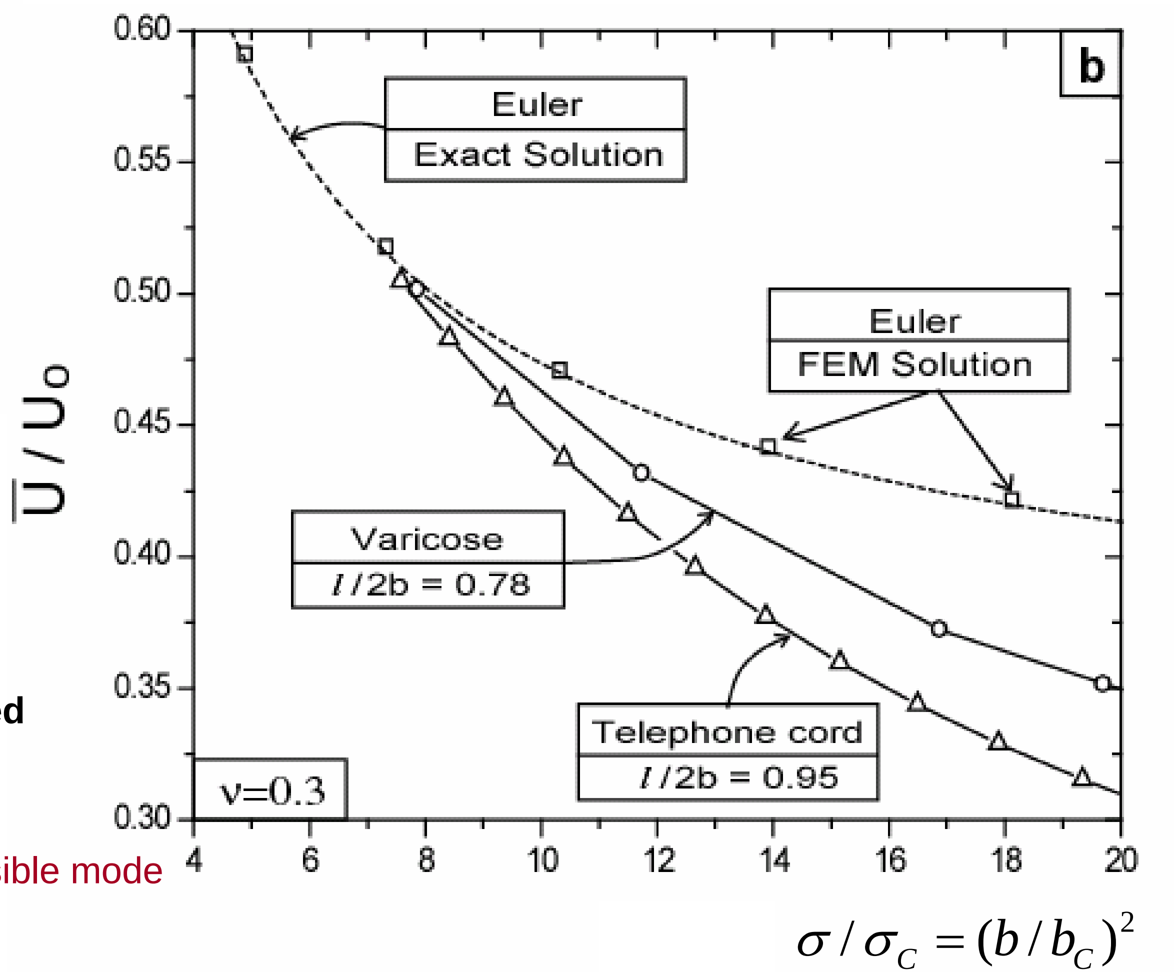


DLC on silicon—tapered low adhesion interface: propagates from right to left

Film under equi-biaxial stress

Energy/area: $U_0 = \dfrac{\sigma^2 h}{E(1-\nu)}$

Energy/area in buckled film averaged over one full wavelength: $\bar{U}$



For $\sigma / \sigma_C < 6$:

Euler (straight-sides) mode is only possible mode

For $\sigma / \sigma_C > 7.5$:

Telephone cord morphology has lowest energy and releases the most energy/area.

Moon et al 2004

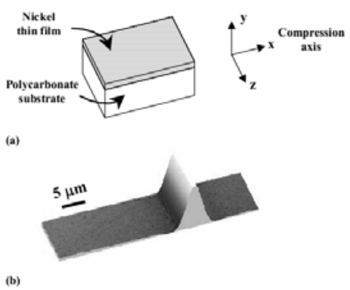
Metal or Ceramic Films on Compliant Substrates (Polymer or Elastomer)

Cotterell & Chen, 2000; Yu & Hutch, 2002; Parry, et al.,2005

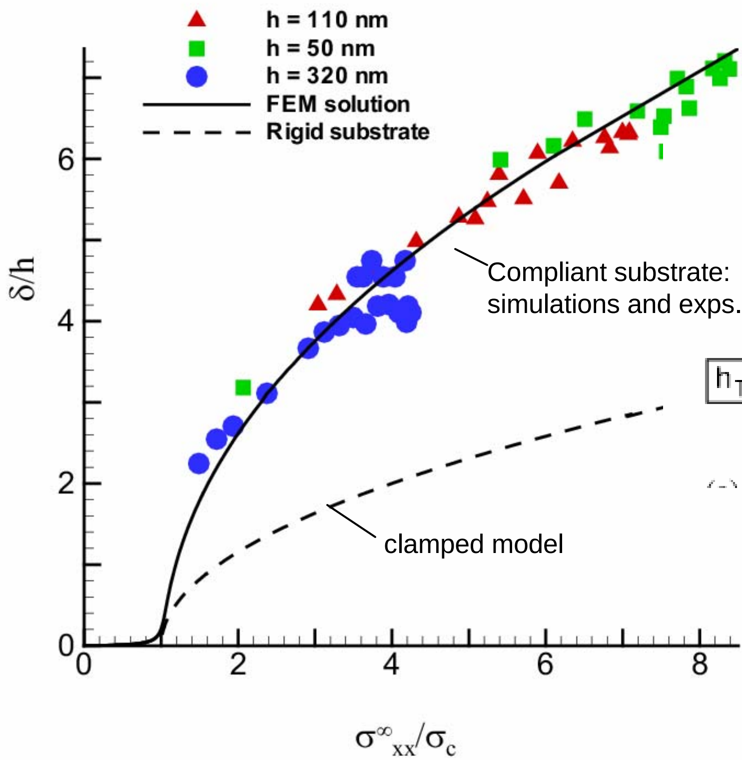
Analytical Fact: Edges of buckle delamination is effectively clamped if substrate modulus is larger than 1/3 of film modulus (i.e. clamped plate model is valid)

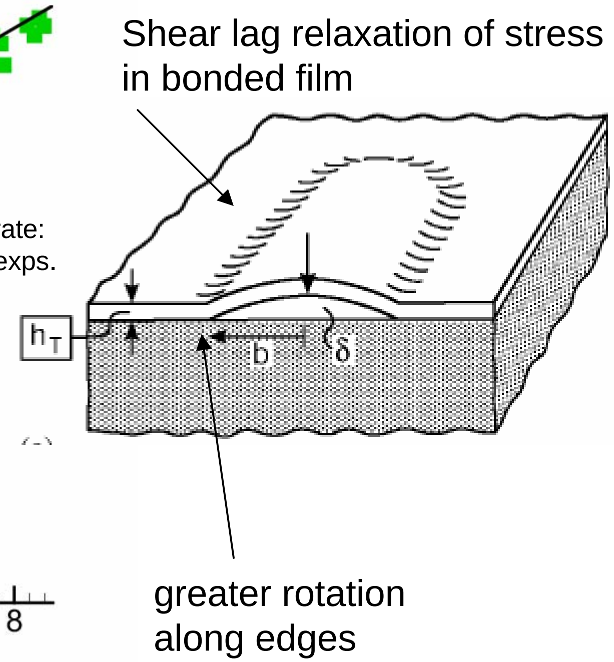
Highly compliant substrate has **three effects:**

1) Stabilizes straight-sided buckle delamination and tends to eliminate telephone cord morphology.
2) Significant film rotation occurs at edges of delamination and larger buckling deflections.
3) Relaxation of stress along bonded edges of delamination (shear lag effect) amplifies energy released.



Ni films on polycarbonate substrates (Parry, et al.)





DELAMINATION MECHANICS
Supplementary Notes and References

Page numbers refer to the slide page. A limited reference list is given on the last page.

Much of the mechanics outlined in the slides was developed around 1990 and is summarized in the article by Hutchinson and Suo (1992). Two other basic references with emphasis on interfaces are those by Evans and Hutchinson (1995) and Evans, Hutchinson and Wei (1995). For the student first getting acquainted with delamination mechanics, the notes, "Mechanics of thin films and multilayers", by Hutchinson (1996) cover some of the basic aspects in an assessable manner. It is assumed that the reader has a basic familiarity with fracture mechanics. Aspects of interface fracture mechanics are important in the developments, and if the author is not acquainted with this subject it might be good to start with Section II.C of Hutchinson and Suo (1990).

The slides also cover topics, in particular, extensions and applications, studied in the past few years. References are provided. It should be noted that the references listed on the last page are not intended to be comprehensive—they are primarily those of the author and his colleagues. These references will permit the reader access to other contributors and to the wider literature. The book on thin films by Freund and Suresh (2003) also provides excellent coverage of some delamination topics.

Page 1. This side provides a pictorial overview of the types of problems considered.

Page 2. The two-layer elasticity solution of Suo and Hutchinson (1990) for isotropic layers with differing moduli and Poisson's ratios has many applications. Dundurs' two dimensionless elastic mismatch parameters characterize the solution: in the slides they have been given for planes strain. Refer to the literature for plane stress definitions. The energy release rate can be obtained by simple methods simply by accounting for the difference between the energy well ahead and well behind the crack tip—see the notes by Hutchinson (1996). One reason for the usefulness and robustness of the energy release results from this solution derive elementary energy accounting. The relative proportion of mode II to mode I, as measured by ψ, requires a the full elasticity solution given by Suo and Hutchinson (1990). The examples in the slides are all based on the limiting case shown where the layer below the interface is very thick compared to the layer (or layers) above the interface, and the limit is for an infinitely deep layer below the interface.

If the second Dundurs mismatch parameter, β_D, is zero, the stresses in the singularity field characterizing the behavior near the tip of an interface have precisely the same form as in the homogeneous case with

$$\sigma_{\alpha\beta} = K_I \sqrt{\frac{1}{2\pi r}} f^I_{\alpha\beta}(\theta) + K_{II} \sqrt{\frac{1}{2\pi r}} f^{II}_{\alpha\beta}(\theta)$$

where r and θ are planar polar coordinated centered at the tip. The functions f^I and f^{II} are the same as those for the homogeneous material. If β_D is not zero, the stresses associated with the crack tip singularity are more complicated—a so-called oscillatory singularity. In all the examples considered in the slides we will take $\beta_D = 0$ since this captures most of the essential features of the phenomena of interest. Students interested in pursuing the effect of non-zero β_D can start off by looking at Section II.C.

Page 3. This is the basic result which will be used throughout the slides.

Page 4. To see the effect of friction on the mode II edge delamination crack see the reference by Balint and Hutchinson (2001).

Page5. Results for $\omega(\alpha_D)$ for α_D near unity (i.e. for stiff films on very compliant substrates such as metals on polymers) have not been published and do not appear to be available.

Page 6. See Evans and Hutchinson (1995) and Evans, Hutchinson and Wei (1999) for discussion of interface toughness and other systems.

Page 8. Reference on delamination in presence of temperature and stress gradients: Evans and Hutchinson (2006).

Page 9. The basic solution for an isolated crack in a homogeneous material subject to a temperature gradient was given by Sih (1962). See Hutchinson and Lu (1995), Hutchinson and Evans (2002) and Evans and Hutchinson (2006) for work specifically related to temperature gradients and their role in delamination of coatings.

Page 10. Reference: Evans and Hutchinson (2006).

Page 11-13. There is now a large literature on buckling delamination covering both theoretical and experimental aspects. Basic mechanics covered in the slides is given in Hutchinson and Suo (1992), Section VI, and Hutchinson, Thouless and Liniger (1992).

More recent references are Moon et al. (2002) and Moon et al (2004); additional references are cited in these papers.

Page 14. This approach has been developed in Faulhaber et al. (2006) with application to delamination of thermal barrier coatings on curved substrates. The approach has also been extended in this paper to delaminations with ridge cracks.

Page 15. Theoretical and experimental work for straight-sided buckle delaminations for stiff films on polymeric substrates have been published in Cotterell & Chen (2000) Yu & Hutchinson (2002); Parry et al. (2005).

Page16. The reference for this slide is Moon et al. (2004).

Page 17. The movie of the real time evolution of a buckle delamination was supplied by M-Y. Moon. See the work of A. Volinsky for many interesting examples of buckle delamination.

Page 18. These and other related results are given by Yu, He and Hutchinson (2001).

Page 19. Three-dimensional results for delamination of thin film strips are presented in Yu and Hutchinson (2003).

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