IS THERE NO PULL-OFF FOR ADHESIVE FRACTAL SURFACES?

Michele Ciavarella, Politecnico di Bari, mciava@poliba.it

Abstract: In this short note we remark that the theory of Fuller & Tabor for the adhesive contact of rough random surfaces, once applied to isotropic fractal surfaces, gives a limiting zero pull-off force, for all fractal dimensions or amplitudes of roughness.

Introduction

That roughness "destroys" adhesion is a well known experimental evidence, and the most interesting result captured by the classical paper by Fuller and Tabor (1975), FT in the following, which is basically an extension of the Greenwood and Williamson (1966), corresponding case without adhesion (GW in the following), including the JKR model (Johnson, Kendall and Roberts, 1971) for the single asperity contact case. As such, the FT model As it was pointed out by Tabor (1977) the JKR theory applies to large, compliant spheres, i.e., at large values of μ, while the DMT theory would appear more appropriate for small, stiff spheres, i.e., at low values of μ, the Tabor parameter. This is therefore a first strong assumption in the model, if we apply it to fractal surfaces, where the smallest scales certainly are problematic in these respects. The FT model also inherits from GW that there is no interaction, that the surface is described by parabolic asperities, which moreover are identical. Moreover, asperities theories cannot deal with the possibility of full contact, are intrinsically separated contacts. In the GW model without adhesion, the limit contact area is zero, as noticed by various authors, is the correct limit for elastic contact of fractal surfaces (Borri-Brunetto et al.,1998; Manners,2000; Greenwood,1996; Ciavarella et al., 2000, 2001; Persson, 2001a,b; Greenwood and Wu, 2001) within of course the framework of elastic infinitesimal deformation theory. In particular, Ciavarella et al. (2000) show that the contact area in a plane contact case,at large enough resolutions tends to have a fractal dimension of 2 D where D is the dimension of the profile. Hence, the higher the dimension of the profile, the smaller and more ''lacunar ''is the contact area. The final conclusion is that the ''true ''contact area for elastic contact between two perfectly fractal surfaces consists of an infinite number of contact spots of zero size, which are subjected to infinite contact pressure, and have a vanishing total area of contact. Ciavarella and Demelio (2001) have found that indeed this was already intrinsic in the old Archard paper with the spheres on spheres fractal "ante-literam" model (1957).

It becomes interesting to see if adhesion gives less paradoxical conclusions. As GW, the FT model needs only three geometrical parameters for the solution of the contact problem: the standard deviation σ of the summit height distribution which is assumed to be Gaussian, the assumed constant radius R_m of the spherical summits and the area density of summits D_{sum}. For an isotropic surface the three parameters depend on the *bandwidth parameter,*

$$\alpha = \frac{m_0 m_4}{m_2^{\,2}}, \tag{1}$$

where m_0, m_2, m_4 are the zeroth, second and forth order moments of the PSD (Power Spectrum Density), known to be the variance of heights, slopes and curvatures, respectively. A much used simple description of fractal profiles is the Weierstrass series (Ciavarella et al. 2000)

$$W(x) = g_0 \sum_{i=0}^{n} \gamma^{(D-2)i} \cos(q_0 \gamma^i x + \phi_i) \tag{2}$$

where g_0 is the amplitude of the first sinusoid, $q_0 = \dfrac{2\pi}{\lambda_0}$ is the circular frequency given by the wavelength of the first sinusoid, λ_0, and γ is a parameter (sometimes chosen to be 1.5 to have a

non-periodic profile, but this is not sufficient to make the scales separated enough to apply the Archard procedure like in Ciavarella et al 2000). Finally, ϕ_i are random phases (not strictly needed for the Weierstrass series to be a fractal). For a Weierstrass series with n terms,

$$m_{0,W} = \left\langle W^2(x) \right\rangle = \frac{1}{2}(g_0)^2 \sum_{i=0}^{n} \gamma^{2(D-2)i} \qquad , \qquad m_{2,W} = \left\langle W'^2(x) \right\rangle = \frac{1}{2}\left(2\pi \frac{g_0}{\lambda_0}\right)^2 \sum_{i=0}^{n} \gamma^{2(D-1)i}$$

$$m_{4,W} = \left\langle W''^2(x) \right\rangle = \frac{1}{2}\left[\left(\frac{2\pi}{\lambda_0}\right)^2 g_0\right]^2 \sum_{i=0}^{n} \gamma^{2(D)i} \tag{3}$$

$$\alpha_W = m_{0,W}\, m_{4,W}\, / m_{2,W}^{\,2} . \tag{4}$$

Using these parameters we can define the standard deviation of the summit height distribution

$$\sigma = \sqrt{\left(1 - \frac{0.8968}{\alpha}\right)m_{0,W}} \ , \tag{5}$$

the constant radius of the spherical summits

$$R_m = \frac{3\pi^{1/2}}{8 m_{4,W}^{1/2}} \tag{6}$$

and the area density of summits

$$D_{SUM} = \frac{m_{4,W}\, / m_{2,W}}{6\pi \cdot 3^{1/2}} . \tag{7}$$

In figure 1, we plot, in Log graphs, the three parameters in eqt. (5), (6) and (7), using the moments in eqt. (3) and the bandwidth parameter in eqt. (4), as a function of the number of terms n of a Weierstrass series with $\gamma=2$ for three different fractal dimensions D=1.05, 1.5, 1.95.
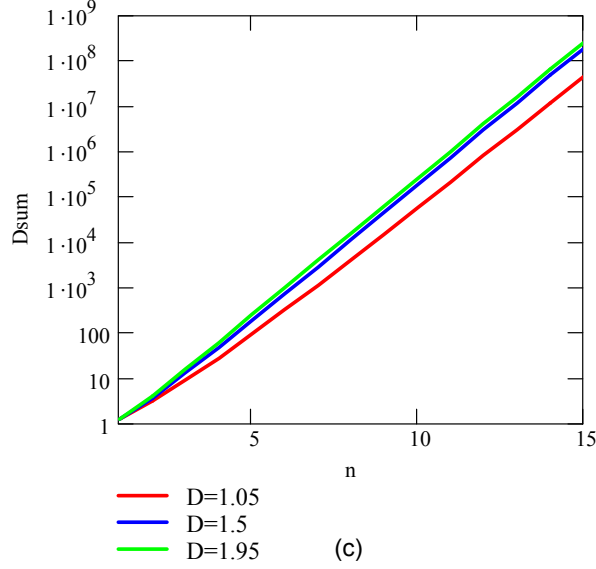

(a)

(b)
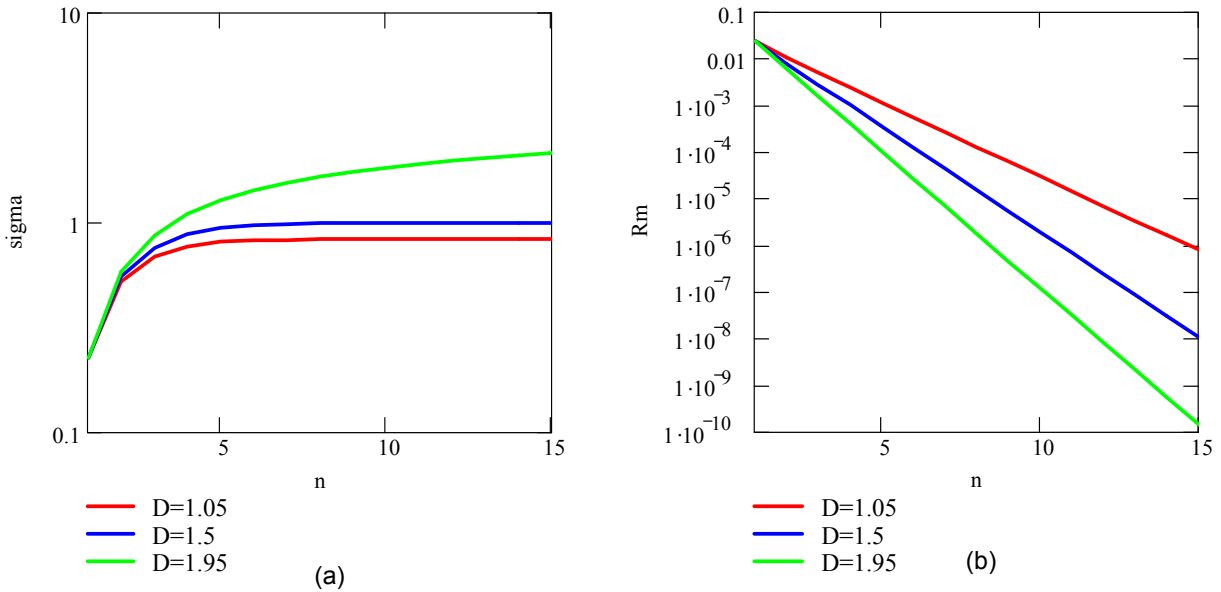
Figure 1 - The standard deviation σ of the summit heights, the constant radius R_m of the spherical summits and the area density of summits D_{sum} as a function of the number of terms n (from 1 to15) of a Weierstrass series with γ=2 for three different fractal dimensions D=1.05, 1.5, 1.95.

For the standard deviation of the summit height distribution we have a rapid convergence to the asymptotic value: for the fractal dimensions D=1.05, 1.5 we need only 6 terms in the series while for D=1.95 we need all the 15 terms. Even for the large fractal dimension of D=1.95, the difference between the value for a single sinusoid, and the converged value is only a factor 2. The constant radius of the spherical summits decrease at increasing number of scales, for the superposition of terms with lower amplitude and wavelength, and it decrease also at increasing fractal dimension for the presence of a higher number of asperities. The area density of summits increases with the number of scales and it is the only geometrical parameter which is not influenced much by the fractal dimension.

Considering the model of Fuller & Tabor, for a Gaussian distribution of the asperity heights the total force exerted between the two solids is the sum of those one exerted by the single asperity, so the expression of the load is

$$P = N \int_{d-\delta_c}^{\infty} P_i \varphi_s(z_s) dz = \frac{NP_c}{\sigma\sqrt{2\pi}} \int_{d-\delta_c}^{\infty} g\left(\frac{\delta_i}{\delta_c}\right) \exp\left(-\frac{z^2}{2\sigma^2}\right) dz, \qquad (8)$$

where N is the number of summits in the nominal area, P_c is the pull-off force for the JKR model $P_c = \frac{3}{2}\pi\omega R_m$, where ω is the energy of adhesion and the relation for the force P_i exerted on an asperity as a function of its approach δ_i is $\frac{P_i}{P_c} = g\left(\frac{\delta_i}{\delta_c}\right)$ which is the inverse relation (numerically evaluated) of

$$\frac{\delta}{\delta_c} = (3\chi - 1)\left(\frac{1+\chi}{9}\right)^{1/3}, \ \chi \geq 0, \ \frac{\delta}{\delta_c} \geq -3^{-2/3} \qquad (9)$$

$$\frac{\delta}{\delta_c} = -(3\chi + 1)\left(\frac{1-\chi}{9}\right)^{1/3}, \ 0 \leq \chi \leq \frac{1}{3}, \ \frac{\delta}{\delta_c} \leq -3^{-2/3}$$

where $\chi = \sqrt{1 + \frac{P}{P_c}}$.

In the next figures, we show the trends of the ratio between the total load and the product of the number of the summits in the nominal area N and P_c as a function of the dimensionless separation $d^*=d/\sigma$ for a rubber having $E=6.8*10^5$ N/m^2 and $\omega=40$ mJ/m^2. The results shown are obtained for Weierstrass series with $\gamma=2$ at increasing number of scales $n=4, 6, 8, 10, 12, 14$ for three fixed fractal dimensions D=1.05, 1.5, 1.95 and at three fixed n with increasing D.

The graphs show very well the effect of the increasing roughness of the surface, adding new scales at fixed fractal dimension or increasing D at fixed number of scale, which cause a reduction of the adhesive pull off force, particularly for large fractal dimensions. Although the latter part was expected, because the "rougher" the surface, the more likely roughness becomes effective, the limit of zero pull-off force, theoretically even with arbitrarily low amplitude of roughness, is perhaps counterintuitive.

From the plots in figure 2 we obtain also the locus of minima of the ratio P/NP_c and of the corresponding value for the separation distance d^* for each fractal dimension. The values of the ratio are obtained from the integral equation

$$\int_{-\delta_c^*}^{\infty} g\left(\frac{\delta^*}{\delta_c}\right)(d^*+\delta^*)\exp\left(-\frac{(d^*+\delta^*)^2}{2}\right)d\delta^* = 0 \tag{10}$$

obtained by differentiating eqt. (9) after a change of variables.

In figure 3 we plot in Log graphs the trends of the two quantities as a function of the number of scales and in Log-Log graphs the same quantities are represented as a function of the parameter $\alpha=\sigma/\delta_c$, where $\delta_c = \left(\frac{P_c^2}{3K^2R}\right)^{1/3}$ and $K = \frac{4}{3}\frac{E}{1-v^2}$. Clearly, the plots look exactly those of the original paper of the FT model, and indeed a truely significant decrease of the pull-off is only likely for high fractal dimensions --- notice the curves in Fig. 3 in terms of α stop early in the plot, suggesting the paradoxical trend only occurs very slowly in the limit, when other factors may stop the fractal "process" well before the original FT assumptions break down anyway.
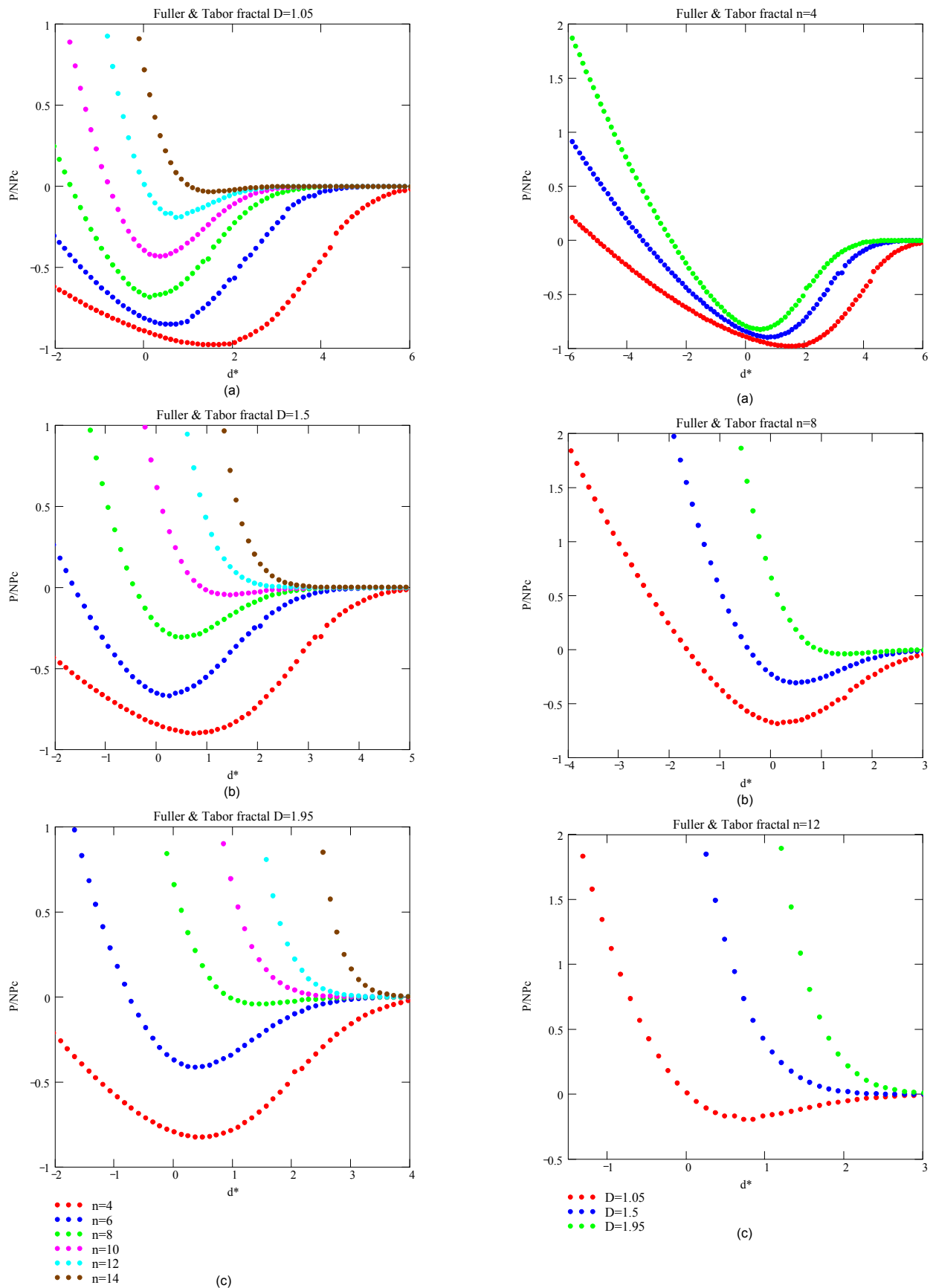
Figure 2 - The trends of the ratio P/NP_c as a function of the dimensionless separation $d*$ for a rubber having $E=6.8*10^5 \ N/m^2$ and $\omega=40 \ mJ/m^2$. The results shown are obtained for Weierstrass series with $\gamma=2$ at increasing number of scales $n=4,6,8,10,12,14$ for three fixed fractal dimensions D=1.05, 1.5, 1.95 and at three fixed n with increasing D.
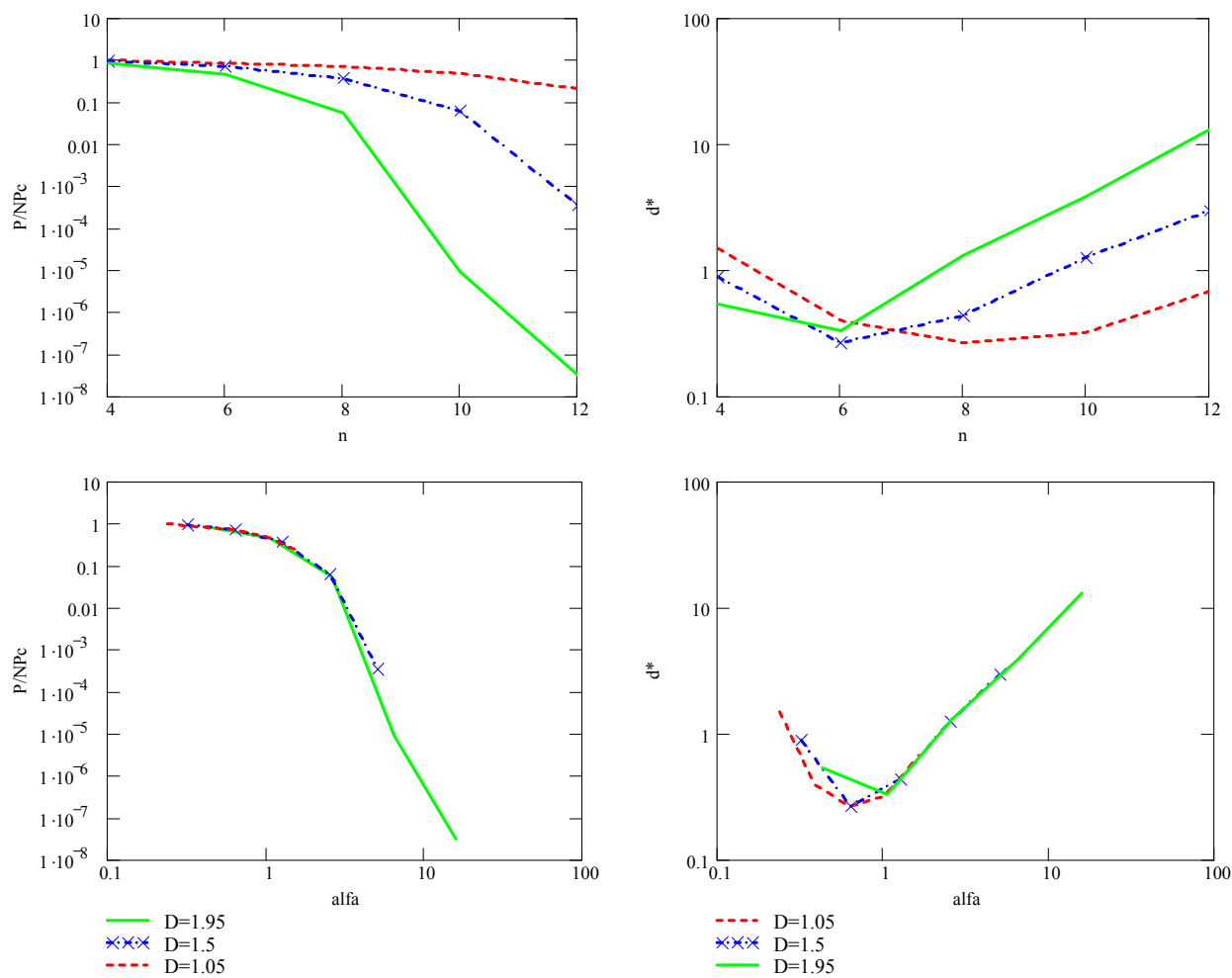
Figure 3 – The trends of the locus of minima of the ratio P/NP_c and of the respective values of d^* as a function of the number of scales n and of the parameter $alfa=\sigma/\delta_c$, for the fractal dimensions $D=1.05, 1.5, 1.95$.

Discussion & conclusion

The FT model gives paradoxical results, although mostly in terms of the fractal limit (and not in a practical application if one defines the smallest scale in some physically "correct" sense), as already found for the non-adhesive case in the GW model. The extension of Fuller and Tabor to deal with the Maugis-Dugdale model by Morrow et al (2003), may give some improvement by considering the local behaviour to deviate from JKR, but it is not necessarily sufficient. Other models using the assumption of arbitrarily defining asperities from the geometrical intersection at a given height of the surface (usually, the Weierstrass-Mandelbrot series), assuming it follows the power-law Korcak law, and integrating on the various sizes, have been developed by Sahoo and Chowdhury (1996) or Komvopoulos (2003), Morrow and Lovell (2005), but seem not capturing the actual physics. Other theories have been proposed (Persson, 2002, Persson and Tosatti, 2001), but they seem to be based on other strong assumptions and are not rigorous as they appear at first (see Manners and Greenwood, 2006).

The question remains largely unsolved. It would be nice to open a discussion into iMechanica!

References

Archard, J.F., 1957, "Elastic deformation and the laws of friction", Proc. Roy. Soc. Lond., **A 243**, pp.190-205.

Borri-Brunetto, M., Carpinteri, A. and Chiaia, B., 1998, "Lacunarity of the contact domain between elastic bodies with rough boundaries" in *Probamat-21st Century: Probabilities and Materials*, Frantziskonis G. ed., Kluwer, Dordrecht, pp.45-64.

Ciavarella, M.; Delfine, V.; Demelio, G. A "re-vitalized" Greenwood and Williamson model of elastic contact between fractal surfaces Journal of the Mechanics and Physics of Solids Volume: 54, Issue: 12, December, 2006, pp. 2569-2591

Ciavarella, M., Demelio, G., Barber, J.R. and Jang, Y.H., 2000, "Linear elastic contact of the Weierstrass profile", Proc. Roy. Soc. Lond., **A456**, pp. 387-405.

Ciavarella, M., Demelio, G., 2001, "Elastic Multiscale Contact of Rough Surfaces: Archard's Model Revisited and Comparisons With Modern fractal Models", ASME J. of Appl. Mech., **68**, pp. 496-498.

Fuller KNG, Tabor D, Effect of surface roughness on adhesion of elastic solids, Proc. Roy Soc London A 345 (1642): 327-342, 1975

Greenwood, J.A.,1996.Contact pressure fluctuations.P.I.Mech.Eng.J-J.Eng.,210 –281.

Greenwood, J.A.,Wu,J.J.,2001.Surface roughness and contact:an apology.Meccanica 36 (6),617 –630.

Greenwood, J.A. & Williamson, J.B.P., 1966, "The contact of nominally flat surfaces", Proc. Roy. Soc. Lond. **A295**, pp. 300-319.

Johnson,K.L., Kendall,K., Roberts,A.D.,1971 Surface energy and the contact of elastic solids.Proc.R. Soc.London A 324,301 –313.

Komvopoulos, K. 2003, Adhesion and friction forces in microelectromechanical systems: mechanisms, measurement, surface modification techniques, and adhesion theory, J of Adhesion Science and Technology, 17 (4), 477 - 517

Manners,W.,2000.Pressure required to flatten an elastic random rough pro file.Int.J.Mech.Sci.42 (12), 2321 –2336.

Manners, W.; Greenwood, J.A. , Some observations on Persson's diffusion theory of elastic contact Wear Volume: 261, Issue: 5-6, September 20, 2006, pp. 600-610

Morrow C, Lovell M and Ning X, A JKR–DMT transition solution for adhesive rough surface contact J. Phys. D: Appl. Phys. 36 No 5 (7 March 2003) 534-540

Morrow CA, Lovell MR, A solution for lightly loaded adhesive rough surfaces with application to MEMS, J of Trib Trans ASME 127 (1): 206-212, 2005

Persson,B.N.J.,2001a.Elastoplastic contact between randomly rough surfaces.Phys.Rev.Lett.87 (11)art.no.116101.

Persson,B.N.J.,2001b.Theory of rubber friction and contact mechanics.J.Chem.Phys.115 (8),3840 – 3861.

Persson BNJ, Adhesion between elastic bodies with randomly rough surfaces, Phys Rev Lett 89 (24): Art. No. 245502 DEC 9 2002

Persson BNJ, Tosatti E, The effect of surface roughness on the adhesion of elastic solids , J of Chem Phys 115 (12): 5597-5610, 2001

Persson,B.N.J.,Bucher,F.,Chiaia,B.,2002.Elastic contact between randomly rough surfaces:comparison of theory with numerical results. Phys.Rev.B 65,184106-1 –184106-7.

Sahoo P., Roy Chowdhury S.K., A fractal analysis of adhesion at the contact between rough solids, J. Eng. Tribol.: Proc. Instn. Mech. Eng. 210 (1996) 269–279.

Tabor D., Surface forces and surface interactions J. Colloid Interface Sci. 58 (1977) 2-13