Newsletter for the ASME Committee on Constitutive Equations

Summer 2008

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Introduction

Welcome to the first edition of the newsletter for the ASME AMD-MD Joint Committee on Constitutive Equations. The purpose of this newsletter is to publicize the activities of the committee and the committee members. The committee meets each year at the ASME International Mechanical Engineering Congress and Exposition. If you are interested in membership, please contact the chair of the Committee, George Voyiadjis (voyiadjis@eng.lsu.edu).

Homogenization and Size of Representative Volume Element (RVE)

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The need for homogenization has been driven by recognition that matter is (highly) heterogeneous while the conventional fluid and solid mechanics rely on a homogeneous continuum concept. In fluids problems the heterogeneous nature of a medium needs to be accounted for scales comparable to the mean free path. [The distinction between continuum and discrete problems is then established with the help of the *Knudsen number* = the molecular mean free path divided by the representative length scale.] In solids problems the situation is complicated by the presence of many length scales (atomistic, dislocation fields, polycrystalline, etc.) as well as the possibility of change of a material as it evolves, such as going from elastic to plastic behavior, aging or fracture. By and large, all the homogenization studies are confined to one particular situation, and so, there is a whole body of literature dealing with, say, scaling laws in fracture of disordered materials (lattices, composites, polycrystals, etc.). The term *homogenization*, however, typically connotes a passage from a fine level in some microstructure to a coarser level, so that an effective continuum-type constitutive response can be established. There are several ways in which one can classify such homogenization theories. The first one is to distinguish either those models predicting an effective medium response (via so-called *mean field theories*, e.g. a self-consistent approximation), or those rigorously bounding such a response, e.g. [1,2,3,4]. Another way to classify these theories is according to whether they are deterministic or probabilistic in character. Yet another classification would ask whether a given theory pertains to a static or a dynamic behavior of the system.

The challenge of establishing a homogenized material behavior is closely linked to the postulate of *separation of scales* on which every continuum theory hinges. This postulate involves two inequalities

$$d \ll L \ll L_{macro} \tag{1}$$

where we distinguish three scales (Fig. 1):

· the *microscale d* (such as a crystal size, when trying to homogenize a polycrystal)
· the *mesoscale L*, size of the *Representative Volume Element* (RVE)
· the *macroscale* L_{macro}, i.e. the macroscopic body size.

The inequality on the right defines a continuum body in the sense that L is a mathematical point relative to the macroscopic body dimensions. For example, we may model a very large polycrystalline sheet by a biharmonic equation with an Airy stress function, under a tacit understanding that we do not resolve any local details at the crystal level. How far above the crystal level we actually need to go is expressed by the left inequality, and it must be immediately noted that, perhaps, $d < L$ may be sufficient in a particular problem.

This leads us to the issue of size of a *Representative Volume Element* (RVE): What should the non-dimensionalized mesoscale $\delta = L/d$ be in a specific problem? The answer is provided by considering the *Hill* (or *Hill-Mandel*) *condition* of micromechanics

$$\overline{\sigma_{ij}\varepsilon_{ij}} = \overline{\sigma}_{ij}\,\overline{\varepsilon}_{ij} \tag{2}$$

where the overbars indicate volume average. Here we recognize that the material is random in the sense that we are dealing with an ensemble of deterministic specimens, so that (2) applies to any $B(\omega)$ of the ensemble $B = \{B(\omega); \omega \in \Omega\}$. Note: (2) assures that a theoretician's interpretation of the constitutive response of a microstructured material (as expressed through the left-hand-side) corresponds to that of an experimentalist's interpretation (right-hand-side). For an unbounded space domain (δ→∞), Hill's condition is trivially satisfied, but for a finite body it requires that $B(\omega)$ be loaded in a specific way on its boundary $\partial B(\omega)$. Here we have the following classical result:

$$\overline{\sigma_{ij}\varepsilon_{ij}} = \overline{\sigma}_{ij}\,\overline{\varepsilon}_{ij} \Leftrightarrow \frac{1}{V_0}\int_{S_0}\left(t_i - \overline{\sigma}_{ij}n_j\right)\left(u_i - \overline{\varepsilon}_{ij}x_j\right)\,dV = 0 \tag{3}$$

which, clearly, is satisfied by three different types of uniform boundary conditions on the mesoscale: *displacement*, *traction*, and *displacement-traction* (also called orthogonal-mixed), e.g. [5]. It follows from (i) the assumption of spatial ergodicity and statistical homogeneity of the material as well as (ii) the extremum principles of elasticity, that the displacement condition provides an upper (stiffer) bound on the effective stiffness tensor C_{ijkl}^{eff}, while the traction condition a lower (softer) bound. Applying these conditions on various mesoscales and carrying out ensemble averaging over $\{B(\omega); \omega \in \Omega\}$, one obtains scale-dependent (i.e. in function of δ)

bounds on C_{ijkl}^{eff}. In other words, this gives scaling towards the RVE, which allows one to say whether we actually need $d \ll L$ or $d < L$ in (1).

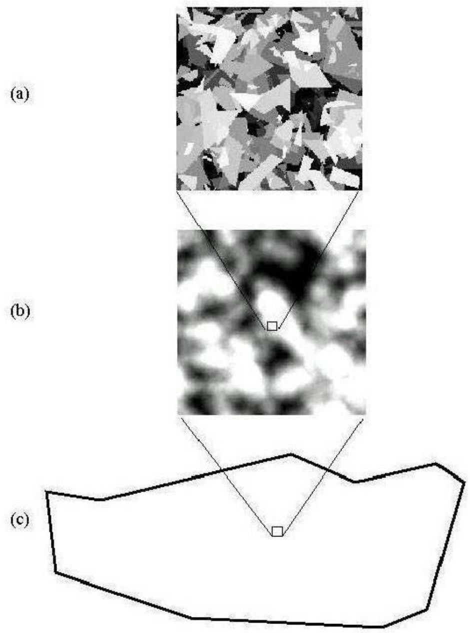

Fig. 1. Passage from a microscale of dimension d (a) via a mesoscale of dimension L (b) to a macroscale (c) of characteristic dimension L_{macro}; *from [5].*

Providing (i) the Hill condition is generalized to nonlinear and/or inelastic microstructures – such as viscoelasticity, elasto-plasticity, finite elasticity, and even flow in porous media – and (ii) there are extremum principles available, one can use the same approach to obtain scale-dependent bounds and scaling laws for a wide range of other materials [5]. In all these problems, just like in linear elastic ones, one can alternatively employ periodic boundary conditions. The latter necessitate an artificial change of the material in a boundary zone, and display some scale dependence without (!) any bounding property. One has to increase the domain until finally that dependence is close to nil. Clearly, both approaches have their pros and cons.

If one uses a mesoscale *L* smaller than necessary (within some precision) to homogenize the microstructure into a uniform continuum, one is faced with fluctuations such as seen in Fig. 1(b). In the language of stochastic mechanics, this figure shows one realization of a continuum random field smoothing the material on a length scale smaller than the scale of RVE. Now, the mesoscale domain of Fig. 1(a) plays the role of a *Statistical Volume Element* (SVE) of random field theories. This is akin to seeing grayscale fluctuations in a sheet of paper, and then having these fluctuations vanish as the sheet is pulled, say, 2 meters away from one's eyes. Clearly, our eye is smoothing (i.e. homogenizing) a cellulose fiber microstructure (millimeter scale) and its flocculation (centimeter scale) depending on its distance from the sheet.

The Hill condition also provides a way to formulate continuum random fields of constitutive properties on the basis of micromechanics, as opposed to simply postulating them as is commonly done in the field of so-called *stochastic finite elements* (SFE). Another challenge which can be handled through a suitably generalized approach is the choice of a non-classical (e.g. micropolar) rather than a classical continuum on the mesoscale. Other typical situations where one wants/has to deal with random mirostructures are: wavefronts (taken as zones of finite thickness relative to grain size, rather than as idealized singular surfaces), cracks and crack tips, FGM, small scale devices (MEMS/NEMS), and fractals. These and related issues in stochastic mechanics and geometry are treated in [5]. Extensive information on the passage from images to models within the general framework of mathematical morphology can be found in [6,7].

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[2] Torquato, S. (2001), *Random Heterogeneous Materials - Microstructure and Macroscopic Properties*, Springer-Verlag.

[3] Markov, K. (2000), Elementary micromechanics of heterogeneous media, in *Heterogeneous Media*, Markov, K. and Preziosi, L. (eds.), Birkhäuser.
[4] Milton, G.W. (2002), *The Theory of Composites*, University Press.
[5] Ostoja-Starzewski, M. (2008), *Microstructural Randomness and Scaling in Mechanics of Materials*, Chapman & Hall/CRC Press.
[6] Serra, J. (1982), *Image Analysis and Mathematical Morphology*, Academic Press.
[7] Jeulin, D. (2001), Random structure models for homogenization and fracture statistics, in *Mechanics of Random and Multiscale Microstructures* (eds. Jeulin, D. and Ostoja-Starzewski, M.), CISM Courses and Lectures **430**, Springer, 33-91.

Annoucements

George Voyiadjis Awarded Newmark Medal

Professor George Voyiadjis, the chair of the committee and Professor of Civil Engineering at Louisiana State University, has received the 2008 Nathan M. Newmark Medal. Prof. Voyiadjis was chosen by the Structural Engineering Institute and the Engineering Mechanics Institute. The award citation reads: "For his outstanding contributions to the fields of structural mechanics and geomechanics, his fundamental research in constitutive modeling and characterization of damage mechanisms in metals, composites, and soils, and his pioneering contributions in multi-scale modeling and localization problems." The selection committee particularly noted his development of a number of widely used nonlinear constitutive models for steel as well as ceramic and composite materials. Prof. Voyiadjis will receive the medal during the Engineering Mechanics Institute's inaugural International Conference, May 18-21, 2008 in Minneapolis, MN.

Gregory Odegard to Receive Beer and Johnston Award

Dr. Gregory Odegard has been chosen as a recipient of the 2008 Ferdinand P. Beer and E. Russell Johnston, Jr. Outstanding New Mechanic Educator Award. Established in 1992, this award is given annually to up to three individuals who have shown a strong commitment to mechanics education. The award consists of a $200 cash prize and a plaque. The award will be presented at the ASEE Mechanics Division's awards banquet at the ASEE Annual Conference in Pittsburgh on June 24, 2008.

New Book by Martin Ostoja-Starzewski

Professor Martin Ostoja-Starzewski has published a book "Microstructural Randomness and Scaling in Mechanics of Materials," Chapman & Hall/CRC Press/Taylor & Francis. The book develops and reviews a number of stochastic models and methods useful in mechanics of random media, a field at the intersection of solid mechanics, materials science, stochastic mathematics and statistical physics. The first six of 11 chapters include exercise problems, making the book suitable for a graduate course. Among the book's unique features are:

- Basic coverage of random geometry and continuum random fields

- Review of truss-type and beam-type lattices, and construction of corresponding classical and non-classical continua
- Theory and consequences of stress invariance in planar classical and micropolar elasticity
- Introduction to statistical continuum theories, including thermomechanics of random media
- Scaling to Representative Volume Element (RVE) in conductivity, linear or finite elasticity and thermoelasticity, elasto-plasticity, flow in porous media, …
- Methods for problems below the RVE – i.e., those lacking the separation of scales – via e.g. stochastic finite elements
- A study of effects of microscale material randomness on waves in (in)elastic/non-linear media, with focus on wavefronts

Special sessions in memory of Thomas S. Gates

Dr. Thomas "Tom" Gates passed away on April 18, 2008. Tom work at NASA Langley Research Center for 18 years conducting research on multiscale modeling of nanostructured materials, characterization of viscoelastic materials, and experimental testing of polymer-composites. In memory of his contribution to developing constitutive models for a host of materials, a series of special sessions are being organized for the 50th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference in Palm Springs, CA, May 4-7, 2009. Please contact Greg Odegard (gmodegar@mtu.edu) for more information.