Computational Analytical Micromechanics.

Background, Opportunities, and Prospective

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The most popular methods of analytical micromechanics of random structure matrix composites being considered are based just on a few basic concepts. The effective field hypothesis (EFH, also called the **H1a** hypothesis, see p. 253 in [1]) is apparently the most fundamental, most prospective, and most exploited concept of micromechanics (see [1] where other references can be found). This concept has directed a development of micromechanics over the last sixty years and made a contribution to their progress incompatible with any another concept. The idea of this concept dates back to Mossotti (1850) who pioneered the introduction of the effective field concept as a local homogeneous field acting on the inclusions and differing from the applied macroscopic one. Among a few hypotheses used by Mossotti (1850), one of the most important ones was the quasi-crystalline approximation (closing hypothesis **H2a**, p. 264 in [1], see also its multiparticle generalization, hypothesis **H2b**, p. 255 in [1]) proposed 100 years later by Lax (1952) in a modern concise form. The idea of the effective field and quasi-crystalline approximation was added by the hypothesis **H3** of "ellipsoidal symmetry" (p. 265 in [1]) for the distribution of inclusions just for providing the applicability of EFH. As a tool for concrete applications of the concepts mentioned, Eshelby (1957) solution was used although the Eshelby's theorem has a fundamental conceptual sense rather than only an analytical solution of some particular problem for the ellipsoidal homogeneous inclusion. The concept of the EFH (even if this term is not mentioned) in combination with subsequent assumptions totally dominates (and creates the fundamental limitations) in all four groups of analytical micromechanics in physics and mechanics of heterogeneous media: model methods, perturbation methods, self-consistent methods [e.g., Mori-Tanaka method (MTM), and the Method of Effective Field, MEF], and variational ones (see for refs. [1]).

However, one shows that the EFH is a central one and other concepts play a satellite role providing the conditions for application of the EFH. Moreover, one shows that all mentioned hypotheses are not really necessary and can be relaxed. The first attack on a citadel called EFH was produced by creation in [2, 3, 4] (see also pp. 607–610 in [1]) the exact general integral equation

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \langle\varepsilon\rangle(\mathbf{x}) + \int [\mathbf{U}(\mathbf{x}-\mathbf{y})\boldsymbol{\tau}(\mathbf{y}) - \langle\mathbf{U}(\mathbf{x}-\mathbf{y})\tau(\mathbf{y})\rangle(\mathbf{y})]d\mathbf{y}. \tag{1}$$

where $\boldsymbol{\tau}(\mathbf{x}) \equiv \mathbf{L}_1(\mathbf{x})\boldsymbol{\varepsilon}(\mathbf{x})$ (see [1, 3] for complete notations). Equation (1) was obtained without any auxiliary assumptions such as, e.g., the version of the EFH $\langle\mathbf{U}(\mathbf{x}-\mathbf{y})\tau(\mathbf{y})\rangle(\mathbf{y}) = \mathbf{U}(\mathbf{x}-\mathbf{y})\langle\tau(\mathbf{y})\rangle(\mathbf{y})$ (hypothesis **H1b**, p. 253 in [1]) implicitly exploited in the known centering methods and reducing Eq. (1) for statistically homogeneous media subjected to the homogeneous boundary conditions to the known one

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \langle\varepsilon\rangle + \int \mathbf{U}(\mathbf{x}-\mathbf{y})[\boldsymbol{\tau}(\mathbf{y}) - \langle\tau\rangle]d\mathbf{y}, \tag{2}$$

which goes back to Lord Rayleigh (1892) (see for refs. [1, 4]). One demonstrates (see [3, 4]) that Eq. (2), erroneously recognized as an exact one after the proofs by Shermergor (1977) and by O'Brien (1979), is correct only after the additional asymptotic assumption **H1b**.

A fundamental deficiency of Eq. (2) is a dependence of the renormalizing term $\mathbf{U}(\mathbf{x}-\mathbf{y})\langle\boldsymbol{\tau}\rangle(\mathbf{y})$ [obtained in the framework of the asymptotic approximation of the hypothesis **H1b**] only on the statistical average $\langle\boldsymbol{\tau}\rangle(\mathbf{y})$ while the renormalizing term $\langle\mathbf{U}(\mathbf{x}-\mathbf{y})\boldsymbol{\tau}\rangle(\mathbf{y})$ in Eq. (1) explicitly depends on details distribution $\langle\boldsymbol{\tau}|v_i,\mathbf{x}_i\rangle(\mathbf{y})$ ($\mathbf{y}\in v_i$). What seems to be only a formal trick is in reality a new background of micromechanics defining a new field of micromechanics called computational analytical micromechanics (CAM). CAM makes it possible to abandon basic concents of analytical micromechanics **H1a**, **H1b**, **H3** (with more [5] or less [6, 7] comletness) in

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the framework of hypotheses either **H2a** [5, 7] or **H2b** [8, 9] used for truncation of hierarchies of averaged Eq. (1) by exployting of any avaliable numerical method (VIEM [5], BEM, FEA [6, 7], hybrid FEA-BEM, multipole expansion method, complex potential method, and other, see for refs. [1]).

So, even in the case of statistically homogeneous media subjected to homogeneous boundary conditions (see [5, 7]), new effects have been found even in the framework of the hypothesis **H2a**. The final classical representations of the effective properties obtained by both the MEF and MTM (see for details [1]) depend only on the average strain concentrator factor $\mathbf{A}_i$, with $\langle \boldsymbol{\varepsilon} \rangle_i = \mathbf{A}_i \langle \overline{\boldsymbol{\varepsilon}} \rangle_i$, while the effective properties estimated by the new approach (1) implicitly depend on the inhomogeneous tensor $\mathbf{A}_i(\mathbf{x})$. The detected dependence allows us to abandon the hypothesis **H1b** whose accuracy is questionable for inclusions of noncanonical shape. We obtained a fundamental conclusion that effective moduli in general depend not only on the strain distribution inside the referred heterogeneity (describing by the tensor $\mathbf{A}_i(\mathbf{x})$, $\mathbf{x} \in v_i$) but also on the strains in the vicinity of heterogeneity i.e. extension of $\mathbf{A}_i(\mathbf{x})$, $\mathbf{x} \notin v_i$ is necessary. Then the size of the excluded volume as well as the binary correlation function will impact on the effective field even in the framework of hypothesis **H2a**.

It is expected to get a larger difference (with the change of the sign of predicted local stresses, see [5, 7]) between the results obtaining the use of either Eqs. (1) or (2) for composites reinforced by heterogeneities demonstrating greater inhomogeneity of stress distributions inside heterogeneities. This inhomogeneity can be produced by the different contributors (see for details [5]): *a*) peculiarities of heterogeneities manifested even in the framework of the hypothesis **H1a**, *b*) multiparticle interaction of heterogeneities (even for homogeneous ellipsoidal ones), *c*) special features of both the microstructure and applied loading. Many of the next linear statical problems were solved (partially, of course, see for refs. [1]) in the framework of the EFH and can be recast in the framework of the CAM with detection of significant improvement of predicted accuracy by the use of Eq. (1) instead of Eq. (2):

a). Composites with either nonellipsoidal [6, 7], coated, or continuously inhomogeneous [5] heterogeneities with, perhaps, either nonideal interface (including sliding, debonding, cohesive phenomena, as well as surface stress and surface tension ones) or nonlocal constitutive law (p. 581 in [1]).

b) Inhomogeneity of statistical moments of stresses for homogeneous ellipsoidal heterogeneities detected for binary interacting heterogeneities [8, 9].

c) Any nonlocal problem (inhomogeneous remote loading, functionally graded materials, clustered materials, bounded media, contact of microinhomogeneous media, macro-inhomogeneity insde microinhomogeneous medium, nonlocal constitutive laws either inside or outside the heterogeneities).

d) Variational methods currently postulated homogeneity of polarization tensors inside the heterogeneities (this assumption is even more restrictive then EFH) can be renewed by using (1) instead of (2) for the case a).

The solutions of mentioned problems obtained in the framework of the EFH were used as the basic elements in analyses of wide classes of dynamic, nonlinear, and coupled problems (see for refs. [1]). The generalization of these schemes can be also easy performed in the framework of the CAM in a straitforward manner.

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