Minimum compliance design problem

Andew Seagraves, Lei Qiao

Professor Radovitzky's CSM Group
Massachusetts Institute of Technology

Outline

An example

Taken from Sigmund and Bendsøe 2004

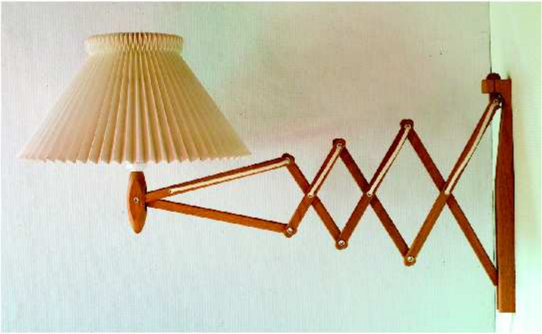


Figure: a hanging device for a lamp

An example

Taken from Sigmund and Bendsøe 2004

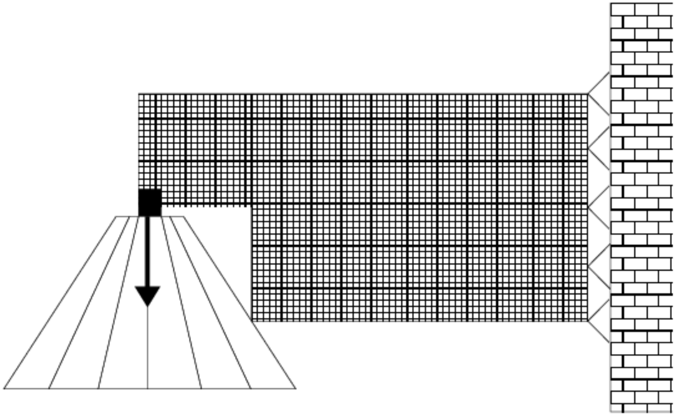


Figure: modelling

Background

Goal: To determine the optimized 'shape'

1. the first step to design a mechanical component
   - to identify the holes in a given domain
2. a constrained optimization problem
   - object functional, such as weight and elastic energy
   - constraints, such as BCs and volume fraction
   - $$\begin{array}{ll} \min & \text{elastic energy} \\ s.t. & \nabla \cdot \sigma + f = 0 \, , \forall x \in \Omega \\ & \sigma \cdot n = t, \quad x \in \Gamma_t \\ & u = 0, \quad x \in \Gamma_u \\ & \int_\Omega 1_{\Omega^{mat}} dx \leq V \; . \end{array}$$

BCs and constraints

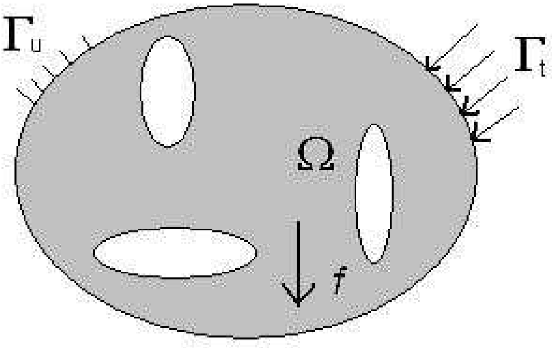


Figure: modelling

basic functionals

- $$a_C(u,v) = \int\limits_{\Omega} C_{ijkl}(x)\varepsilon_{ij}(u)\varepsilon_{kl}(v)dx \ ,$$

  where elastic energy equals $\frac{1}{2}a_C(u,u)$,

- $$l(u) = \int\limits_{\Omega} fudx + \int\limits_{\Gamma_t} tuds \ .$$

original formulation

$$\min_{u \in U, C} \quad \frac{1}{2} a_C(u,u)$$
$$s.t. \quad a_C(u,v) = l(v) \;, \; \forall v \in U \;,$$
$$C \in \mathbf{C}_{ad} \;,$$

- $$U = \{u \in \mathbf{H}^1(\Omega) : u|_{\Gamma_u} = 0\}$$
- $$\mathbf{C}_{ad} = \{C_{ijkl} \in \mathbf{L}^{\infty} : C_{ijkl} = 1_{\Omega^{mat}} C^0_{ijkl}, \int_{\Omega} 1_{\Omega^{mat}} dx \leq V\}$$

relaxed formulation

- original

$$C_{ijkl}(x) = 1_{\Omega^{mat}}(x) C^0_{ijkl}(x)\ ,\ \forall x \in \Omega$$

$$\int_\Omega 1_{\Omega^{mat}} dx \leq V$$

- relaxed

$$C_{ijkl}(x) = \rho(x) C^0_{ijkl}(x)\ ,\ \forall x \in \Omega$$

$$\int_\Omega \rho(x) dx \leq V\ , 0 < \rho_{min} \leq \rho(x) \leq 1$$

$$\min_{\rho} \quad F(\rho) = \mathbf{U}^T\mathbf{F} = \mathbf{U}^T\mathbf{K}\mathbf{U} = \sum_{e=1}^{N} (\rho_e)^p \mathbf{u}_e^T \mathbf{k}_e \mathbf{u}_e$$

$$s.t. \quad \sum_{e=1}^{N} \rho_e v_e \le V$$

$$0 < \rho_{min} \le \rho_e \le 1, \; e = 1, 2, ..., N$$

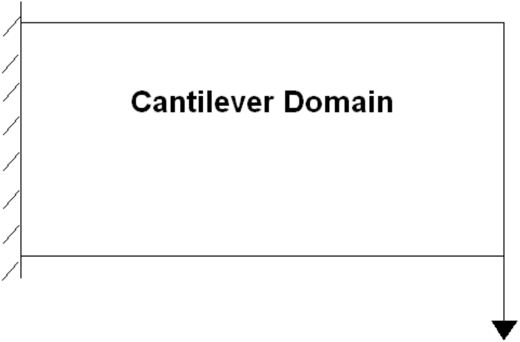
- Vectors **K**, **U** and **F** satisfy

$$\mathbf{K}\mathbf{U} = \mathbf{F} \, .$$

- Exponent p is used as penalty. Usually $p = 2, 3$.
- Solver adopts Method of Moving Asymptotes by Svanberg 2002.[1]

[1] Thank Prof. Svanberg for his MMA code!

Model to show the numerical difficulty



Numerical difficulty of mesh dependence

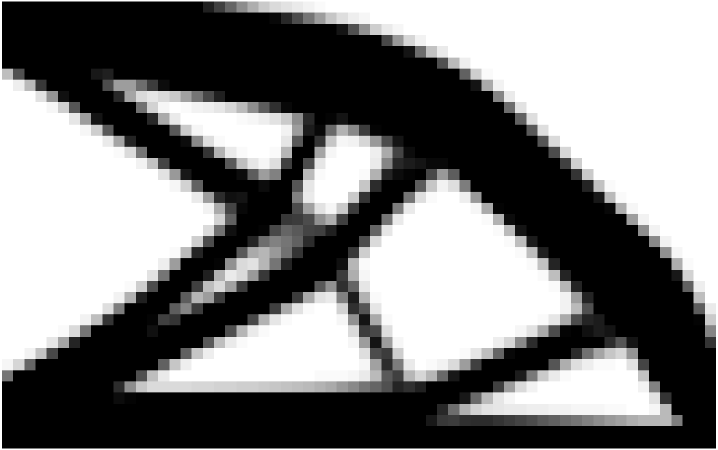


Figure: [2] $64 \times 40, p = 2$

[2] This figure is got by the code of Sigmund 2001.

Numerical difficulty of mesh dependence

Figure: [3] $128 \times 80, p = 2$

[3] This figure is got by the code of Sigmund 2001.

Use filter to control the result

In computation, we use the derivatives after filter instead of the original ones

$$\widehat{\frac{\partial F(\rho)}{\partial \rho_e}} = \frac{1}{\rho_e \sum_{f=1}^{N} H_{e,f}} \sum_{f=1}^{N} H_{e,f} \rho_f \frac{\partial F(\rho)}{\partial \rho_f}$$

where

$$H_{e,f} = \max\{0, r_{min} - dist(e,f)\} \ , \ e, f = 1, 2, ..., N \ ;$$

$dist(e,f)$ is the distance between the centers of e and f, r_{min} is the radius of filter.

Example, bridge like structure

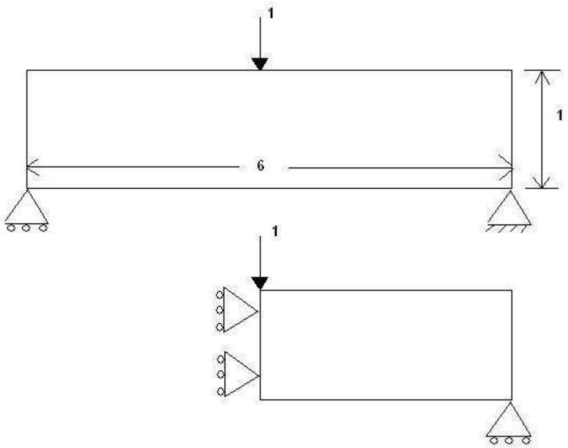


Figure: computational domain

Example, bridge like structure

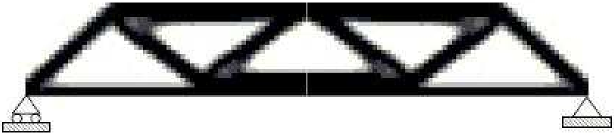


Figure: result from literature

Example, bridge like structure

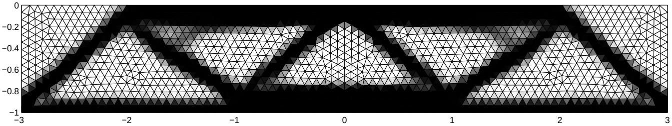


Figure: 691 nodes, 1272 elements

Example, bridge like structure

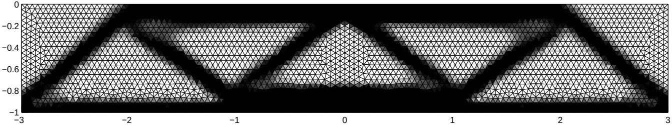


Figure: 1476 nodes, 2790 elements

Example, bridge like structure

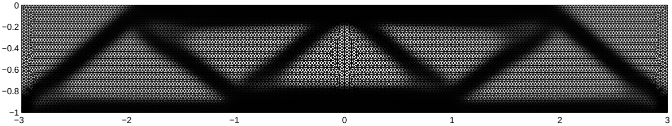


Figure: 5690 nodes, 11,058 elements

Example, cylinder under pressure

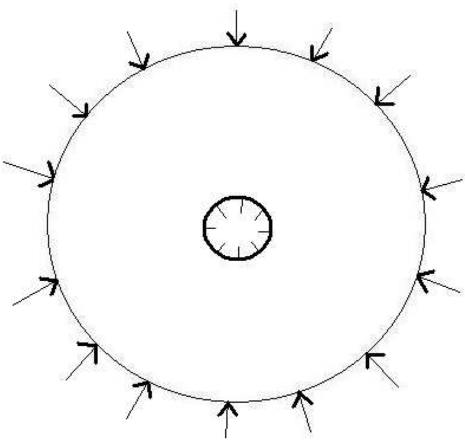


Figure: cylinder with unit pressure at outer boundary

Example, cylinder under pressure

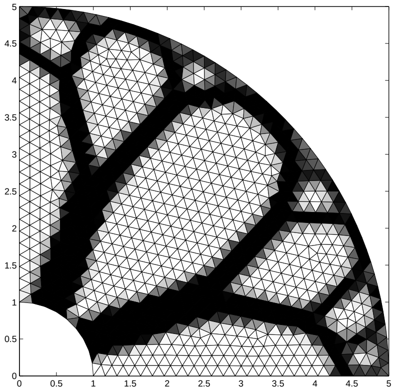


Figure: a quarter with unit pressure at outer boundary

Example, cylinder under pressure

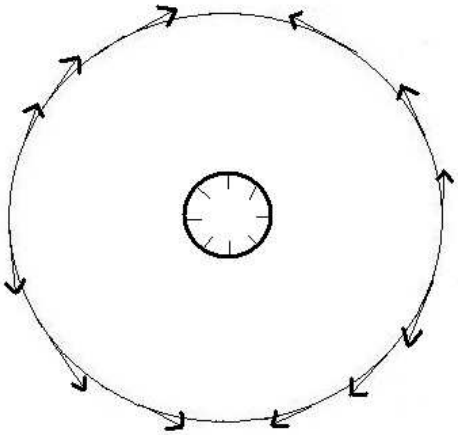


Figure: cylinder with unit tangential traction along outer boundary

Example, cylinder under tangential traction

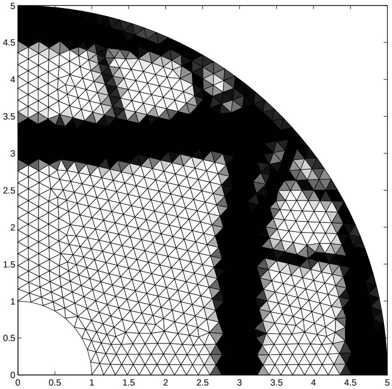


Figure: a quarter with unit tangential traction along outer boundary

Conclusion

- Interesting and challenging theoretical/computational problem
- Important practical applications:
  - Optimal design of structural components/supports
  - composites
  - microstructures
- Computational topology optimization increasingly important
- Achievable with simple MATLAB routines
- Two new structures optimized

The end

Thank you!