Nonlocal Micromechanics of Composites of both Random and Periodic Structures. (Background, Opportunities and Prospects)

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In locally elastic theory, the numerous methods of micromechanics, inspired by Eshelby, can be classified, according to Willis, into four broad categories (see for Refs. [1]): perturbation methods, self-consistent methods of truncation of a hierarchy, variational methods, and the model methods among which there are no rigorous boundaries. Apart from these methods exploiting microtopological information of the composite materials (CM), there are the general results establishing the links between the effective properties (effective elastic moduli, effective thermal expansion, and effective specific heat) and the corresponding mechanical and transformation ifluence functions which were inspired by Hill and Levin. In parallel with micromechanics of random structures, conceptually different methods of micromechanics of periodic structure composites can be subdivided on two-scale expansion methods (apparently coined by Babuska) and the methods of computational homogenization inspired by Suquet. We will demonstrate that the mentioned basic directions of locally elastic micromechanics are generalized to the nonlocal micromechanics (so-called strongly nonlocal ones by Eringen and peridynamics by Silling).

In contrast to these classical local and nonlocal theories, the peridynamic equation of motion introduced in Silling [2] (see also [3]) is free of any spatial derivatives of displacements, where each material point interacts directly with other material points separated from it by a finite distance. In bond-based approach considered, these interactions only occur between pairs of material points within a horizon. Wide expansion of the methods of locally elastic micromechanics into nonlocal phenomena was supported by a critical generalization of micrmechanics. Namely, the author (see for Refs. [4, 5], and [6-11]) proposed the general integral equation GIE of microinhomogeneous media forming the second background of micromechanics (the first one is based on the effective field hypothesis, EFH, proposed by Faraday, Poisson, Mossotti, Clausius, and Maxwell (1830-1880), see for Refs. [1]). The critical features of the new GIE [4, 5] are the absent of a direct dependence of GIE on both the Green function and the constitutive law (either local or nonlocal) without restrictions of the conventional micromechanics (such as, e.g., acceptance of the EFH and ellipsoidal symmetry of microtopology) that offers opportunities for a fundamental jump in multiscale and multiphysics researches with drastically improved accuracy of local field estimations (even to the point of correction of a sign, see [5]) that is critical for advanced material development (especially for nonlinear modeling). It was established formal similarity of the operator forms of GIEs for both locally elastic CMs and nonlocal ones (in the senses of either Eringen [12-14] or Silling [15-21]) that opens the opportunities to straightforward generalization of their solutions for locally elastic CMs [1] to their nonlocal counterparts.

For statistically homogeneous thermoperistatic media subjected to homogeneous volumetric macro boundary loading, one proposed the background principles and proved [16, 21] that the effective behavior of this media is governing by conventional effective constitutive equation which is intrinsic to the local thermoelasticity theory (see [1]). The general results establishing the links between the effective properties (effective elastic moduli, effective thermal expansion) and the corresponding mechanical and transformation influence functions (do not miss with the influence functions in peridynamics) are obtained by the use of both the decomposition of local fields into the load and residual fields as well as extraction from the material properties a constituent of the matrix properties. The energetic definition of effective elastic moduli is proposed. A detected similarity of results for both the locally elastic (and so-called strongly nonlocal) and peristaltic composites is explained by the fundamental reasons because the methods used for obtaining of the mentioned results widely exploit the Hill's condition proved and the self-adjointness of the peristatic operator. This similarity opens a way for straightforward expansion of analytical micromechanics tools (see [1]) to the new area of random structure peridynamic composites.

Effective moduli are expressed through the introduced new notions of both the micropolarization tensor and the average local polarization tensor (although the local polarization tensor is not defined) [16, 21]. The average is accomplished over the surface of the extended inclusion phase (that was performed by a straightforward generalization of interphase integral technique proposed by the author in locally elastic micromechanics [22-27]) rather than over an entire space. Any spatial derivatives of local displacement fields are not required. The basic hypotheses of locally elastic micromechanics (in the version of the second background of micromechanics [4, 5]) are generalized to their peristatic counterparts. In particular, in a dilute approximation method [28] (belonging to the class of perturbation methods), the volume fraction of the particles is small and their mutual interaction are neglected, i.e. each particle can be thus imagined as single, immersed into an unbounded matrix (this problem is solved in [29]). In self-consistent methods [30], e.g. in the generalized effective field method (EFM) proposed, the effective field is evaluated from self-consistent estimations by the use of closing of a corresponding integral equation in the framework of the quasi-crystalline approximation. In so doing, the classical EFH is relaxed, and the hypothesis of the ellipsoidal symmetry of the random structure of CMs is not used. One demonstrates some similarity and difference with respect to other methods (such as the dilute approximation and Mori-Tanaka approach). Effective nonlocal properties of statistically homogeneous peristatic CMs subjected to arbitrary self-equilibrated strongly inhomogeneous body-force density are estimated as a straightforward generalization of locally elastic micromechanics (see Chapter 8 in [1]). The generalized effective field method is realized in the iteration scheme without the EFH. For the dilute approximation [31], the problem is reduced to the solution for one inhomogeneity subjected to a set of body-force densities obtained by a parallel transition from the fixed original body-force.

The variational methods represent the most rigorous trend and security aspect of micromechanics of statistically homogeneous media. The admissible displacement and force fields are defined in [32]. The theorem of work and energy, Betti's reciprocal theorem, and the theorem of virtual work are proved. Principles of minimum of both potential energy and complimentary energy are generalized. The strain energy bounds are estimated for both the displacement and force homogeneous volumetric boundary conditions. Generalized Hill's bounds on the effective elastic moduli of peridynamic random structure composites are obtained. In contrast to the classical Hill's bounds, in the new bounds, comparable scales of the inclusion size and horizon are taken into account that lead to dependance of the bounds on both the size and shape of the inclusions. Generalized Hashin-Shtrikman variational principle and bounds can be obtained as a straightforward combination of variational principles proposed and the new version of Hashin-Shtrikman bounds [33, 34] obtained by the author in locally elastic micromechanics of CM with noncanonical shape of inclusions in the framework of the EFH.

The background principles of computational homogenization (initiated by Suquet for locally elastic micromechanics) of micromechanics of periodic structure CMs with the peristatic mechanical properties of constituents are proposed [35-39]. One introduces new volumetric periodic boundary conditions (PBC, [37, 39]) at the interaction boundary of a representative unite cell whose local limit implies the known locally elastic periodic boundary conditions. The classical representations of effective elastic moduli through the mechanical influence functions for elastic CM are generalized to the case of peristatics, and the energetic definition of effective elastic moduli is proposed. A generalization of the Hill's equality to peristatic composites is proved analogously to the theory for the peristatic random structure CM. Due to the volumetric displacement periodicity the special traction boundary condition is not required for establishment of the micro-to-macro displacement relationship although this classical antiperiodic traction condition at the geometrical boundary of the UC is needed for estimations of both the overall stresses and effective moduli. The discretization of the equilibrium equation acts as a macro-to-micro transition of the deformation-driven type, where the overall deformation is controlled. One shows numerically the convergence of the effective moduli estimations by the peristatic model to their locally elastic counterpart. Estimation of effective moduli of the medium with periodically distributed damage (i.e. with the broken bonds) is considered [38]. Asymptotic exactness of Kachanov formula is proved for uniformly damaged peridynamic medium. Proposal of the new volumetric Bloch conditions opens the opportunities for analyses of a wide class of wave propagation in periodic peridynamic composites (e.g. metamaterials and phononic crystals).

The mentioned problems were considered for the bond-based approaches, which can be easy generalized to the linear state-based approaches in a straightforward manner. Linear solutions are used as the basic elements in analyses of wide classes of dynamic, nonlinear, and coupled problems in the framework of the second background micromechanics proposed by the author (see for Refs. [4, 5]).

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