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THREE-POINT BENDING TEST PROCEDURE: RECTANGULAR CROSS SECTION

1. *Determination of the stiffness (compliance) of the testing system*: K_s (C_s). The test is carried out applying the load to the specimen supported on a rigid surface. The stiffness is determined in the linear zone, when the curved zone due to indentation effects dissappears. It is enough to testing several times one specimen of a given material. For a testing machine, K_s depends mainly on the material properties and on the load cell stiffness. Fig.1 shows the curve obtained according to the following procedure: " the specimen was placed on a steel bar with square section of side 40 mm. The bar was placed on the bending support at the minimum span and load–displacement curves were obtained. The displacement rate was 1.4 mm/min and the maximum load was approximately 7000N"[1]. Fig. 2 shows the testing configuration.

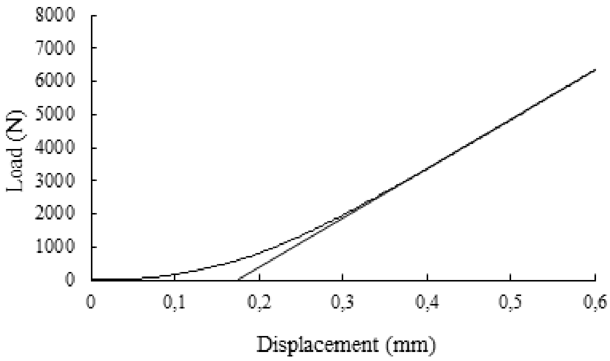

Fig. 1. Load-displacement curve of an indentation test

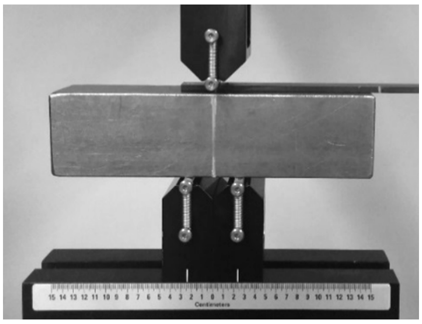

Fig. 2. Example of indentation test

2. *The actual displacement* δ of the middle section is

$$\delta = \delta_{\exp} - PC_s \tag{1}$$

$\delta_{\exp}$: experimental displacement measured by the testing machine

P: applied load measured by the testing machine

3. *Strain chanel is defined as a function of the load P*:

$$\varepsilon = \frac{3PL_0}{2E_{ap}bh^2} \tag{2}$$

L_0: support span
E_{ap}: approximate value of the modulus. Its obtention is explained below
b: width
h: thickness

4. The **chord slope** *m* of the load-displacement curve is determined between two points 1 and 2 fixed by their strains. For instance: $\varepsilon_1 = 0.001$, $\varepsilon_2 = 0.003$

$$m = \frac{P_2 - P_1}{\delta_2 - \delta_1} \tag{3}$$

5. *The usual three point bending modulus* is determined knowing the chord slope *m* by means of:

$$E_{3p} = \frac{mL_0^3}{4bh^3} \tag{4}$$

6. The flexural modulus, including shear effects and the span variation due to the contact point variation between the specimen and the support rollers, is given by [2]:

$$E_f = E_{3p}\left\{1 - \frac{9}{4} f\left(\varepsilon_1 + \varepsilon_2\right) + \frac{6}{5}\frac{E_f}{G_{13}}\left(\frac{h}{L_0}\right)^2 \left[1 - f\left(\varepsilon_1 + \varepsilon_2\right)\right]\right\} \tag{5}$$

$f = \frac{R}{h}$: being *R* the support roller radius
b: width
h: thickness
The factor $\frac{6}{5}$ corresponds to a rectangular cross section

7. Taking into account Eqs. (4) and (5) it is obtained the equation of a straight line:

$$y = Ax + B \qquad \text{where}$$
$$y = E_{3p}^{-1} \qquad x = \left(\frac{h}{L_0}\right)^2 \tag{6}$$
$$A = E_f^{-1}\left[1 - \left(\frac{9}{4} f + \frac{3}{4}\right)\left(\varepsilon_1 + \varepsilon_2\right)\right] \qquad B = \frac{6}{5}G_{13}^{-1}\left[1 - f\left(\varepsilon_1 + \varepsilon_2\right)\right]$$

8. Tests are carried out at several spans, obtaining experimental values of E_{3p}. Then, A and B are obtained by linear regression. The range of L_0/h values has to be enough wide in order to determine accurate values of the out-of-plane shear modulus G_{13}. For instance:

$$(L/h)_{min}=10; \ (L/h)_{max} = 40$$

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NOTES

Note 1: Determination of strains

1. As E_{ap} is necessary for determining strains, the first approximation can be obtained in two steps:

   1.1. For a great (L/h) value, the flexural modulus without taking into account shear effects is obtained from Eq. (5) as:

$$E_{ap0} = E_{3p}\left\{1 - \frac{9}{4} f\left(\varepsilon_1 + \varepsilon_2\right)\right\} \tag{7}$$

In this test, strains for defining the strain range are determined by the experimental displacements:

$$\varepsilon = \frac{6h\delta_{\exp}}{L_0^2} \tag{8}$$

   1.2. Two tests are carried out at minimum and maximum spans determining strains with Eq. (2) and E_{ap0} obtained in the previous step. Then, a first approach of E_{f0} and G_{130} are obtained, being $E_{ap} = E_{f0}$. In order to obtain accurate values of E_f and G_{13} other intermediate spans are tested.

It is worth noting that, in the case that G_{13} is unknown, shear effect in step 1.1 is ignored, as its influence is not known. The suitability of that assumption is checked in the step 1.2.

Note 2: Displacement rate

The displacement rate of the load nose $\dot{\delta}_{\exp}$ is determined including shear and system stiffness, using the approximate values E_{f0} and G_{130}, with the following expression:

$$\dot{\delta}_{\exp} = \frac{\dot{\varepsilon}L_0^2}{6h}\left[1 + \frac{6}{5}\frac{E_{f0}}{G_{130}}\left(\frac{h}{L_0}\right)^2 + 4\frac{E_{f0}b}{K_s}\left(\frac{h}{L_0}\right)^3\right] \tag{9}$$

being $\dot{\varepsilon} = 0.01\ \text{min}^{-1}$ according to ISO 14125 and ASTM D790.

In step 1.1 of Note 1, shear and system stiffness terms are not considered in the determination of the displacement rate:

$$\dot{\delta}_{\exp} = \frac{\dot{\varepsilon}L_0^2}{6h} \tag{10}$$

Note 3: Indentation effects

The indentation of the load roller in the specimen is easily detected by a curved zone at the beginning of the load-displacement curve, as shown in Fig. 1. The minimum span has to be large enough to avoid indentation effects. Anyway, the chord slope has to be determined out of the indentation zone.

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Note 4: Preload

In order to ensure the contact between the specimen and the load application roller, an initial load is applied at the beginning of the test. Then, being *t* the time of the test:

$$t = 0 \qquad P = P_0 \qquad \delta = 0$$

In spite of the measurement of the displacement is not exact, this fact does not affect the determination of the modulus, as in Eq. (3) the error due to the initial displacement disappears.

Note 5: Measurement of the support span

The actual support span has to be determined accurately with a precision of 0.1 mm. This is critical in the case of the lesser spans. After fixing the nominal span according to the marks of the testing fixture, it is recommended to determine the distance between two reference points by a caliper, after knowing the distance from those reference points to the centre of the support roller, as shown in Fig. 3.

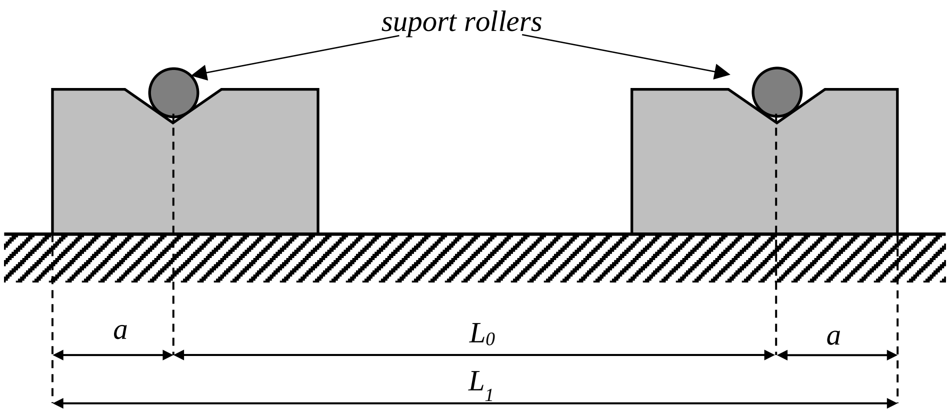

Fig. 3. Determination of the actual support span

a: known distance of the testing fixture, with a precision of 0.1 mm

L_1: determined by a caliper with a precision of 0.1mm

$L_0 = L_1 - 2a$: actual support span

REFERENCES

1 Mujika F (2007). On the effect of shear and local deformation in three-point bending tests, *Polymer Testing*, **26**: 869-877.

2 Mujika F, Arrese A, Adarraga I, Oses U (2016). New correction terms concerning three-point and four-point bending tests, Polymer Testing, **55**: 25-37.