

Universal features in "stickiness" criteria for soft adhesion with rough surfaces

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Abstract

A very interesting recent paper by Dalvi *et al.* has demonstrated convincingly with adhesion experiments of a soft material with a hard rough material that the simple energy idea of Persson and Tosatti works reasonably well, namely the reduction in apparent work of adhesion is equal to the energy required to achieve conformal contact. We demonstrate here that, in terms of a stickiness criterion, this is extremely close to a criterion we derive from BAM (Bearing Area Model) of Ciavarella, and not very far from that of Violano *et al.* It is rather surprising that all these criteria give very close results and this also confirms stickiness to be mainly dependent on macroscopic quantities.

Keywords:

Adhesion, JKR model, DMT model, soft matter, roughness models.

1. Introduction

The role of adhesion in contact mechanics has seen an explosion of interest in recent years, due to the enormous interest in soft materials technology, nano-systems, cell adhesion, and the understanding of bio-attachments and the idea to imitate their solutions (Creton *et al.*, 1996, Kendall, 2001, Kendall *et al.*, 2010, Autumn *et al.*, 2002). Ciavarella *et al.*(2018) discuss some aspects of various methods of solution, stemming from the seminal paper of Johnson, Kendall, and Roberts (JKR, 1971) who introduced an energy balance calculation like that of Griffith in fracture mechanics (Maugis, 2013).

The presence of surface roughness is so important that for a long time it made impossible to measure adhesion between hard materials, until JKR experimented on rubber, and Fuller and Tabor (1975) were able to first measure the role of roughness. For nominally flat bulk solids, it appears that the main solution to maintain stickiness is to reduce the elastic modulus, as already suggested by the empirical Dahlquist (1969a, 1969b) criterion, which sets the threshold at the elastic Young modulus of about 1 MPa.

Fuller and Tabor (1975) theory is based on asperities and its adhesion parameter contains the mean asperity radius, which is not well defined for "fractal" surfaces as today we consider, for which the "stickiness" would tend to zero if we included extremely small wavelengths. Various other theories have been proposed more recently (see the review by Ciavarella *et al.*(2018) for a general presentation), and there is debate still about the applicability of each. Numerical solutions have clarified some aspects, and a remarkable effort was made with the state-of-the-art Müser's recent 'Contact Challenge' (Müser *et al.*, 2018, Ciavarella, 2018b). However, they typically describe surfaces with PSDs spanning only about three decades — e.g. nanometer to micrometer scales, similarly to Pastewka and Robbins (2014). Instead, the real "broadness" of the band of roughness is likely to span many more decades of wavelength.

Indeed, in a very interesting recent paper by Dalvi *et al.* (2019), the authors describe topography across more than seven orders of magnitude, including down to the Ångström-scale, and the Power Spectrum Density (PSD) follows almost a power law despite the broadness of the band (see fig.S2 which gives the 2D isotropic PSD). The "stickiness" criteria generated by interpolating numerical results (Pastewka and Robbins, 2014, Müser, 2016), seem to depend critically on the truncation of the PSD, so further investigation is important, and in particular, experiments. In these respects, the very interesting recent paper by Dalvi *et al.* (2019) reports extensive adhesion measurements for soft elastic polydimethylsiloxane (PDMS) hemispheres with elastic modulus ranging from 0.7 to 10 MPa in contact with four different polycrystalline diamond substrates, and their careful experimental effort corroborates ideas originally suggested by Persson (2002) and Persson & Tosatti (2002) inspired by the JKR energy balance concepts (Johnson, Kendall and Roberts, 1971) of fracture mechanics applied to adhesion of elastic bodies. We shall therefore further elaborate on the Persson and Tosatti's ideas, and compare the results with other recent criteria, in particular those proposed by Ciavarella (2018), and Violano *et al.* (2018). We shall find surprisingly

universal results, despite the very different origin of the various proposals we compare.

1.1. Persson-Tosatti

Persson (2002) and Persson & Tosatti (2002) argue with a energy balance between the state of full contact and that of complete loss of contact that the effective energy available at pull-off with a rough interface is

$$\Delta\gamma_{eff} = \frac{A}{A_0}\Delta\gamma - \frac{U_{el}}{A_0} \quad (1)$$

where A is not the real contact area, but rather an area in full contact, increased with respect to the nominal one A_0 , because of an effect of roughness-induced increase of contact area, $\frac{A}{A_0} > 1$. Also, U_{el} is the elastic strain energy stored in the halfspace having roughness with isotropic power spectrum $C(q)$ when this is squeezed flat¹

$$\frac{U_{el}(\zeta)}{A_0} = \frac{\pi E^*}{2} \int_{q_0}^{q_1} q^2 C(q) dq = E^* l(\zeta) \quad (2)$$

where we have integrated over wavevectors in the range q_0, q_1 , and $E^* = E/(1 - \nu^2)$ is the plane strain elastic modulus, where ν is Poisson's ratio. We have introduced in (2) a length scale $l(\zeta)$ where $\zeta = q_1/q_0$ is the so called "magnification". The elastic energy $U_{el}(\zeta)$ is unbounded for surfaces with fractal dimension $D \geq 2.5$, in the fractal limit $\zeta \rightarrow \infty$ (see Ciavarella *et al.*, 2018) so this theory would predict that such surfaces could never adhere, even for arbitrarily small rms height h_{rms} . This result may be in contrast with the theory by Joe *et al.* (2017, 2018), and should be further investigated. By contrast, for $D < 2.5$ the energy converges in the fractal limit $\zeta \rightarrow \infty$ and hence full contact is expected to be possible regardless of h_{rms} . Also, the simple theory has been shown to be a reasonable approximation experimentally by Dalvi *et al.* (2019). We shall return on the Persson-Tosatti's idea to derive a stickiness criterion later.

1.2. BAM theory

The BAM model (Ciavarella, 2017) takes its inspiration from the DMT solution for a single sphere (Derjaguin *et al.*, 1975), completely different

¹Notice we use the original Persson's convention and notation for $C(q)$ and not Dalvi *et al.* (2019) which is $C^{iso}(q) = 4\pi^2 C(q)$.

from the JKR energy approach, but makes a geometric interpretation of it. Hence, it doesn't follow any of the classical DMT calculations (neither the thermodynamic method, nor the sum of adhesive forces in separation regions of the adhesionless solution as done by Persson and Scaraggi, 2014). BAM assumes the simplified Maugis-Dugdale force-separation law with a given interface energy $\Delta\gamma$, for which the tensile stress is defined as a function of gap u as

$$\begin{aligned}\sigma_{ad}(u) &= \sigma_0, & u \leq \epsilon \\ \sigma_{ad}(u) &= 0, & u > \epsilon\end{aligned}\quad (3)$$

where $\sigma_0 = \frac{\sigma_{th}}{16/(9\sqrt{3})} \simeq \sigma_{th}$ (the theoretical strength of the material, for a crystalline solid, and anyway the peak of tensile stress in a true Lennard-Jones potential), ϵ is the range of attraction, and $\Delta\gamma = \sigma_0\epsilon$. BAM makes an independent estimate for the repulsive and adhesive components of the load. It has the big advantage to be very simple to implement, particularly for rough surfaces, as it results in closed form equations. The attractive area A_{ad} is defined as

$$A_{att}(\Delta) \approx B(\Delta + \epsilon) - B(\Delta), \quad (4)$$

where $B(\Delta)$ is the classical *bearing area*, namely the area over which the bodies taken as rigid, would interpenetrate each other when moved together through a distance Δ . For a Gaussian nominally flat surface, this results in

$$\frac{A_{ad}}{A_0} = \frac{1}{2} \left[\text{Erfc} \left(\frac{\bar{u} - \epsilon}{\sqrt{2}h_{rms}} \right) - \text{Erfc} \left(\frac{\bar{u}}{\sqrt{2}h_{rms}} \right) \right] \quad (5)$$

where \bar{u} is the mean separation of the surfaces, h_{rms} is rms amplitude of roughness. The total force is obtained by superposition of the repulsive pressure at indentation Δ which is easily obtained with Persson's theory (Persson, 2007) which, for the simplest power law PSD, and $D \simeq 2.2$ gives

$$\frac{p_{rep}(\bar{u})}{E^*} \simeq q_0 h_{rms} \exp \left(\frac{-\bar{u}}{\gamma h_{rms}} \right) \quad (6)$$

where $\gamma \simeq 0.5$ is a corrective factor. Therefore, summing up repulsive (6) and attractive ($\sigma_0 A_{ad}(\bar{u})$) contributions, BAM gives

$$\frac{\sigma(\bar{u})}{\sigma_0} \simeq q_0 h_{rms} \frac{E^*}{\sigma_0} \exp \left(\frac{-\bar{u}}{\gamma h_{rms}} \right) - \frac{1}{2} \times \left[\text{Erfc} \left(\frac{\bar{u} - \epsilon}{\sqrt{2}h_{rms}} \right) - \text{Erfc} \left(\frac{\bar{u}}{\sqrt{2}h_{rms}} \right) \right] \quad (7)$$

which obviously results in a pull off finding the minimum as a function of \bar{u} . Notice that $\frac{E^*}{\sigma_0} \simeq \frac{E^*}{\Delta\gamma/\epsilon} = \frac{\epsilon}{l_a}$ where $l_a = \Delta\gamma/E^*$ defines a characteristic adhesion length which for the typical Lennard Jones description of an interface between crystals of the same material is $l_a \simeq 0.05\epsilon$. The theoretical strength in this case, $\sigma_0 = \frac{\Delta\gamma}{\epsilon} = \frac{l_a E^*}{\epsilon} = 0.05E^*$. However, when considering contamination, one can estimate that l_a is reduced by orders of magnitude. The results show that the pull-off traction is principally determined by h_{rms}, q_0 and upon increasing the "magnification" of the surface, $\zeta = q_1/q_0$, converges rapidly, as in the adhesionless load-separation relation (6). We shall return later on BAM to derive a stickiness criterion also from it.

1.3. *Violano et al. criterion*

Inspired by some concepts originally introduced by Pastewka and Robbins (2014), namely about the presence of a boundary layer near the edge of contact where gaps could be described by universal asymptotic expressions, Violano *et al.* (2018) obtained the probability density function of gaps with Persson and Scaraggi's DMT theory (Persson and Scaraggi, 2014, see also Afferrante *et al.*(2018)) and found that it converges with increasing magnification ζ , thus, in the fractal limit, any DMT theory should not depend on the PSD wavenumber cutoff q_1 — thereby showing a different extrapolation to broad band roughness than Pastewka and Robbins (2014) who had numerically explored only up to $\zeta \simeq 10^3$. Violano and co-authors showed that the area-load slope, at the origin (which becomes vertical when we move from sticky to unsticky), depends in a pure power law PSD only on well-defined macroscopic quantities, such as h_{rms} and the lowest wavenumber q_0 , and in particular that for low fractal dimension ($D \simeq 2.2$) rough surfaces stick for

$$\frac{h_{rms}}{\epsilon} < \left(\frac{9 \sigma_0 / E^*}{4 \epsilon q_0} \right)^{3/5} \quad (8)$$

which we are going to use for comparative purposes.

2. New stickiness criteria

2.1. *A new Persson-Tosatti stickiness criterion*

Let us start from obtaining a stickiness criterion from the Persson-Tosatti's idea of the effective surface energy (1). If we take a typical power law PSD $C(q) = Zq^{-2(1+H)}$ for $q > q_0 = \frac{2\pi}{\lambda_L}$, where H is the Hurst exponent (equal to

3 - D where D is the fractal dimension of the surface), the integral of the full contact energy (2) depends on whether $H > 0.5$ or not. Specifically, as $Z = \frac{H}{2\pi} \left(\frac{h_0}{q_0}\right)^2 \left(\frac{1}{q_0}\right)^{-2(H+1)}$ where $h_0^2 = 2h_{rms}^2$ (see again Persson, 2002), for $H \neq 0.5$

$$l(\zeta) = \frac{\pi}{2} \int_{q_0}^{q_1} q^2 C(q) dq = \frac{\pi Z}{2} \int_{q_0}^{q_1} q^{-2H} dq = \pi \frac{h_{rms}^2}{\lambda_L} H \frac{\zeta^{-2H+1} - 1}{-2H + 1} \quad (9)$$

For the usual case of $H > 0.5$ (low D) (see Persson 2014) the integral converges quickly, is relatively insensitive to high wavevector truncation and indeed for practical purposes we can use the limit value

$$l(\infty)_{lowD} = \pi \frac{h_{rms}^2}{\lambda_L} \frac{H}{2H - 1} \quad (10)$$

which shows the energy is mainly stored in the long wavelength components. Summarizing, and neglecting the effect of the term A/A_0 , we can simplify the effective surface energy (1) as

$$\Delta\gamma_{eff} = \Delta\gamma - E^* \pi \frac{h_{rms}^2}{\lambda_L} \frac{H}{2H - 1} \quad (11)$$

We can then obtain a new "Persson-Tosatti" stickiness criterion, by imposing $\Delta\gamma_{eff} = 0$ in (11) obtaining in terms of roughness amplitude, the condition

$$h_{rms} < \sqrt{\frac{\Delta\gamma}{E^*} \lambda_L \frac{2H - 1}{\pi H}} \quad (12)$$

which we shall compare with other criteria.

2.2. A new BAM stickiness criterion

We have not obtained in the original BAM paper (Ciavarella, 2018), a true criterion for stickiness, and this does not seem to be obtained in closed form. One can obtain from eqt (7) the decay of the pull-off tension (fig.1) as a function of rms roughness amplitude. Given the abrupt decay in pull-off values, stickiness is defined (for example) when $-\sigma_{min}/\sigma_0 = 10^{-8}$ finding this by numerical routines. Moreover, defining the threshold from the exact minimum of the tension-mean gap curve (solid lines in Fig.1), or defining it from the curves obtained at $\bar{u}/\epsilon = 2$ (dashed lines) is the same as clearly demonstrated

by the Fig.1, so that one can find the threshold for stickiness also directly from solving the following equation $f(\lambda_L/\epsilon, (h_{rms}/\epsilon)_{thresh}, l_a/\epsilon) = 10^{-8}$ where

$$f(\lambda_L/\epsilon, (h_{rms}/\epsilon)_{thresh}, l_a/\epsilon) = \frac{2\pi}{l_a/\epsilon} \frac{(h_{rms}/\epsilon)_{thresh}}{\lambda_L/\epsilon} \exp\left(\frac{-2}{\gamma (h_{rms}/\epsilon)_{thresh}}\right) - \frac{1}{2} \times \left[\text{Erfc}\left(\frac{1}{\sqrt{2} (h_{rms}/\epsilon)_{thresh}}\right) - \text{Erfc}\left(\frac{2}{\sqrt{2} (h_{rms}/\epsilon)_{thresh}}\right) \right] \quad (13)$$

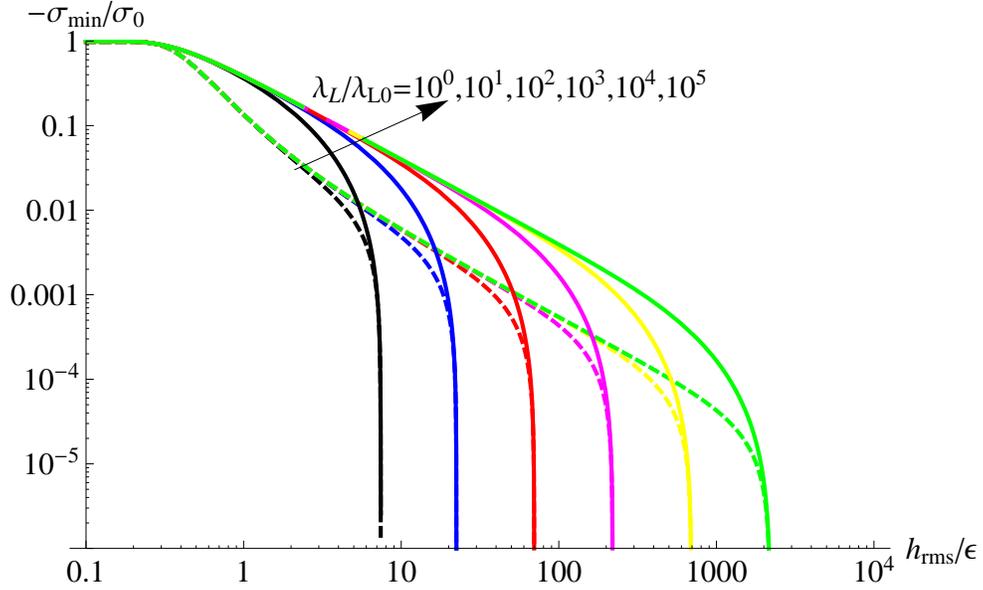


Fig.1- Curves of decay of pull-off normalized pressure $-\sigma_{min}/\sigma_0$ as a function of normalized rms roughness amplitude h_{rms}/ϵ (solid lines represent the true pull-off point, while dashed lines represent an approximation computed at $\bar{u}/\epsilon = 2$). Here, the reference long wavelength cutoff $\lambda_{L0} = \frac{q_0}{2\pi} = 2048\epsilon$ and the curves shift to the right with increasing $\lambda_L/\lambda_{L0} = 10^0, 10^1 \dots 10^5$.

Hence, we can explore the threshold $(h_{rms}/\epsilon)_{thresh}$ so obtained in Fig.2, where solid lines represent the actual solutions to eqt. (13), and dashed lines represent power law approximations of the type

$$(h_{rms}/\epsilon)_{thresh} = \alpha \left(\frac{l_a}{\epsilon}\right) (\lambda_L/\lambda_{L0})^{1/2} \quad (14)$$

which provides a very reasonable fit over various orders of magnitude of (λ_L/λ_{L0}) , taking as reference $\lambda_{L0}/\epsilon = 2048$. Supposing ϵ in the Angstrom range, $10^{-10}m$, $\lambda_L/\lambda_{L0} = 10^7$ means we are effectively plotting up to mm range, similarly to the broadness of roughness measured in Davli *et al.* (2019).

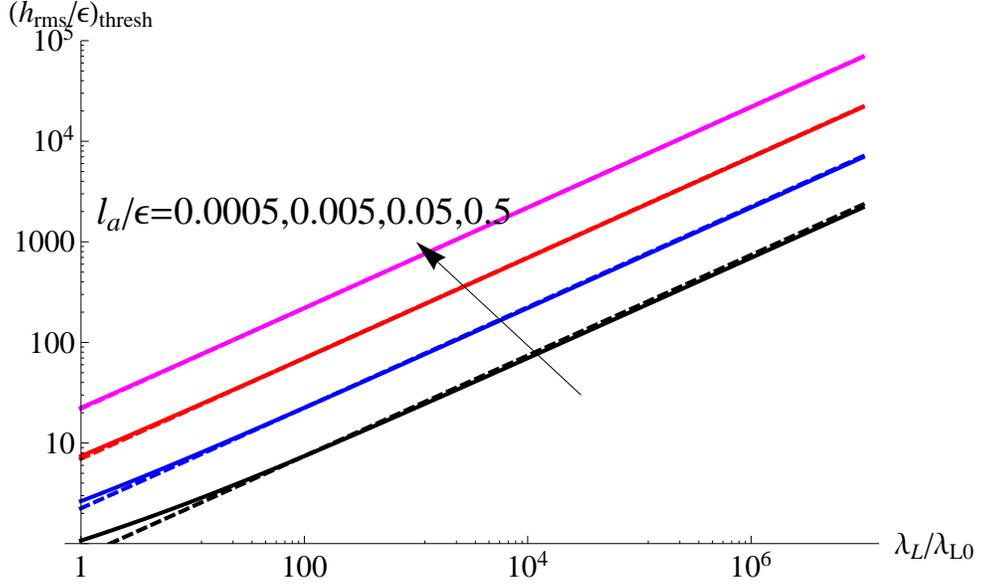


Fig.2- The threshold for stickiness $(h_{rms}/\epsilon)_{thresh}$ as a function of λ_L/λ_{L0} across 7 orders of magnitude where $\lambda_{L0} = \frac{g_0}{2\pi} = 2048\epsilon$ shows extremely good power law behaviour for all values of $(\frac{l_a}{\epsilon})$. Solid lines represent the actual solutions to eqt. (13), and dashed lines represent power law approximations (14)

The constant $\alpha(\frac{l_a}{\epsilon})$ is further studied in Fig.3, showing that even this quantity has a very good power law fit across many orders of magnitude of $\frac{l_a}{\epsilon}$, namely we can write (shown as dashed line)

$$\alpha\left(\frac{l_a}{\epsilon}\right) \simeq 35\left(\frac{l_a}{\epsilon}\right)^{0.5} \quad (15)$$

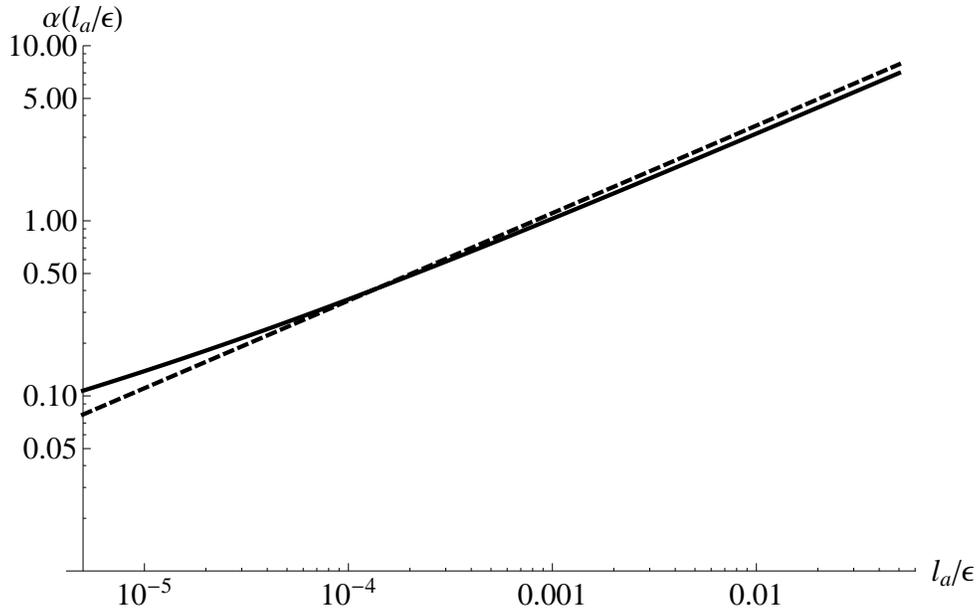


Fig.3- The constant of proportionality $\alpha\left(\frac{l_a}{\epsilon}\right)$ as a function of $\frac{l_a}{\epsilon}$ across 4 orders of magnitude shows also extremely good power law behaviour (15).

Summarizing, using (14, 15), the dependence on ϵ disappears and we have obtained for stickiness

$$h_{rms} < (0.6l_a\lambda_L)^{0.5} \quad (16)$$

which we shall compare with the other criteria.

3. Comparison between the three stickiness criteria

We have obtained, for the example case of a pure power law PSD roughness (for the typical case of $H \simeq 0.8$) that Persson-Tosatti and BAM predict stickiness (12) (16) with exactly the same qualitative form

$$h_{rms} < \sqrt{\beta l_a \lambda_L} \quad (17)$$

where $\beta_{PT} = 0.24$ and $\beta_{BAM} = 0.6$ which are even *quantitatively* close — even closer they will appear considering the factor A/A_0 as we demonstrate in the discussion. The result is really unexpected, given the two simple criteria are obtained with completely different routes, one being a simple energy balance

without considering details of the contact mechanics, and the other a mix of Persson's solution for repulsive pressure, and a geometrical estimate for the adhesive pressure.

Violano's DMT criterion instead (8) contains a slightly different qualitative dependence on material properties, since we can write it in the form

$$h_{rms} < \epsilon^{-1/5} (0.36l_a\lambda_L)^{3/5} \quad (18)$$

which shows the product $l_a\lambda_L$, instead of the power 1/2, is raised to the power 3/5, and this is due to a weak apparent dependence on the range of attractive forces ϵ .

We can rewrite all three criteria by introducing the parameter ϵ (which is purely a normalization factor for Persson-Tosatti and BAM, whereas it is a true dependent parameter for Violano's), in the form

$$\frac{h_{rms}}{\epsilon} < \sqrt{0.24 \frac{l_a \lambda_L}{\epsilon \epsilon}} \quad ; \quad \text{Persson-Tosatti} \quad (19)$$

$$\frac{h_{rms}}{\epsilon} < \sqrt{0.6 \frac{l_a \lambda_L}{\epsilon \epsilon}} \quad ; \quad \text{BAM} \quad (20)$$

$$\frac{h_{rms}}{\epsilon} < \left(0.358 \frac{l_a \lambda_L}{\epsilon \epsilon}\right)^{3/5} \quad ; \quad \text{Violano} \quad (21)$$

and a comparison is shown in Fig.4, where Persson-Tosatti is reported in black solid line, BAM as blue solid line, and Violano as red solid line. Clearly, considering the three criteria have so different origin, it is remarkable that they give so close results.

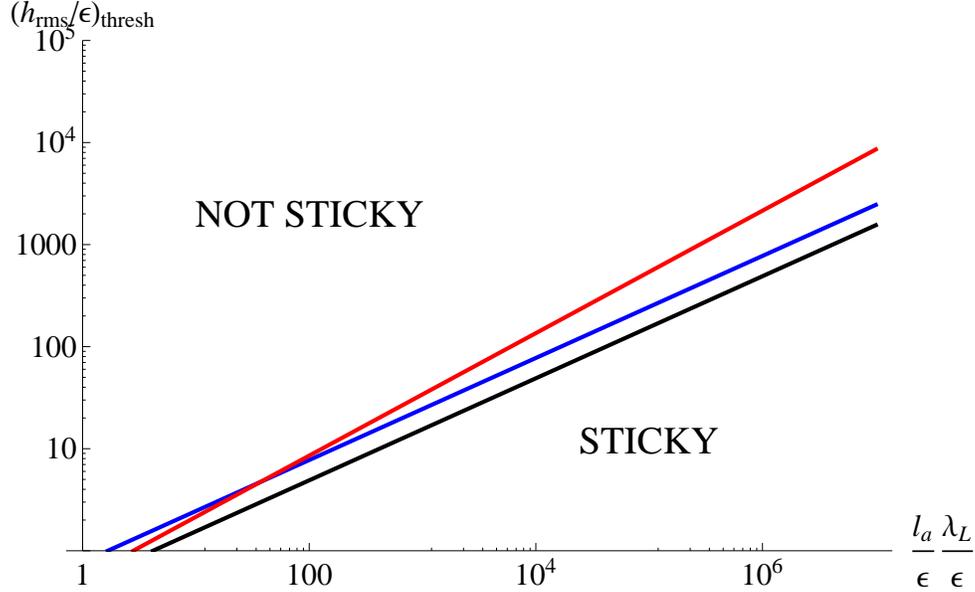


Fig.4. A comparison of the three derived stickiness criteria: Persson-Tosatti (black line) (19), BAM (blue solid line) (20) and Violano (red line) (21), in terms of the rms amplitude of roughness. All results are for $H = 0.8$ ($D = 2.2$)

Further comparisons between the different criteria would require very sophisticated experiments, and those of Davli *et al.* (2019) are relevant to a spherical geometry, whereas all three criteria are in principle obtained from theories on contact about nominally flat surfaces. Writing $h_{rms} \simeq \sqrt{\pi Z/H} q_0^{-H}$, taking the new Persson-Tosatti criterion in the reelaborated following form

$$Z < 1.16 \frac{\Delta\gamma}{E^*} \left(\frac{1}{\lambda_L} \right)^{0.6} \quad (22)$$

we find for $\Delta\gamma = 37 \text{ mJ/m}^2$ and $E^* = 0.7 - 10 \text{ MPa}$, with $\lambda_L \simeq 10 \text{ mm}$, while from the plots in Davli *et al.* (2019), we can estimate an approximate power law with $Z \simeq C(q) q^{3.6} \simeq 2.5 \times 10^{-10} m^{0.4} < 1.16 \frac{\Delta\gamma}{E^*} \left(\frac{1}{\lambda_L} \right)^{0.6} = 6.8 \times 10^{-8} m^{0.4}$, all data should be sticky, as it appears the case.

4. Discussion

4.1. Persson-Tosatti and BAM even closer when considering surface area increase

Dalvi *et al.* (2019) make a discussion about the term true surface area, which they estimate as

$$\frac{A(\zeta)}{A_0} = 1 + \frac{\sqrt{\pi}}{2} h'_{rms} \exp\left(\frac{1}{h'^2_{rms}}\right) \text{Erfc}\left(\frac{1}{h'_{rms}}\right) / h'_{rms}$$

which complicates the model slightly. This terms will modify the Persson-Tosatti criterion introducing a magnification-dependence

$$h_{rms} < \sqrt{\frac{A(\zeta)}{A_0} \frac{\Delta\gamma}{E^*} \lambda_L \frac{2H-1}{\pi H}} \quad (23)$$

For example, let us consider the usual case $H = 0.8$, this will make the stickiness criterion look like

$$h_{rms} < \sqrt{\beta_{PT}(\zeta) l_a \lambda_L} \quad (24)$$

where the $\beta_{PT} = 0.24 \frac{A(\zeta)}{A_0}$ term increase only with a quite large h'_{rms} as plotted in Fig.5. Incidentally this will get even closer to the $\beta_{BAM} = 0.6$ despite for a different reason. An almost perfect coincidence between Persson-Tosatti criterion and BAM will occur at some $h'_{rms} \simeq 2.51$ which is a realistic value for a limit true slope — above which many other assumptions of linear elasticity, of geometrical description of atomic structures etc. would be violated.

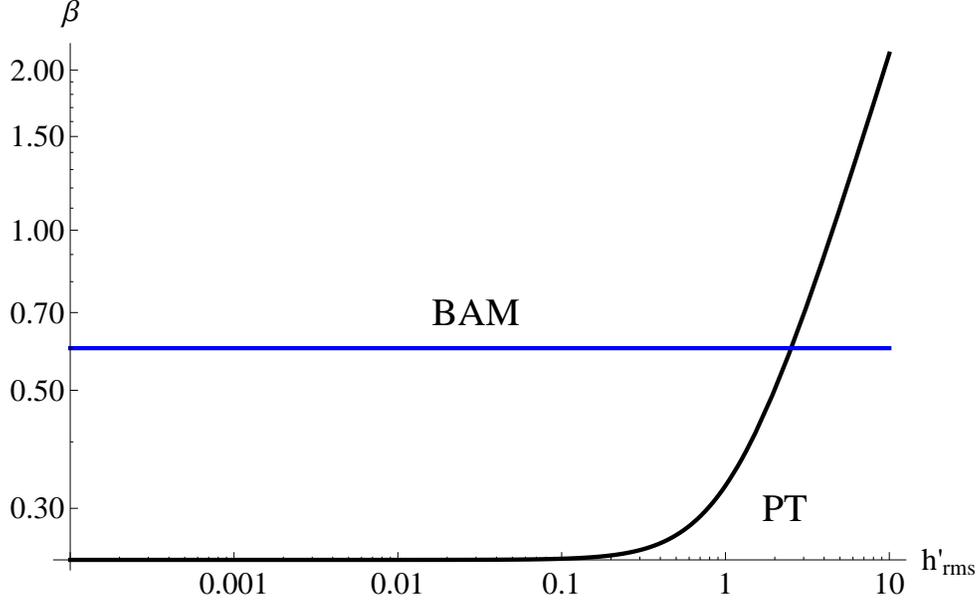


Fig.5. The coefficient β in the Persson-Tosatti and BAM stickiness criteria $h_{rms} < \sqrt{\beta(\zeta) l_a \lambda_L}$ as a function of the rms slope

The correction $\frac{A(\zeta)}{A_0}$ was discussed also in Persson's own later experiments (Peressadko *et al.*, 2005).

4.2. Other criteria

We haven't so far commented on the Pastewka and Robbins (2014) stickiness criterion, which we can rewrite it in the form (for power law tail of the PSD)

$$\frac{\epsilon}{l_a} \left(\frac{h_{rms}}{\epsilon} \right)^{2/3} < \frac{3 a_V(\zeta)}{2 q_0 h_{rms}} \quad (25)$$

where

$$[a_V(\zeta)]_{PR} = 1.4622 q_0 h_{rms} \left(\frac{h_{rms}^2}{h_{rms}^7} \right)^{1/3} h_{rms}^{2/3} \quad (26)$$

Now for power law PSD, estimating $h_{rms} = \sqrt{m_0} = \sqrt{2\pi Z q_0^{-2H} \left(\frac{\zeta^{-2H-1}}{-2H} \right)} \simeq \sqrt{\frac{\pi Z}{H}} q_0^{-H}$ and $h'_{rms} = \sqrt{2m_2} \simeq \sqrt{\frac{\pi Z}{1-H}} q_1^{1-H}$, $h''_{rms} = \sqrt{8m_4/3} \simeq \sqrt{2\frac{\pi Z}{4-2H}} q_1^{2-H}$, where m_0, m_2, m_4 are spectral moments of the PSD, we obtain

$$[a_V(\zeta)]_{PR} = 1.4622 \frac{(1-H)^{7/6}}{(2-H)^{1/3} H^{5/6}} \zeta^{\frac{5}{3}H-1} \quad (27)$$

which for $H = 0.8$ leads to $[a_V(\zeta)]_{PR} = 0.253$ and therefore using again (25)

$$\frac{h_{rms}}{\epsilon} < \left(0.06 \frac{l_a \lambda_L}{\epsilon \epsilon} \right)^{3/5} \zeta^{1/5} \quad (28)$$

Now, this shows a (weak) dependence on magnification, which remains even for high ζ , unlike the other criteria. If we plot the Pastewka and Robbins criterion as in Fig.4, we obtain fig.6 for $\zeta = 10^3, 10^4, \dots, 10^7$ (where we add to the three previous lines dashed lines corresponding to Pastewka and Robbins criterion for increasing ζ , increasing as indicated by arrow). It is evident that the PR criterion corresponds very closely to the Violano criterion for low $\zeta < 1000$ (which is where it was obtained), but departs for higher ζ . Hence, in practical cases shown by Davli *et al.* (2019) who have $\zeta \simeq 10^7$, it is safer to use the other three criteria which all do not show this dependence, probably found spuriously from the limited numerical experiments. More specifically, we don't really need to measure surface roughness down to atomic scale, since the three criteria (Persson-Tosatti, BAM and Violano), all do not require very precise informations about small scale details to be defined.

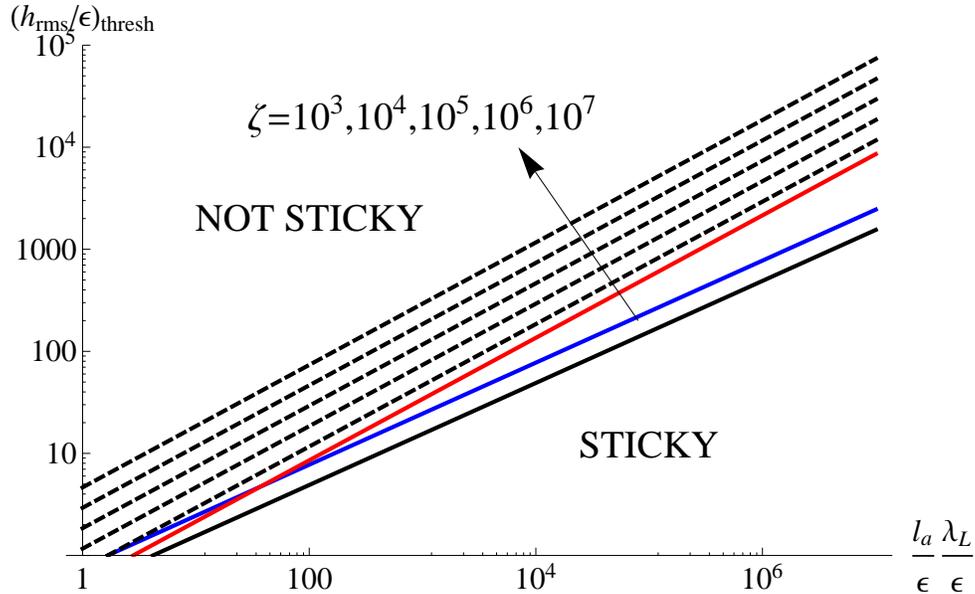


Fig.6. A further comparison of the three derived stickiness criteria, Persson-Tosatti (black line), BAM (blue solid line) and Violano (red line), with a fourth criterion, that of Pastewka and Robbins (2014) which was obtained only by interpolation of numerical results for $\zeta < 1000$ — this

shows that there is good agreement even with the PR criterion but only for low magnifications, and beyond this (as in practical cases shown by Davli *et al.* who have $\zeta \simeq 10^7$), it is safer to use the other three criteria.

Similarly to Pastewka and Robbins (2014), Müser (2016) also defines a stickiness criterion interpolating numerical results, defines a "dimensionless surface energy",

$$\Delta\gamma_{rss} = \frac{\Delta\gamma \tanh(\mu_T)}{E^* (h'_{rms})^3} \quad (29)$$

where h'_{rms} is the root mean-square gradient of the surface, \tanh is introduced as an empirical fitting between the "correct" asymptotics in the two limits of small and large Tabor generalized coefficients μ_T (see Müser (2016) for details). For the power law PSD spectrum $h'_{rms} \sim \zeta^{1-H}$, $h''_{rms} \sim \zeta^{2-H}$, and hence $\tanh(\mu_T) \sim (h''_{rms})^{-1/3}$ so that

$$\Delta\gamma_{rrs} \sim \zeta^{(2-H)\frac{2}{3}-3+3H} \sim \zeta^{(7H-5)/3}, \quad \zeta \rightarrow \infty \quad (30)$$

which means that for $H = 0.8$ that $\Delta\gamma_{rrs} \rightarrow \zeta^{0.2}$, and again this shows a magnification dependence for all ζ similarly to Pastewka and Robbins (2014), but in contrast with the Persson-Tosatti, BAM, and Violano criteria.

5. Conclusions

We have obtained two new stickiness criteria, originated from the theories of Persson-Tosatti, and from BAM. These two, which have completely different origin (one being a simple energy balance concept, and the other a mix of Persson's adhesiveless solution with a geometric estimate of adhesive forces), together with the DMT criterion of Violano *et al.* which in turn is based on the elaborated DMT theory of Persson and Scaraggi, seem to differ only by prefactors (Persson-Tosatti vs BAM), or by a small difference in the power laws exponent, due to a weak apparent dependence on the range of attractive forces, emerging in the Violano's criterion. However, all three criteria show the main factors for stickiness are the low wavevector cutoff of roughness, $q_0 = \frac{2\pi}{\lambda_L}$, the rms amplitude of roughness h_{rms} and the ratio between the work of adhesion and the plane strain Young modulus. We find this result rather surprising and hence a robust indication now that small scale features (such as local slopes or curvature, which are hard to define down to perhaps

atomic scale) do not affect stickiness. For adhesion to various levels of macroscopic roughness, the only characteristic which can be changed easily is the elastic modulus, in qualitative and quantitative agreement with Dahlquist criterion.

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